International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 9 Issue 7 ǁ 2021 ǁ PP. 49-56

Evolutionary computing technique for multi-level inventory Lot Sizing problem

V.V.D. Sahithi[1], M. Srinivasa Rao[2], C.S.P. Rao[3]
[1]*Research Scholar, JNTU Hyderabad, India*
[2] *Professor, JNTU Hyderabad, India*
[3] *Director, NIT, Warangal, India*

Abstract
In this research paper, we present a binary invasive weed optimization algorithm (BIWOA) which is inspired from colonizing weeds, to solve the multi-level lot-sizing optimization problem (MLLSOP). The MLLSOP deals with material requirement planning (MRP) in the Production systems with a Finite Planning horizon, to optimize the production cost. The MLLSOP aims to find out the optimum production plan which takes the minimization of total setup cost (SC) and inventory holding costs (HC). Here in our work,we developed BIWOA programming techniques to optimize the cost of Multi-level (ML) assembly structures with reasonable CPU time. The effectiveness of the Algorithms was investigated by comparing these algorithms simulation results with a genetic algorithm (GA), Wagner-Whitin (WW), and other evolutionary algorithms experimental results. In this comparison both BIWOA was proven to reach the optimum solution for ML problem with very less computational time.
Keywords*: Multi-level Lot sizing, Material Requirement Planning, Evolutionary algorithms*

Date of Submission: 10-07-2021 Date of acceptance: 26-07-2021

I. INTRODUCTION

MRPplays a very crucial role in Coordinating replenishment decisions for finished goods. The MLLSOP in MRP belongs to those problem that industry manufacturers face daily in organizing their overall production plans [1]. The main aim of Lot sizing (LS) is to find out the optimum order quantities to satisfy the master production schedule (MPS). Depending on the context of the problem, orderquantity means the amount of product to buy or produce at the starting of each time bucket in the given planning horizon [2]. These MLLSOP are the NP hard problems which cannot be solved using with simple polynomial equations [3].

Lot of researchers concentrated on these optimization techniques to solve these lot sizing (LS) problems of single level(SL) and multi-level(ML), because of their quick and efficient methodology to solve NP hard problem. The initial days of research to solve MRP challenges, economic order quantity (EOQ) technique was used. But it is applicable only when the orders are in steady state. But when the orders are not same in each period then EOQ assumptions are not applicable in that case. So many researchers worked on these dynamic demand scenarios. In 1958 Wagner&Whitin proposed an algorithm to solve the dynamic version of lot sizing problem and successful in reaching the optimum solution for single level problem [4], and later on so many researchers tried to find out a optimization technique for solving MLproblems. In1973 Silver and Meal proposed a heuristic technique to solve these multi stage problems by minimizing the sum of set up and inventory holding costs periodically[5].In 1990 Kuik & Salomon proposed a Simulated annealing(SA) evolutionary heuristic technique for solving SL Problems with independent demand items as well as ML systems with independent & dependent demand items[6].Here in ML systems Independent demand for final item comes from MPS and dependent demand for constituent items were generated from the BOM structure .In the year 1999 Hernandez & Suer Proposed an evolutionary genetic Algorithm to solve single level problem[7]. In 2000 Dellaert, N. and Jaunet, successfully applied genetic algorithm for solving multi-level inventory problem without any capacity restrictions and assuming that the costs are not changing with respect to time [8].in 2011, Klorklear Wajanawichakon and Rapeepan Pitakaso applied binary version of Particle swarm optimization (PSO) algorithm for solving unconstrained ML inventory problem with hybrid selection mechanism [9]. Alfares et. all. proposed an algorithm in 2016 proposed a general model to solve SL problem with quantity discounts, backorder ordering in, multiple supplier environment [10]. M. Hrouga proposed a heuristic algorithm to solve the capacitated problem with lost sales and back ordering environment. It is a combination of GA and fix and optimise heuristic [11]. In 2019 Meng You et. all proposed a heuristic approach called fix and optimize to solve the problem with rapidly changing cost data with capacity limitations [12]. Seyed Ashkan Hoseini Shekarabiin

2019 solved a lot sizing issue in supply chains to optimize the inventory cost using outer approximation technique.[13]. In 2020 Sahithi V.V.D.and Rao C.S.P. successfully applied harmonic search algorithm for multi-level lot sizing problem with and without capacity restrictions [14].

In this paper authors considered the complex MLassembly structure from the literature and solved the same using BIWOA,and then to know the effectiveness of the algorithm, results were compared withGA, WW Algorithm& other evolutionary algorithms. Results achieved were very much promising in case of solution value as well as computational time.

The next sections of the paper are as follows: section (2) represents the mathematical modelling required to solve the MLLSOP and different parameters included in the problem were defined. Section (3) explains the procedure to create the initial basic feasible solution for ML inventory problem and the procedure to calculate the total variable cost before applying the actual algorithm. Section (4) Explains about the pseudo code & procedure to optimize the ML problem using BIWOA after creating the initial solution. Section (5) experimental framework and results were presented. Finally, the section (6) was dedicated to conclusions.

II. Mathematical Formulation:

In MLLSOP, there arethree major types of Product structures. They are assembly,absorbent,and general structures[15-19]. In this problem bill of materials structure is represented by an acyclic graph.Here in that type of graphs each item is represented by index k,each edge between node k & m indicates that item k is directly required to assemble item m.Γ^{-1} (k), Γ(k)are used to present immediate predecessor, immediate successor of node k [20].

K item index

$C_{k,m}$ Number of k items required to produce item m

H_k Inventory holding cost to produce 1 unit of item m

K_k Setup cost for 1 unit of itemk

l_k Lead time

$l_{k,t}$item k inventory level at the end of period t

$a_{k,t}$ binary values(0 or 1)

$D_{k,t}$ item k requirement in period t

$P_{k,t}$production of item I in t period

M a big integer value

N1 Total items

T length of the planning horizon

Binary value $a_{k,t}$ ϵ(0,1) represents weather the item k is produced in period 't' or not. If '0' is assigned to $a_{k,t}$ that means the item was not ordered (or produced) in that period t. If $a_{k,t}$ =1 that means the order is placed in that period t for item k.

$$A = \left[\begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,T} \\ a_{21} & a_{2,2} & \cdots & a_{2,T} \\ \vdots & \vdots & \vdots & \vdots \\ a_{N,1} & a_{N,2} & \cdots & a_{N,T} \end{pmatrix}\right] \quad (2.1)$$

Mathematical model forML lot sizingproblem with assembly type of productstructure was presented here[

$$Minimize\ \sum_{k=1}^{N1}\sum_{t=1}^{T} H_k \times l_{k,t} + K_k \times a_{k,t} \quad (2.2)$$

Subjected to following constraints

$$l_{k,t=}l_{k,t} + P_{k,t} - D_{k,t} \quad (2.3)$$

$$\sum_{k=1}^{N1} C_{k,m} \times P_{m,t+l_m} \quad (2.4)$$

$$P_{k,t=}a_{k,t} \times D_{k,t} + \sum_{s=t+1}^{T}\left(a_{k,s+1} D_{k,s} \prod_{u=k+1}^{s}(1-a_{k,u})\right) \quad (2.5)$$

$$P_{k,t} - Ma_{k,t} \le 0 \quad a_{k,t} \epsilon \{0,1\} \quad (2.6)$$

$$P_{k,t} \ge 0 \quad l_{k,t} \ge 0 \quad (2.7)$$

To solve MLLSOP problem using any optimization algorithm we have to first initialize the problem by creating random basic feasible solution matrix (2.1). The Fitness function (2.2) is the sum of inventory HC and SC. The remaining Equations represents the constraints for this problem.Equation (2.3) represents the balanced equation of demand and replenishment inventory.Equation (2.4) is used to calculate the internal demand of sub items, Equation (2.5) guarantee the replenishment quantity of a particular period depend on the setup decisions. The equation 2.6 represents that no backlogging is allowed and 2.7 is non negativity restriction which allows the production to be only positive or zero.

III. Model Calculations

To Solve MLLSOP with any Metaheuristic optimization Algorithm, we have to Map the problem with the Algorithm Frame work. In this section explanation is Given for the creation of Initial basic feasible solution and the total cost calculation for MLLSOP.

To solve the MLLSOP with evolutionary algorithms, firstwe have tocreatethe initialbasic feasible solution (Abinary matrix). These binary values in the A matrix represents weather the order is placed or notin particular period in the entire planning horizon. If ‘1’ is assigned, then it means that the order is placed in that particular prriod, and ‘0’ is assigned that means order is not placed in that period for that particular item.D is the allocation tablecreated based on A matrix requirements. Costcalculations are made according to Dmatrix and HC, SC.

Here in the following example problem Fig1. represents the model BOM structure with 4 items in which, item 1 is having independent demand which is shown in Table1, the cost information of item 1 is given by table 2. The remaining items demands were dependent on the item1 demand

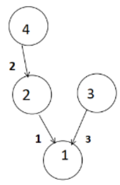


Fig.1: ML Lot-Sizing Problem with 4 items

Table1: Demandrequirements of item1

t	1	2	3	4	5	6
Demand	32	41	148	36	120	28

Table2: HC, SCof BOMitems

Item Number	1	2	3	4
Holding cost	1	1	1	1
Setup Cost	130	120	25	30

Here the following A matrix is generated randomly. Here binary values represent weather the order is placed or not. Each row in matrix is related to each item in BOM structure and the number of columns represents the planning horizon.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.1)$$

By considering item1 demand values given in table1, the dependent items from BOM structures the demands allocation matrix (D) was calculated according to the random solution matrix (3.1)

$$D = \begin{bmatrix} 32 & 41 & 148 & 36 & 120 & 28 \\ 32 & 41 & 304 & 0 & 0 & 28 \\ 96 & 123 & 444 & 108 & 444 & 0 \\ 810 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.2)$$

By considering the demand allocation matrix D (3.2) and the cost values from Table2 Cost Calculation MatrixC (3.3) was generated

$$\text{Cost (C)}=\begin{bmatrix} 780 \\ 1032 \\ 209 \\ 3186 \end{bmatrix} \quad (3.3)$$

By adding all the variable cost values from cost matrix C, the total variable cost value was calculated.
Total cost (TC)= 5207
This calculated TC was a initial solution from one randomly generated solution matrix. In this way a greater number of solutions were randomly generated and the total costs were calculated as the initialization step in further sections. Afterinitialization, the algorithms were applied to calculate the optimum solution.

IV. Binary Invasive weed optimization Algorithm procedure and pseudocode:

BIWOA algorithm is one of the natures inspired population-based evolutionary metaheuristic. This optimization Algorithm is inspired from the robust colonized weeds which are very robust in nature. Because of invasive growth they are threat to the desirable plants in agriculture. Thus, capturing weeds robust and adaptive properties forms a very good optimization algorithm. This BIWOA algorithm finds its application in wide range of scientific research [22].

BIWOA pseudo code

```
%% Step1-Begin
Set basic parameters;
%% Step1-Initialization phase
Generate Initial Population randomly
Calculate cost values from random population
%% Step1-BIWOA Main Loop
for it = 1:P  % here P represents Maximum number of iterations.
sigma = ((P - it)/(P - 1))^Exponent * (σinitial - σ_final) + σ final;
CostValues = [population.Cost]; % to find out the best and worst solution values
BestCost = min(CostValues);
WorstCost = max(CostValues);
%% Initialize Population
%%Evaluate Population
newpopulation = [newpopnewsol];  % create new population
%% Merge both old and new Populations
   pop = [popnewpop];
% Sort the merged Population
   [~, SortOrder]=sort([pop.Cost]);
   pop = pop(SortOrder);
% %delete the unwanted extra population
% % Update-Best Solution
end
```

V. Experimental Framework and computational results of MLLSOP:

In thisexperimental section, we present various experimentsto test our algorithms BIWOA&.These algorithms were coded in MATLAB R2018a, platform used is PC with 4GB RAM & 2.8 GHZ CPU. The maximum iteration rule is adopted as stopping criteria. Each experiment is conducted 100 times for all the different problems.

The product structure of 7×6 shown in Fig.2.is used for simulation of first experiment.Here 7×6 represents the problem with 7 items BOM structure with the planning horizon of 6 periods.Here both BIWOA techniques were applied and results were compared with other evolutionary algorithms like GA,SA,tabu search and Lagrangean relaxation algorithms(LR) [23].Results were presented in Table 5 and Fig.3. is the graphical representation of the praposed algorithm behaviour in terms of solution efficiency as well as the computational effectiveness when compared with other evolutionary algorithms.

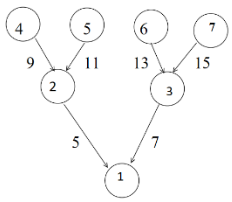


Fig.2. Product structure of 7×6 MLLSOP

Table1: External demands of end items and available capacity

Period	1	2	3	4	5	6
Item 1 demand	40	0	100	0	90	10
Capacity availability	10000	0	5000	5000	1000	1000

Table4: SC and HC of items

Item number	1	2	3	4	5	6	7
SC	400	500	1000	300	200	400	100
HC	12	0.6	1	0.04	0.03	0.04	0.04

Table5: Simulation results of 7×6 problem

Problem	Algorithm	Optimum cost	Time to find out Optimum solution(s)
7×6 problem	BIWOA	8320	<0.2
	GA	9245	10.10
	SA	10740	9.90
	TS	9620	8.80
	LR	9239	34.10

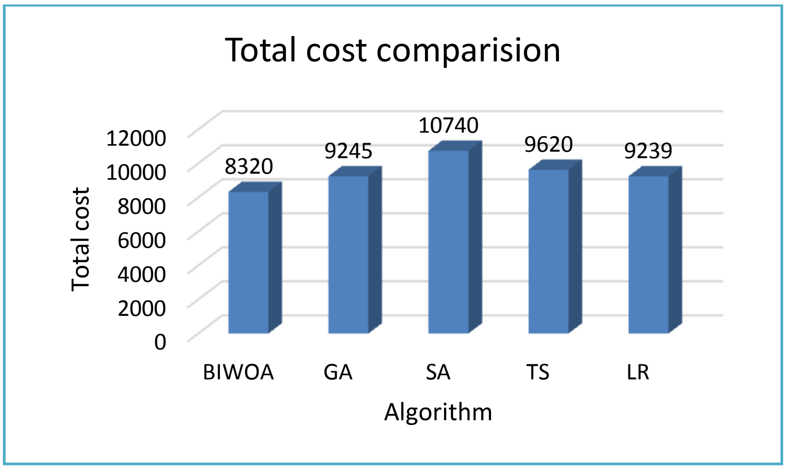


Fig.3. 7×6 problem Simulation results comparison

Fig.2. (A, B) used for remaining Experiments [11]. Table3, Table4 are the input parameters for experiment1 and table5 represents simulation results. Table6,7 are the input parameters for the 6 items and 9 items problem with different planning horizons and the results were listed in table 8.

The Product structures shown in Fig.4.A, B gives the information about BOM structure with 6 items ,9 items. Table 6,7 gives the information about input parameters of the problem. Here the problem is divided into different experiments with different planning horizons. Here 6,9 items problem were divided into 3 experiment each with 10,12,15-timebuckets. Simulation results were shown in Table 8,9. And Fig5,6 illustrate the algorithms behaviour in different scenarios [20].

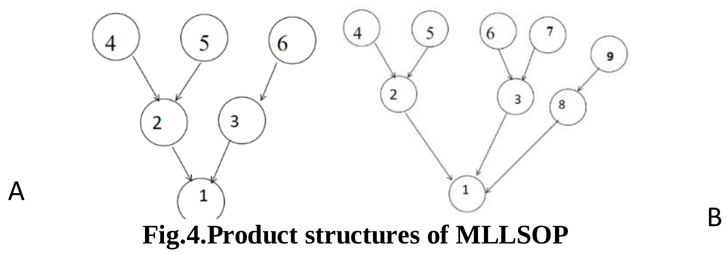


Fig.4.Product structures of MLLSOP

Table6: Parameters used for the given MLLSOP

Item Number	Γ(k)	C (k, Γ(k))	Holding cost of 1 unit	Setup Cost
1	0	0	1	130
2	1	1	2	120
3	1	3	1	25
4	2	2	2	30
5	2	4	3	30

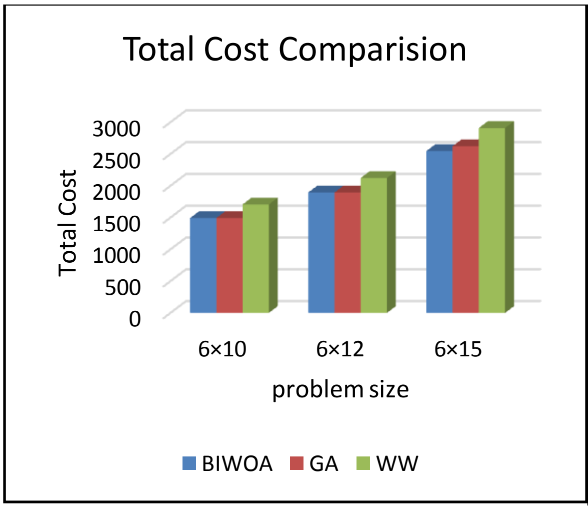
6	3	2	1	40
7	3	1	1	130
8	1	2	2	120
9	8	1	1	25

Table7: External Demand values of MLLSOP

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D1, t	32	41	148	36	120	28	32	12	30	10	32	41	148	36	120

Table 8: Computational Results of MLLSOP with 6 items and different planning horizons

Size of Problem	Algorithm	Optimum cost	Time to find out Optimum solution(s)
6×10	BIWOA	1493	2
	GA	1493	5.6
	WW	1707	<0.1
6×12	BIWOA	1895	5.1
	GA	1895	8.0
	WW	2123	<0.1
6×15	BIWOA	2546	6.5
	GA	2623	10.7
	WW	2909	<0.1



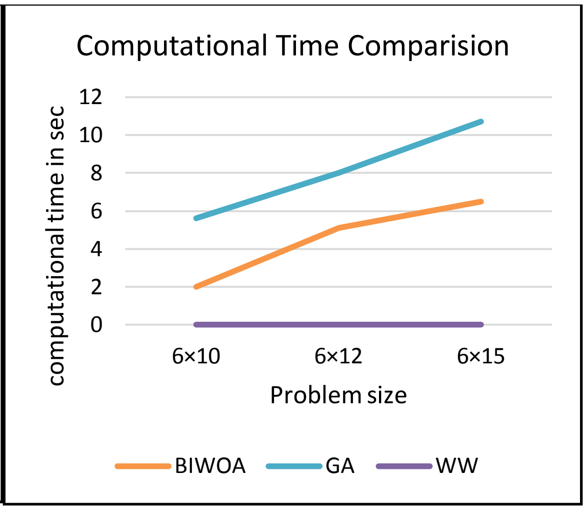
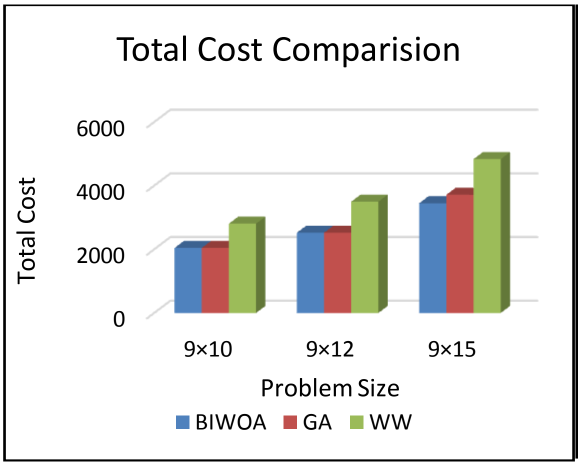


Fig.5. cost and computational performance comparisons for 6 items problem

Table9: simulation Results of MLLSOP with 9 items and different planning horizons

Size of Problem	Algorithm	Optimum cost	Time to find out Optimum solution(s)
9×10	BIWOA	2043	6.5
	GA	2043	10.1
	WW	2807	<0.1
9×12	BIWOA	2522	7.1
	GA	2522	12.9
	WW	3498	<0.1
	BIWOA	3448	12

9×15	GA	3714	16.3
	WW	4834	<0.1



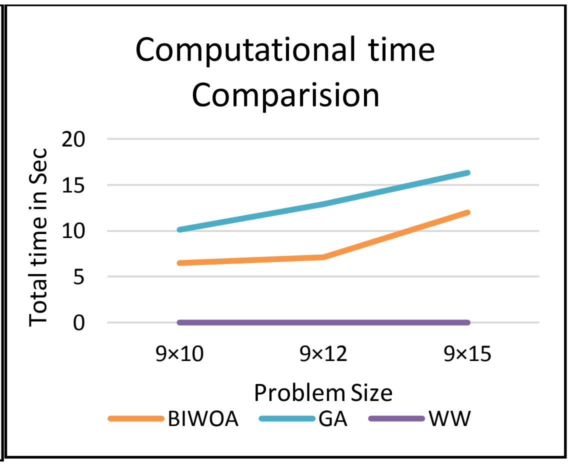


Fig.6. cost and computational performance comparisons for 9 items problem

VI. Conclusions:

In this research paper, BIWOA was used to solve MLLSOP.Both algorithms were compared with different Evolutionary computing heuristics.

- In this simulation results comparison BIWOA is giving better results when compared to other meta heuristic algorithms.
- In case of very small problem like 7×6 problem, which is having only 6 period planning horizon,BIWOA algorithmis reaching optimum with in negligible time.
- In BOM structures with 6 items and 9 items, as the number of periods is increasing in planning horizon the computational time is rapidly increasing.
- In almost all the cases BIWOA is more efficient in terms of computational time.

References:

[1]. Dellaert, N. &Jeunet, J. &Jonard, N., 2000. "**A genetic algorithm to solve the general multi-level lot-sizing problem with time-varying costs**," International Journal of Production Economics, Elsevier, vol. 68(3), pages 241-257, December.

[2]. W. Hernandez and G. A. Suer, "Genetic algorithms in lot sizing decisions," *Proceedings of the 1999 Congress on Evolutionary Computation-CEC99 (Cat. No. 99TH8406)*, Washington, DC, USA, 1999, pp. 2280-2286 Vol. 3, doi: 10.1109/CEC.1999.785558.

[3]. Klorklear Wajanawichakon & Rapeepan Pitakaso (2011) Solving large unconstrainted multi level lot-sizing problem by a binary particle swarm optimization, International Journal of Management Science and Engineering Management, 6:2, 133-140, DOI: 10.1080/17509653.2011.10671156

[4]. Wagner, H. M. and T. M. Whitin, "Dynamic Version of the Economic Lot Size Model," Management Science, Vol. 5, 1958.

[5]. Silver, E. and Meal, H. (1973). A heuristic for selecting lot size requirements for the case of a deterministic time-varying demand rate and discrete opportunities for replenishment. Production and Inventory Management, 14(2):64–74.

[6]. Kuik, R. & Salomon, M., 1990. "Multi-level lot-sizing problem: Evaluation of a simulated-annealing heuristic," European Journal of Operational Research, Elsevier, vol. 45(1), pages 25-37, March.

[7]. W. Hernandez and G. A. Suer, "Genetic algorithms in lot sizing decisions," Proceedings of the 1999 Congress on Evolutionary Computation-CEC99 (Cat. No. 99TH8406), Washington, DC, USA, 1999, pp. 2280-2286 Vol. 3, doi: 10.1109/CEC.1999.785558.

[8]. Dellaert, N. and Jeunet, J. (2000). A genetic algorithm to solve the general multi-level lot sizing problem with timevarying costs. International Journal of Production Economics, 68(3):241–257.

[9]. Klorklear Wajanawichakon & Rapeepan Pitakaso (2011) Solving large unconstrainted multi level lot-sizing problem by a binary particle swarm optimization, International Journal of Management Science and Engineering Management, 6:2, 133-140, DOI: 10.1080/17509653.2011.10671156

[10]. Alfares, Hesham &Turnadi, Rio. (2016). General Model for Single-item Lot-sizing with Multiple Suppliers, Quantity Discounts, and Backordering. Procedia CIRP. 56. 199-202. 10.1016/j.procir.2016.10.054.

[11]. Hrouga, Mustapha &Godichaud, Matthieu & Amodeo, Lionel. (2016). Heuristics for multi-product capacitated disassembly lot sizing with lost sales. IFAC-PapersOnLine. 49. 628-633. 10.1016/j.ifacol.2016.07.749.

[12]. You, Meng & Xiao, Yiyong& Zhang, Siyue& Zhou, Shenghan& Yang, Pei & Pan, Xing. (2019). Modeling the Capacitated Multi-Level Lot-Sizing Problem under Time-Varying Environments and a Fix-and-Optimize Solution Approach. Entropy. 21. 377. 10.3390/e21040377.

[13]. Seyed Ashkan Hoseini Shekarabi, AbolfazlGharaei& Mostafa Karimi (2019) Modelling and optimal lot-sizing of integrated multi-level multi-wholesaler supply chains under the shortage and limited warehouse space: generalised outer approximation, International Journal of Systems Science: Operations & Logistics, 6:3, 237-257, DOI: 10.1080/23302674.2018.1435835

[14]. Sahithi V.V.D., Rao C.S.P., Srinivasa Rao M. (2020) An Extensive Study of Multi-level Inventory Lot Sizing Optimization Problem. In: Voruganti H., Kumar K., Krishna P., Jin X. (eds) Advances in Applied Mechanical Engineering. Lecture Notes in Mechanical Engineering. Springer, Singapore. https://doi.org/10.1007/978-981-15-1201-8_126

[15]. P. Afentakis, B. Gavish, V. Karmarkar, Computationally efficient optimal solutions to the lot-sizing problem in multi-stage assembly systems, Management Science 30 (1984) 222–239.

[16]. H. Tempelmeier, M. Derstroff, A lagrangean-based heuristic for dynamic multilevel multiitem constrained lotsizing with setup times, Management Science 42 (1996) 738–757.

[17]. R. Pitakaso, C. Almeder, K.F. Doerner, R.F. Hartl, A MAX–MIN ant system for unconstrained multi-level lot-sizing problems, Computers and Operations Research 34 (2007) 2533–2552.

[18]. Mishra.S.K , Sahithi, V.V.D , Rao. C. (2016). A hybrid binary particle swarm optimization for large capacitated multi itemmulti level lot sizing (CMIMLLS) problem. IOP Conference Series: Materials Science and Engineering. 149. 012040. 10.1088/1757-899X/149/1/012040.

[19]. Xiao, Y.; Kaku, I.; Zhao, Q.; Zhang, R. A variable neighborhood search based approach for uncapacitated multilevel lot-sizing problems. Comput. Ind. Eng. 2011, 60, 218–227.

[20]. Yi Han, Jiafu Tang, Iko Kaku, LifengMu,Solvinguncapacitated multilevel lot-sizing problems using a particle swarm optimization with flexible inertial weight,Computers& Mathematics with Applications,Volume 57, Issues 11–12,2009,Pages 1748-1755,ISSN 0898-1221,https://doi.org/10.1016/j.camwa.2008.10.024.

[21]. Woo Z, Hoon J, Loganathan GV. A New Heuristic Optimization Algorithm: Harmony Search. *SIMULATION*. 2001;76(2):60-68. doi:10.1177/003754970107600201

[22]. Mehrabian, A.R. & Lucas, Cameron. (2006). A novel numerical optimization algorithm inspired from weed colonization. Ecological Informatics. 1. 355-366. 10.1016/j.ecoinf.2006.07.003.

[23]. J. Xie, J.F. Dong, Heuristic genetic algorithm for general capacitated lot-sizing problems, Computer and Mathematics with Applications 44 (2002) 263–276.