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Upper and Lower level Sets of Absolute Direct Product of Doubt Intuitionistic Fuzzy K-ideal of BCK / BCI-algebra

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Abstract-*In this paper, we investigate some properties of level sets of absolute direct product of doubt intuitionistic fuzzy K-ideals.*
Keywords: *BCK/BCI-algebra , doubt intuitionistic fuzzy K-ideal, Absolute Direct Product of Doubt Intuitionistic Fuzzy K-ideals*

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I. INTRODUCTION

The notion of BCK/ BCI-algebras was proposed by Imai and Iseki in 1966 as a generalization of the concept of set-theoretic difference and proportional calculi. In 1991, Xi applied the concept of fuzzy sets to BCK-algebras. In 1993, Jun and Ahmad applied it to BCI-algebras. After that Jun, Meng Liu and several researchers investigated further properties of fuzzy BCK-algebras and fuzzy ideals . In 1999, Khalid and Ahmad introduced fuzzy H-ideals in BCI-algebras. In 2010, Satyanarayana introduced intuitionistic fuzzy H-ideals in BCK-algebras. In this paper, we investigate some properties of level sets of absolute direct product of doubt intuitionistic fuzzy K-ideals.

II. PRELIMINARIES

In this section , some elementary aspects that are necessary for this paper are included.

Definition 2.1: Let $A = (\sigma_{A,}\rho_A)$ and $B = (\sigma_{B,}\rho_B)$ be two intuitionistic fuzzy sets in BCK/BCI- algebras X and Y respectively. Then direct product of intuitionistic fuzzy sets A and B is denoted by A × B = $(\sigma_{A\times B}, \rho_{A\times B})$ where

$\sigma_{A\times B}(x, y) = \min\{\sigma_A(x), \sigma_B(y)\}$ and
$\rho_{A\times B}(x, y) = \max\{\rho_A(x), \rho_B(y)\}$ for all (x ,y) ϵ X × Y

Definition 2.2 : Let X and Y be two BCK/BCI-algebras and let $A = (\sigma_{A,}\rho_A)$ and $B = (\sigma_{B,}\ \rho_B)$ be two doubt intuitionistic fuzzy sets in X and Y respectively. Then the absolute direct product of doubt intuitionistic fuzzy sets A and B is defined by

A × B = $(\sigma_{A\times B}, \rho_{A\times B})$ where $\sigma_{A\times B}: X \times Y \to [0,1]$ is given by
$\sigma_{A\times B}(x, y) = max\{\sigma_A(x), \sigma_B(y)\}$ and $\rho_{A\times B}: X \times Y \to [0,1]$ is given by
$\rho_{A\times B}(x, y) = min\{\rho_A(x), \rho_B(y)\}$ for all (x ,y) ϵ X × Y

Definition 2.3 : If A × B = $(\sigma_{A\times B}, \rho_{A\times B})$ is an intuitionistic fuzzy set of BCK / BCI-algebra X × Y is said to be a doubt intuitionistic fuzzy K-ideal of X × Y if it satisfies the following axioms

i) $\sigma_{A\times B}(0,0) \le \sigma_{A\times B}(x, y)$ and $\rho_{A\times B}(0,0) \ge \rho_{A\times B}(x, y)$

ii) $\sigma_{A\times B}\{(x_1, y_1) * ((x_3, y_3) * (x_4, y_4))\}$
$$\le \sigma_{A\times B}\left[(x_{1,}y_1) * \left((x_2, y_2) * ((x_3, y_3) * (x_4, y_4))\right)\right] \vee \sigma_{A\times B}(x_2, y_2)$$

iii) $\rho_{A\times B}\{(x_1, y_1) * ((x_3, y_3) * (x_4, y_4))\}$
$$\ge \rho_{A\times B}\left[(x_{1,}y_1) * \left((x_2, y_2) * ((x_3, y_3) * (x_4, y_4))\right)\right] \wedge \rho_{A\times B}(x_2, y_2)$$

for every $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4) \in X \times Y$

III. MAIN RESULTS

Definition 3.1 : Let A × B = $(\sigma_{A\times B}, \rho_{A\times B})$ be a doubt intuitionistic fuzzy K-ideal of BCK / BCI-algebra X × Y, and $\alpha, \beta \in [0,1]$ then α - level cut of σ and β - level cut of ρ of A × B, is as follows.
$\sigma^{\leq}_{A\times B,\alpha} = \{(x,y) \in X \times Y / \sigma_{A\times B}(x,y) \leq \alpha\}$ and $\rho^{\geq}_{A\times B,\beta} = \{(x,y) \in X \times Y / \rho_{A\times B}(x,y) \geq \beta\}$

Theorem 3.2 : If A × B $= (\sigma_{A\times B}, \rho_{A\times B})$ be a doubt intuitionistic fuzzy K-ideal of X × Y, then $\sigma^{\leq}_{A\times B,\alpha}$ and $\rho^{\geq}_{A\times B,\beta}$ are K-ideals of X × Y for any $\alpha, \beta \in [0,1]$.

Proof: Let A × B = $(\sigma_{A\times B}, \rho_{A\times B})$ be a doubt intuitionistic fuzzy K-ideal of X × Y, and let $\alpha \in [0,1]$.Then we have $\sigma_{A\times B}(0,0) \leq \sigma_{A\times B}(x,y)$ for all $(x,y) \in X \times Y$
But $\sigma_{A\times B}(x,y) \leq \alpha$ for all $(x,y) \in \sigma^{\leq}_{A\times B,\alpha}$.So $\sigma_{A\times B}(0,0) \leq \alpha$
Therefore $(0,0) \in \sigma^{\leq}_{A\times B,\alpha}$
Let $(x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4) \in X \times Y$ be such that
$(x_1,y_1) * \left((x_2,y_2) * \left((x_3,y_3) * (x_4,y_4)\right)\right) \in \sigma^{\leq}_{A\times B,\alpha}$ and $(x_2,y_2) \in \sigma^{\leq}_{A\times B,\alpha}$ then
$\sigma_{A\times B}\left[(x_1,y_1) * \left((x_2,y_2) * \left((x_3,y_3) * (x_4,y_4)\right)\right)\right] \leq \alpha$ and $\sigma_{A\times B}(x_2,y_2) \leq \alpha$
Since $\sigma_{A\times B}$ is a doubt intuitionistic fuzzy K-ideal of X × Y, it follows that,

$$\sigma_{A\times B}\{(x_1,y_1) * ((x_3,y_3) * (x_4,y_4))\} \leq max\{\sigma_{A\times B}\left[(x_1,y_1) * \left((x_2,y_2) * \left((x_3,y_3) * (x_4,y_4)\right)\right)\right], \sigma_{A\times B}(x_2,y_2)\} \leq \alpha$$

And hence $(x_1,y_1) * ((x_3,y_3) * (x_4,y_4)) \in \sigma^{\leq}_{A\times B,\alpha}$ for all
$(x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4) \in X \times Y$.
Therefore $\sigma^{\leq}_{A\times B,\alpha}$ is a K-ideal of $X \times Y$ for $\alpha \in [0,1]$.
let $\beta \in [0,1]$, we have $\rho_{A\times B}(0,0) \geq \rho_{A\times B}(x,y)$ for all $(x,y) \in X \times Y$
But $\rho_{A\times B}(x,y) \geq \beta$ for all $(x,y) \in \rho^{\geq}_{A\times B,\beta}$.So $\rho_{A\times B}(0,0) \geq \beta$
Therefore $(0,0) \in \rho^{\geq}_{A\times B,\beta}$
Let $(x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4) \in X \times Y$ be such that
$(x_1,y_1) * \left((x_2,y_2) * \left((x_3,y_3) * (x_4,y_4)\right)\right) \in \rho^{\geq}_{A\times B,\beta}$ and $(x_2,y_2) \in \rho^{\geq}_{A\times B,\beta}$ then
$\rho_{A\times B}\left[(x_1,y_1) * \left((x_2,y_2) * \left((x_3,y_3) * (x_4,y_4)\right)\right)\right] \geq \beta$ and $\rho_{A\times B}(x_2,y_2) \geq \beta$
Since $\rho_{A\times B}$ is a doubt intuitionistic fuzzy K-ideal of X × Y, it follows that ,

$$\rho_{A\times B}\{(x_1,y_1) * ((x_3,y_3) * (x_4,y_4))\} \geq min\{\rho_{A\times B}\left[(x_1,y_1) * \left((x_2,y_2) * \left((x_3,y_3) * (x_4,y_4)\right)\right)\right], \rho_{A\times B}(x_2,y_2)\} \geq \beta$$

And hence $(x_1,y_1) * ((x_3,y_3) * (x_4,y_4)) \in \rho^{\geq}_{A\times B,\beta}$ for all
$(x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4) \in X \times Y$.
Therefore $\rho^{\geq}_{A\times B,\beta}$ is a K-ideal of $X \times Y$ for $\beta \in [0,1]$.

Theorem 3.3 : A doubt intuitionistic fuzzy set A × B = $(\sigma_{A\times B}, \rho_{A\times B})$ is a doubtintuitionistic fuzzy K-ideal of X × Y if and only if for all $\alpha, \beta \in [0,1]$, $\sigma^{\leq}_{A\times B,\alpha}$ and $\rho^{\geq}_{A\times B,\beta}$ are either empty or K-ideal of X × Y.

Proof: Assume $\sigma^{\leq}_{A\times B,\alpha}$ and $\rho^{\geq}_{A\times B,\beta}$ are either empty or K-ideal of X × Y for $\alpha, \beta \in [0,1]$.
For any $(x,y) \in X \times Y$, let $\sigma_{A\times B}(x,y) = \alpha$ and $\rho_{A\times B}(x,y) = \beta$
Then $(x,y) \in \sigma^{\leq}_{A\times B,\alpha}$ and $\rho^{\geq}_{A\times B,\beta}$, so $\sigma^{\leq}_{A\times B,\alpha} \neq \phi \neq \rho^{\geq}_{A\times B,\beta}$
Since $\sigma^{\leq}_{A\times B,\alpha}$ and $\rho^{\geq}_{A\times B,\beta}$ are K-ideals of$X \times Y$, therefore $(0,0) \in \sigma^{\leq}_{A\times B,\alpha}$ and $\rho^{\geq}_{A\times B,\beta}$
Hence $\sigma_{A\times B}(0,0) \leq \alpha = \sigma_{A\times B}(x,y)$ and $\rho_{A\times B}(0,0) \geq \beta = \rho_{A\times B}(x,y)$ where
$(x,y) \in X \times Y$.
If there exist $(x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4) \in X \times Y$ such that

$$\sigma_{A\times B}\{(x_1,y_1) * ((x_3,y_3) * (x_4,y_4))\} > max\{\sigma_{A\times B}\left[(x_1,y_1) * \left((x_2,y_2) * \left((x_3,y_3) * (x_4,y_4)\right)\right)\right], \sigma_{A\times B}(x_2,y_2)\}$$

Then by taking,

$$\alpha_0 = \frac{1}{2}(\sigma_{A\times B}\left((x_1,y_1) * ((x_3,y_3) * (x_4,y_4))\right) + max\{\sigma_{A\times B}\left[(x_1,y_1) * \left((x_2,y_2) * \left((x_3,y_3) * (x_4,y_4)\right)\right)\right], \sigma_{A\times B}(x_2,y_2)\})$$

We have

$$\sigma_{A\times B}\{(x_1,y_1) * ((x_3,y_3) * (x_4,y_4))\} > \alpha_0 > max\{\sigma_{A\times B}\left[(x_1,y_1) * \left((x_2,y_2) * \left((x_3,y_3) * (x_4,y_4)\right)\right)\right], \sigma_{A\times B}(x_2,y_2)\}$$

Hence $(x_1,y_1) * ((x_3,y_3) * (x_4,y_4)) \notin \sigma^{\leq}_{A\times B,\alpha_0}$,

$\left(x_{1,}y_1\right)*\left((x_2,y_2)*\left((x_3,y_3)*(x_4,y_4)\right)\right)\in\ \sigma^{\leq}_{A\times B,\alpha_0}$ and $(x_2,y_2)\in\ \sigma^{\leq}_{A\times B,\alpha_0}$
That is $\sigma^{\leq}_{A\times B,\alpha_0}$ is not a K-ideal of $X\times Y$.
Which is a contradiction.
Therefore $\sigma_{A\times B}\{(x_1,y_1)*\left((x_3,y_3)*(x_4,y_4)\right)\}$

$$\leq max\{\sigma_{A\times B}\left[\left(x_{1,}y_1\right)*\left((x_2,y_2)*\left((x_3,y_3)*(x_4,y_4)\right)\right)\right],\sigma_{A\times B}(x_2,y_2)\}$$

for any $(x_1,y_1),(x_2,y_2),(x_3,y_3),(x_4,y_4)\in X\times Y$

Now assume there exist $(x_1,y_1),(x_2,y_2),(x_3,y_3),(x_4,y_4)\in X\times Y$ such that
$\rho_{A\times B}\{(x_1,y_1)*\left((x_3,y_3)*(x_4,y_4)\right)\}$

$$< min\{\rho_{A\times B}\left[\left(x_{1,}y_1\right)*\left((x_2,y_2)*\left((x_3,y_3)*(x_4,y_4)\right)\right)\right],\rho_{A\times B}(x_2,y_2)\}$$

Then by taking,

$$\beta_0=\frac{1}{2}(\rho_{A\times B}\left((x_1,y_1)*\left((x_3,y_3)*(x_4,y_4)\right)\right) + min\{\rho_{A\times B}\left[\left(x_{1,}y_1\right)*\left((x_2,y_2)*\left((x_3,y_3)*(x_4,y_4)\right)\right)\right],\rho_{A\times B}(x_2,y_2)\})$$

We have

$$min\{\rho_{A\times B}\left[\left(x_{1,}y_1\right)*\left((x_2,y_2)*\left((x_3,y_3)*(x_4,y_4)\right)\right)\right],\rho_{A\times B}(x_2,y_2)\}>\beta_0 >\rho_{A\times B}\{(x_1,y_1)*\left((x_3,y_3)*(x_4,y_4)\right)\}$$

Hence $(x_1,y_1)*\left((x_3,y_3)*(x_4,y_4)\right)\notin\ \rho^{\geq}_{A\times B,\beta_0}$,
$\left(x_{1,}y_1\right)*\left((x_2,y_2)*\left((x_3,y_3)*(x_4,y_4)\right)\right)\in\ \rho^{\geq}_{A\times B,\beta_0}$ and $(x_2,y_2)\in\ \rho^{\geq}_{A\times B,\beta_0}$
That is $\rho^{\geq}_{A\times B,\beta_0}$ is not a K-ideal of $X\times Y$. Which is a contradiction.
Therefore $\rho_{A\times B}\{(x_1,y_1)*\left((x_3,y_3)*(x_4,y_4)\right)\}$

$$\geq min\{\rho_{A\times B}\left[\left(x_{1,}y_1\right)*\left((x_2,y_2)*\left((x_3,y_3)*(x_4,y_4)\right)\right)\right],\rho_{A\times B}(x_2,y_2)\}$$

for any $(x_1,y_1),(x_2,y_2),(x_3,y_3),(x_4,y_4)\in X\times Y$

Conversely assume A × B = $(\sigma_{A\times B},\rho_{A\times B})$ is a doubt intuitionistic fuzzy K-ideal of X × Y.
To prove $\sigma^{\leq}_{A\times B,\alpha}$ and $\rho^{\geq}_{A\times B,\beta}$ are either empty or K-ideal of X × Y.
Suppose that $\sigma^{\leq}_{A\times B,\alpha}\neq\phi$ for any $\alpha\ \epsilon\ [0,1]$.
It is clear that $(0,0)\in\sigma^{\leq}_{A\times B,\alpha}$ and $\rho^{\geq}_{A\times B,\beta}$, since $\sigma_{A\times B}(0,0)\leq\sigma_{A\times B}(x,y)=\alpha$,
$\rho_{A\times B}(0,0)\geq\rho_{A\times B}(x,y)=\beta$
Let $\left(x_{1,}y_1\right)*\left((x_2,y_2)*\left((x_3,y_3)*(x_4,y_4)\right)\right)\in\ \sigma^{\leq}_{A\times B,\alpha}$ and $(x_2,y_2)\in\ \sigma^{\leq}_{A\times B,\alpha}$
$\sigma_{A\times B}\left[\left(x_{1,}y_1\right)*\left((x_2,y_2)*\left((x_3,y_3)*(x_4,y_4)\right)\right)\right]\leq\alpha$ and $\sigma_{A\times B}(x_2,y_2)\leq\alpha$

$$\sigma_{A\times B}\{(x_1,y_1)*\left((x_3,y_3)*(x_4,y_4)\right)\} \leq max\{\sigma_{A\times B}\left[\left(x_{1,}y_1\right)*\left((x_2,y_2)*\left((x_3,y_3)*(x_4,y_4)\right)\right)\right],\sigma_{A\times B}(x_2,y_2)\}\leq\alpha$$

Therefore $\sigma_{A\times B}\{(x_1,y_1)*\left((x_3,y_3)*(x_4,y_4)\right)\}\leq\alpha$
Therefore $(x_1,y_1)*\left((x_3,y_3)*(x_4,y_4)\right)\in\ \sigma^{\leq}_{A\times B,\alpha}$
Hence $\sigma^{\leq}_{A\times B,\alpha}$ are K-ideals of X × Y.
Let $\left(x_{1,}y_1\right)*\left((x_2,y_2)*\left((x_3,y_3)*(x_4,y_4)\right)\right)\in\ \rho^{\geq}_{A\times B,\beta}$ and $(x_2,y_2)\in\ \rho^{\geq}_{A\times B,\beta}$
$\rho_{A\times B}\left[\left(x_{1,}y_1\right)*\left((x_2,y_2)*\left((x_3,y_3)*(x_4,y_4)\right)\right)\right]\geq\beta$ and $\rho_{A\times B}(x_2,y_2)\geq\beta$

$$\rho_{A\times B}\{(x_1,y_1)*\left((x_3,y_3)*(x_4,y_4)\right)\} \geq min\{\rho_{A\times B}\left[\left(x_{1,}y_1\right)*\left((x_2,y_2)*\left((x_3,y_3)*(x_4,y_4)\right)\right)\right],\rho_{A\times B}(x_2,y_2)\}\geq\beta$$

Therefore $\rho_{A\times B}\{(x_1,y_1)*\left((x_3,y_3)*(x_4,y_4)\right)\}\geq\beta$
Therefore $(x_1,y_1)*\left((x_3,y_3)*(x_4,y_4)\right)\in\ \rho^{\geq}_{A\times B,\beta}$
Hence $\rho^{\geq}_{A\times B,\beta}$ are K-ideals of X × Y.

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