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On Problems of Control Spatially Distributed Dynamic Systems

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ABSTRACT: *Non-defined for initial-boundary observations' control problems of spatially- distributed dynamic system, the mathematical model of which is a limited (partially limited) spatial-temporal domain is given by a linear differential equation are formulated and solved. Desirable continuously and discretely defined system's condition is achieved for the root-mean-square criterion in almost any practically justified combination of scale, insufficiently and initially defined outward control exposure to the system. The root-mean-square accuracy is estimated and conditions of the uniqueness of the considered problems' solutions are formulated.*
KEY WORDS: *spatially distributed systems, dynamic problems, control problems, problems with uncertainties.*

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I. INTRODUCTION

The issues of correct formulation and solution of problems of studying the dynamics of distributed spatial-temporal systems are complex and relevant [1]. The complexity of the problems increases while solving problems of controlling such systems. The approach proposed in [2] and summarized in [3,4] allows to solve these problems by the root-mean square criterion. It is important that the solutions are constructed without restrictions on the quantity and quality of observations of the initial-boundary condition of the system, which while solving the problem are also satisfied on a standard basis. This paper is devoted to peculiarities and results of using methods [3,4], concerning this class problems' solutions. Based on the results of mathematical researches [3,4] will be formulated and solved the problems of distributed spatial-temporal system's control on achieving of the function condition. Control factors in this case will act discretely or continuously distributed outward-dynamic factors, the initial and boundary condition of the process, both individually and taken in combination. The assessment of the accuracy of mathematical solutions of the formulated problems will be conducted and also conditions of the uniqueness will be given.

FORMULATION OF THE PROBLEM

Consider a spatial-temporal process, function $y(s)$ of the condition of which in the spatial-temporal domain

$$S_0^T = \{ s = (x,t) : x \in S_0 \subset R^n, 0 \le t \le T \} \qquad (1)$$

is determined by the equation

$$L(\partial_s) y(s) = u(s) \quad (s \in S_0^T), \qquad (2)$$

where $L(\partial_s)$ − linear differential operator, $\partial_s = (\partial_x, \partial_t) = (\partial_{x_1}, \dots, \partial_{x_n}, \partial_t)$, but $u(s)$ − of function distributed outward-dynamic pertubations, that accompany this process.

Through $Y_r^0(x), (x \in S \subset S_0, r = \overline{1, R_0})$ and $Y_\rho^\Gamma(x,t), (x \in \Gamma \subset \Gamma_0, \rho = \overline{1, R_\Gamma})$ denote the initial (at $t = 0$) and boundary (Γ_0 − edge of spatial domain S_0) pertubations, which additionally influence on the process' dynamics. We assume here, that known linear differential operators $L_r^0(\partial_t)$ and $L_\rho^\Gamma(\partial_x)$ are such, that:

$$Y_r^0(x) = L_r^0(\partial_t) y(s)\big|_{t=0}, \quad x \in S, \qquad (3)$$

$$Y_\rho^\Gamma(x,t) = L_\rho^\Gamma(\partial_x) y(s), \quad s \in \Gamma \times [0,T]. \qquad (4)$$

Consider the case, where some of the functions $u(s)$, Y_r^0 $(r=\overline{1,R_0})$, $Y_\rho^\Gamma(x,t)$ $(\rho=\overline{1,R_\Gamma})$ are known, but others are measured unter condition, that

$$\sum_{i=1}^{I}\int_{S_i}(L_i(\partial_s)y(s)-Y_i(s))^2 ds \to \min\ , \qquad (5)$$

$$\sum_{i=1}^{I}\int_{X_i}(L_i(\partial_s)\,y(s)\big|_{t=t_i}-Y_i(x))^2 dx \to \min, \qquad (6)$$

$$\sum_{i=1}^{I}\int_{0}^{T}(L_i(\partial_s)\,y(s)\big|_{x=x_i}-Y_i(t))^2 dt \to \min, \qquad (7)$$

$$\sum_{i=1}^{I}(L_i(\partial_s)\,y(s)\big|_{s=s_i}-Y_i)^2 \to \min \qquad (8)$$

at $t_i\in[0,T]$, $x_i\in X_i$, $s_i\in S_0$.

As in [1], the function $y(s)$ of the system's condition in a restricted spatial-temporal domain S_0^T, define [3,4] by ratio

$$y(s)=y_\infty(s)+y_0(s)+y_\Gamma(s), \qquad (9)$$

in which

$$y_\infty(s)=\int_{-\infty}^{\infty}G(s-s')u(s')ds', \qquad (10)$$

$$y_0(s)=\int_{S^0}G(s-s')u_0(s')ds', \qquad (11)$$

$$y_\Gamma(s)=\int_{S^\Gamma}G(s-s')u_\Gamma(s')ds', \qquad (12)$$

$$S^0=S_0\times(-\infty,0],\ S^\Gamma=(R^n\setminus S_0)\times[0,T],$$

$$G(s-s')=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{1}{L(p)}e^{p(s-s')}dp$$

under $p=(\lambda,\mu)=(\lambda_1,\ldots\lambda_n,\mu)$, $d\lambda=d\lambda_1\ldots d\lambda_n d\mu$, $p(s-s')=\sum_{i=1}^{n}\lambda_i(x_i-x_i')+\mu(t-t')$ and i – imaginary.

Here and then $u_0(s)$, $u_\Gamma(s)$ – functions, which according to the root-mean-square criterion are modelled known initial-boundary outward-dynamical pertubations. These functions, as well as the components $y_0(s)$ and $y_\Gamma(s)$ which are determined through them, will be absent due to the absence of restrictions at the temporal interval $(t\in(-\infty,T])$ and the value of the spatial domain ($x\in R^n$).

Note, however, that for any of the function, according to (11), (12) defined in S^0 and S^Γ, $u_0(s)$, $u_\Gamma(s)$ function $y(s)$, found accordingly to (9), equation (2) satisfies exactly. Therefore, for finding of the modelling functions $u_0(s)$, $u_\Gamma(s)$ we use ratio (3), (4) for observing the system, which are modelled by these functions.

PROBLEMS OF CONTROL UNDER DISCRETELY DEFINED DESIRABLE CONDITION

Based on the results of paper [4], we give the solutions of problem (8) on the achievement of the function $y(s)$ the process condition of discretely determined values Y_i $(i=\overline{1,I})$ for different spatial-temporal domains and different combinations of control outward-dynamic factors.

Problem 1.1. Control of the considered process for achieving according to (8) values Y_i $(i = \overline{1, I})$ is conducted by the function $u(s)$ at the known initial-boundary pertubations $Y_r^0(x)$ $(r = \overline{1, R_0})$, $Y_\rho^\Gamma(x,t)$ $(\rho = \overline{1, R_\Gamma})$. The dynamics of the process, determined according to (8) will be described by ratios (9) − (12), vector-function

$$\bar{u}(s) = col\ (u(s)\ (s \in S_0^T), u_0(s)\ (s \in S^0), u_\Gamma(s)\ (s \in S^\Gamma)) \tag{13}$$

of the control-modelling factors, in which occur under the condition that

$$\Phi = \left\| \int\limits_{(\cdot)} A(s)\bar{u}(s)ds - Y \right\|^2 \to \min_{\bar{u}(s)} \tag{14}$$

(here and further by the symbol $(\cdot)$ indicated integration in the domain of the definition of argument s of the subintegral function) under

$$Y = \begin{pmatrix} Y^* \\ Y^0 \\ Y^\Gamma \end{pmatrix};\ A(s) = \begin{pmatrix} A_{11}(s)\ (s \in S_0^T) & A_{12}(s)\ (s \in S^0) & A_{13}(s)\ (s \in S^\Gamma) \\ A_{21}(s)\ (s \in S_0^T) & A_{22}(s)\ (s \in S^0) & A_{23}(s)\ (s \in S^\Gamma) \\ A_{31}(s)\ (s \in S_0^T) & A_{32}(s)\ (s \in S^0) & A_{33}(s)\ (s \in S^\Gamma) \end{pmatrix} \tag{15}$$

and

$$\begin{aligned} Y^* &= col\ (Y_i, i = \overline{1, I}), \\ Y^0 &= col((Y_r^0(x_l^0), l = \overline{1, L_0}),\ \ r = \overline{1, R_0}), \\ Y^\Gamma &= col((Y_\rho^\Gamma(s),\ l = \overline{1, L_\Gamma}),\ \rho = \overline{1, R_\Gamma}), \end{aligned} \tag{16}$$

$$A_{2i}(s') = col((L_r^0(\partial_t)G(s - s')\Big|_{\substack{s = x_l^0 \\ t=0}}, l = \overline{1, L_0}), r = \overline{1, R_0}), \tag{17}$$

$$A_{3i}(s') = col((L_\rho^\Gamma(\partial_x)G(s - s')\Big|_{s = s_l^\Gamma}, l = \overline{1, L_\Gamma}), \rho = \overline{1, R_\Gamma})$$

$(s' \in S_0^T$ under $i = 1$, $s' \in S^0$ under $i = 2$, $s' \in S^\Gamma$ under $i = 3$, a $x_l^0 \in S_0, s_l^\Gamma \in \Gamma \times [0, T])$.
The solution of problem (14) will be [3,4]

$$\bar{u}(s) = A^T(s)P_1^+(Y + A_v) + \bar{v}(s), \tag{18}$$

where at random intergrates in S_0^T, S^0 and S^Γ functions $v(s)$, $v_0(s)$ and $v_\Gamma(s)$

$$\bar{v}(s) = col(v(s)\ (s \in S_0^T),\ v_0(s)\ (s \in S^0),\ v_\Gamma(s)\ (s \in S^\Gamma)),$$

$$A_v = \int\limits_{(\cdot)} A(s)\bar{v}(s)ds,$$

and by the sign "+" is marked operation of pseudo-inverse matrices

$$P_1 = \int\limits_{(\cdot)} A(s)A^T(s)ds.$$

Hence

$$\min_{u(s)} \Phi_1 = \varepsilon_1^2 = Y^TY - Y^TP_1P_1^+Y. \tag{19}$$

Problem 1.2. The control factor in solving problem (8) is functions $Y_r^0(x)$ $(r = \overline{1, R_0})$ of the initial perturbations. Functions $u(s)$ and $Y_\rho^\Gamma(x,t)$ $(\rho = \overline{1, R_\Gamma})$ are known.
Marking through

$$\bar{Y}^* = col((Y_i - L_i(\partial_s)y_\infty(s)\Big|_{s = s_i}), i = \overline{1, I}),$$

$$\bar{Y}^\Gamma = col(((Y_\rho^\Gamma(s_l^\Gamma) - L_\rho^\Gamma(\partial_x)y_\infty(s)\Big|_{s = s_l^\Gamma}), l = \overline{1, L_\Gamma}), \rho = \overline{1, R_\Gamma}),$$

the problem's solution we get by (3) under $y(s)$, defined by ratios (9) − (12), (18), in which

$$Y = \begin{pmatrix} \overline{Y}^{*} \\ \overline{Y}^{\Gamma} \end{pmatrix}, \; A(s) = \begin{pmatrix} A_{12}(s) & A_{13}(s) \\ A_{32}(s) & A_{33}(s) \end{pmatrix}, \tag{20}$$

The accuracy of the solution is defined by the value

$$\min_{Y_r^0(x)(r=\overline{1,R_0})} \Phi_1 = \varepsilon_1^2. \tag{21}$$

Problem 1.3. Similarly, there is a solution of problem (8) for the case when the known $u(s)$ and $Y_r^0(x)$ $(r = \overline{1, R_0})$ the control factor is $Y_\rho^\Gamma(x,t)$ $(\rho = \overline{1, R_\Gamma})$.

Marking through

$$\overline{Y}^0 = col(((Y_r^0(x_l^0) - L_r^0(\partial t) y_\infty(s)\Big|_{\substack{t=0 \\ x=x_l^0}}), l = \overline{1, L_0}), \rho = \overline{1, R_0}),$$

function $Y_\rho^\Gamma(x,t)$ we find accordingly to (4), (9) $-$ (12) (18), where unlike (20)

$$Y = \begin{pmatrix} \overline{Y}^{*} \\ \overline{Y}^{0} \end{pmatrix}, \; A(s) = \begin{pmatrix} A_{12}(s) & A_{13}(s) \\ A_{22}(s) & A_{23}(s) \end{pmatrix}. \tag{22}$$

Taken into account these changes in value

$$\varepsilon_1^2 = \min_{Y_\rho^\Gamma(x,t)(\rho=\overline{1,R_\Gamma})} \Phi_1$$

define accuracy of the obtained solution.

Problem 1.4. With accuracy

$$\varepsilon_1^2 = \min_{\substack{Y_r^0(x)(r=\overline{1,R_0}) \\ Y_\rho^\Gamma(x,t)(\rho=\overline{1,R_\Gamma})}} \Phi_1 = Y^T Y - Y^T P_1 P_1^+ Y$$

with ratios (3) $-$ (4), (9) $-$ (12), (18), problem (8) will be solved aiso for a known function $u(s)$ and control initial-boundary perturbations. Here, however

$$Y = \overline{Y}^{*}, \; A(s) = (A_{12}(s) \; A_{13}(s)).\,, \tag{23}$$

Problem 1.5. Control of the considered process on achievement by its function of a state's values Y_i $(i = \overline{1, l})$ is conducted by initial and distributed in S_0^T pertubations. Defined according to (8) control $u(s)$ found from (18) determining there

$$Y = \begin{pmatrix} Y^{*} \\ Y^{\Gamma} \end{pmatrix}, \; A(s) = \begin{pmatrix} A_{11}(s) & A_{12}(s) & A_{13}(s) \\ A_{31}(s) & A_{32}(s) & A_{33}(s) \end{pmatrix}. \tag{24}$$

Taking into account the defined by ratio (18) $u_0(s)$ and $u_\Gamma(s)$ from (3), (9) $-$ (12), we also find values for control functions $Y_r^0(x)$ $(r = \overline{1, R_0})$. As determined according to (8) root-mean-square deviation of the found solution from the desired value ε_1^2 written with taking into account adopted in (24) designations.

Problem 1.6. Similarly, the considered problem is solved for the case, when controlling is conducted by a combined action of distributed in S_0^T and concentrated on the contour of the spatial domain by controlling factors $u(s)$ and, $Y_\rho^\Gamma(x,t)$ accordingly. In this case, when

$$Y = \begin{pmatrix} Y^{*} \\ Y^{0} \end{pmatrix}, \; A(s) = \begin{pmatrix} A_{11}(s) & A_{12}(s) & A_{13}(s) \\ A_{21}(s) & A_{22}(s) & A_{23}(s) \end{pmatrix} \tag{25}$$

from (18) we define control $u(s)$ and modelling functions $u_0(s)$, $u_\Gamma(s)$. Taking into account the last of ratios (4), (9) $-$ (12), we find functions $Y_\rho^\Gamma(x,t)$. Taking into account (25), easily will be written, determined according to (8), (14) value ε_1^2.

Problem 1.7. Consider the case, when the control factors in solving problem (8) are all outward-dynamic factors – initial, boundary and distributed spatial-temporal perturbations. The vector of control-modelling functions $u(s)$, $u_0(s)$ and $u_\Gamma(s)$ we find from (18) under

$$Y = \overline{Y}^*, \ A(s) = \left(A_{11}(s) \quad A_{12}(s) \quad A_{13}(s) \right). \qquad (26)$$

Functions $Y_r^0(x)$ $(r = \overline{1, R_0})$ and $Y_\rho^\Gamma(x,t)$ $\rho = (\overline{1, R_\Gamma})$ we find from (3) – (4) taking into account (9) – (12). Under defined according to (26) vector Y and matrix function $A(s)$ from ratio (19) we obtain the accuracy of solving the problem.

Note, that the solutions of the considered above problems will take place under conditions of unlimited spatial domain S_0 and stability of the dynamic process (temporal interval is unlimited on the left). In the first case, there will be absent modelling function $u_\Gamma(s)$ and blocks $A_{3i}(s)$ $(i = \overline{1,3})$ in expressions (15), (20), (24) for the matrix function $A(s)$. Similarly for the second case. There will be absent modelling function $u_0(s)$ and blocks $A_{2i}(s)$ $(i = \overline{1,3})$ in expressions (15), (22) and (25) for the matrix function $A(s)$.

Note, that problems' solutions 1.1–1.7 are unambiguous $(\overline{v} \equiv 0)$, if $\lim\limits_{N \to \infty} \det \left[A^T(s_i) A(s_j) \right]_{i,j=1}^{i,j=N} \succ 0$ for s_i, s_j from the domain of the defined matrix function $A(s)$.

II. PROBLEMS OF CONTROL UNDER CONTINUOUSLY DEFINED DESIRABLE CONDITION

Let us consider [4] on the variants of the formulation and the results of solving problem (5) (we will not dwell on problems (6) and (7) − they are a partial case of problem (5)) on the control of system (2) with the purpose of obtaining by function $y(s)$ of the system's condition values in a root-mean-square way close to the functions $Y_i^0(s)$ $(i = \overline{1, I})$. On the contrary to the above mentioned, we will consider continuously defined initial-boundary outward-dynamic perturbations $Y_r^0(x)$ $(r = \overline{1, R_0})$ and $Y_\rho^\Gamma(x,t)$ $(\rho = \overline{1, R_\Gamma})$ in problems, where they are taken into account. Modelling of these perturbations will be conducted by vectors

$$u^0 = col(u_m^0 = u_0(s_m^0), m = \overline{1, M_0}) \ \ (s_m^0 \in S^0)$$

and

$$u^\Gamma = col(u_m^\Gamma = u_\Gamma(s_m^\Gamma), m = \overline{1, M_\Gamma}) \ \ (s_m^\Gamma \in S^\Gamma)$$

values of modelling functions $u_0(s)$ and $u_\Gamma(s)$. Vector

$$u^* = col(u_m = u(s_m), m = \overline{1, M}) \ \ (s_m \in S_0^T)$$

will determine distributed outward-dynamic pertubation $u(s)$ for cases, where it is control.

With this regard, components $y_\infty(s)$, $y_0(s)$ and $y_\Gamma(s)$ in presentation (9) function $y(s)$ of the system's condition (2) is given in the form of

$$y_0(s) = \sum_{m=1}^{M_0} G(s - s_m^0) u_m^0, \qquad (27)$$

$$y_\Gamma(s) = \sum_{m=1}^{M_\Gamma} G(s - s_m^\Gamma) u_m^\Gamma, \qquad (28)$$

$$y_\infty(s) = \sum_{m=1}^{M} G(s - s_m) u_m \qquad (29)$$

(for the case, when the function $u(s)$ is a control).

We give the solutions [3,4] of problem (5) for different combinations of control factors.

Problem 2.1. Problem (5) on the approximation of function $L_i(\partial_s)y(s)$ to a function $Y_i(s)$ $(i=\overline{1,I})$ is solved for known initial-boundary perturbations $Y_r^0(x)$ $(r=\overline{1,R_0})$, $Y_\rho^\Gamma(x,t)$ $(\rho=\overline{1,R_\Gamma})$ with a control vector u^*.
Vector

$$\bar{u}=col(u^*,u^0,u^\Gamma)$$

of values of control-modelling functions $u(s)$, $u_0(s)$ and $u_\Gamma(s)$, due to which ratios (9), (27) − (29) is determined the state $y(s)$ of the system, we find from the condition, that

$$\Phi_2=\sum_{i=1}^{I}\int_{(\cdot)}\left\|B(s)\bar{u}-Y(s)\right\|^2 ds\to\min. \tag{30}$$

As above mentioned, the integration in (30) is considered on the domain of change of the argument s in the vector and matrix functions $Y(s)$ and $B(s)$, which are determined by the ratios

$$Y(s)=\begin{pmatrix} Y^*(s) & (s\in S_0^T) \\ Y^0(x) & (x\in S_0) \\ Y^\Gamma(s) & (s\in\Gamma\times[0,T]) \end{pmatrix};\ B(s)=\begin{pmatrix} (B_{1i}(s) & s\in S_0^T), & i=\overline{1,3} \\ (B_{2i}(x) & x\in S_0), & i=\overline{1,3} \\ (B_{3i}(s) & s\in\Gamma\times[0,T]), & i=\overline{1,3} \end{pmatrix}, \tag{31}$$

in which

$$Y^*(s)=col(Y_i(s),i=\overline{1,I}),$$
$$Y^0(x)=col(Y_r^0(x),\ r=\overline{1,R_0}), \tag{32}$$
$$Y^\Gamma(s)=col(Y_\rho^\Gamma(x,t),\rho=\overline{1,R_\Gamma}),$$

$$\begin{aligned}
&B_{11}(s)=col(str(L_i(\partial_s)G(s-s_m),m=\overline{1,M}),i=\overline{1,I}),\\
&B_{12}(s)=col(str(L_i(\partial_s)G(s-s_m^0),m=\overline{1,M_0}),i=\overline{1,I}),\\
&B_{13}(s)=col(str(L_i(\partial_s)G(s-s_m^\Gamma),m=\overline{1,M_\Gamma}),i=\overline{1,I}),\\
&B_{21}(x)=col(str(L_r^0(\partial_t)G(s-s_m)\big|_{t=0},m=\overline{1,M}),r=\overline{1,R_0}),\\
&B_{22}(x)=col(str(L_r^0(\partial_t)G(s-s_m^0)\big|_{t=0},m=\overline{1,M_0}),r=\overline{1,R_0}),\\
&B_{23}(x)=col(str(L_r^0(\partial_t)G(s-s_m^\Gamma)\big|_{t=0},m=\overline{1,M_\Gamma}),r=\overline{1,R_0}),\\
&B_{31}(s)=col(str(L_\rho^\Gamma(\partial_x)G(s-s_m),m=\overline{1,M}),\rho=\overline{1,R_\Gamma}),\\
&B_{32}(s)=col(str(L_\rho^\Gamma(\partial_x)G(s-s_m^0),m=\overline{1,M_0}),\rho=\overline{1,R_\Gamma}),\\
&B_{33}(s)=col(str(L_\rho^\Gamma(\partial_x)G(s-s_m^\Gamma),m=\overline{1,M_\Gamma}),\rho=\overline{1,R_\Gamma}).
\end{aligned} \tag{33}$$

The solution of problem (30) will be [3,4]

$$\bar{u}=P_2^+B_Y+\bar{v}-P_2^+P_2\bar{v}, \tag{34}$$

where for random $v\in R^M$, $v_0\in R^{M_0}$, $v_\Gamma\in R^{M_\Gamma}$

$$\bar{v}=col(v,v_0,v_\Gamma),$$
$$P_2=\int_{(\cdot)}B^T(s)B(s)ds,$$
$$B_Y=\int_{(\cdot)}B^T(s)Y(s)ds.$$

Here, as above , the integration is in the domain of the change of argument s, and the sign "+" is designated the operation of pseudo-inversion of the matrix. Hence

$$\min_{u} \Phi_2 = \int_{(\cdot)} Y^T(s)Y(s)ds - B_Y^T P_2^+ B_Y = \varepsilon_2^2 . \qquad (35)$$

Problem 2.2. The root-mean-square according to (5) approximation of the function $L_i(\partial_s)y(s)$ to the function $Y_i(s)$ $(i = \overline{1, I})$ is conducted by initial perturbations $Y_r^0(x)$ at the known boundary and distributed outward-dynamic perturbations $Y_\rho^\Gamma(x,t)$ $(\rho = \overline{1, R_\Gamma})$ and $u(s)$ accordingly. The control functions $Y_r^0(x)$, $(r = \overline{1, R_0})$ we obtain from (3) under $y(s)$ defined by ratios (9) (10) (27) (28), in which

$$Y(s) = \begin{pmatrix} \overline{Y}^*(s) \\ \overline{Y}^\Gamma(s) \end{pmatrix}, \; B(s) = \begin{pmatrix} B_{12}(s) & B_{13}(s) \\ B_{32}(s) & B_{33}(s) \end{pmatrix}, \qquad (36)$$

$$\overline{Y}^*(s) = col(Y_i(s) - L_i(\partial_s)y_\infty(s)) \;\; (i = \overline{1, I}),$$

$$\overline{Y}^\Gamma(s) = col(Y_\rho^\Gamma(x,t) - L_\rho^\Gamma(\partial_x)y_\infty(s)) \;\; (\rho = \overline{1, R_\Gamma}).$$

The accuracy of solving the problem is determined by the value

$$\varepsilon_2^2 = \min_{Y_r^0(x)\,(r=\overline{1,R_0})} \Phi_2 . \qquad (37)$$

Problem 2.3. The problem (5) is solved with known $Y_r^0(x)$ $(r = \overline{1, R_0})$, $u(s)$ and control $Y_\rho^\Gamma(x,t)$ $(\rho = \overline{1, R_\Gamma})$.

Searched $Y_\rho^\Gamma(x,t)$ we will find from (4), (9), (10), (27), (29) putting there

$$Y(s) = \begin{pmatrix} \overline{Y}^*(s) \\ \overline{Y}^0(x) \end{pmatrix}, \; B(s) = \begin{pmatrix} B_{12}(s) & B_{13}(s) \\ B_{22}(s) & B_{23}(s) \end{pmatrix}, \qquad (38)$$

$$\overline{Y}^0(x) = col((Y_r^0(x) - L_r^0(\partial_t)y_\infty(s))\big|_{t=0}) \;\; (r = \overline{1, R_0}).$$

Hence

$$\min_{Y_\rho^\Gamma(x,t)\,(\rho=\overline{1,R_\Gamma})} \Phi_2 = \varepsilon_2^2 ,$$

where ε_2^2 is defined in (35).

Problem 2.4. Under the known function $u(s)$ and control initial-boundary perturbations $Y_r^0(x)$ $(r = \overline{1, R_0})$ and $Y_\rho^\Gamma(x,t)$ $(\rho = \overline{1, R_\Gamma})$ determined according to (9), (10), (27), (28), the condition of the system will be the solution of problem (5), if $Y_r^0(x)$ $(r = \overline{1, R_0})$ and $Y_\rho^\Gamma(x,t)$ $(\rho = \overline{1, R_\Gamma})$ determined by ratios (3), (4), (9), (10), (27), (28), in which

$$Y(s) = \overline{Y}^*(s), \; B(s) = (B_{12}(s), \quad B_{13}(s)). \qquad (39)$$

As above noted, at the same time

$$\varepsilon_2^2 = \min_{\substack{Y_r^0(x)\,(r=\overline{1,R_0}) \\ Y_\rho^\Gamma(x,t)\,(\rho=\overline{1,R_\Gamma})}} \Phi_2 .$$

Problem 2.5. Let us consider the case, when problem (5) is solved under known boundary perturbations. The control vector u^* and modelling vectors u^0, u^Γ found from (34), defining there

$$Y(s) = \begin{pmatrix} \overline{Y}^*(s) \\ \overline{Y}^\Gamma(s) \end{pmatrix}, \; B(s) = \begin{pmatrix} B_{11}(s) & B_{12}(s) & B_{13}(s) \\ B_{31}(s) & B_{32}(s) & B_{33}(s) \end{pmatrix}. \qquad (40)$$

The control initial perturbations corresponding to these vectors are determined by ratios (3) with taking into account (9), (27) – (29). By substitution (40) in (35) we will find the accuracy of solving the problem ε_2^2.

Problem 2.6. Similarly, the problem of control the process under consideration with known initial perturbations is solved. Here, as above, the control-modelling vector $\bar{u}$ is determined according to (34) when

$$Y(s) = \begin{pmatrix} \bar{Y}^*(s) \\ \bar{Y}^0(s) \end{pmatrix}, \; B(s) = \begin{pmatrix} B_{11}(s) & B_{12}(s) & B_{13}(s) \\ B_{21}(x) & B_{22}(x) & B_{23}(x) \end{pmatrix}. \qquad (41)$$

Taking into account (34), (41), by ratio (4), we will define also the control functions $Y_\rho^\Gamma(x,t)$ $(\rho = \overline{1, R_\Gamma})$. By substitution (41) in (35) we will find also ε_2^2.

Problem 2.7. For the case, when solving the problem (5) all external-dynamic factors are control, vector u^* is found from (34), and functions $Y_r^0(x)$ $(\rho = \overline{1, R_0})$ and $Y_\rho^\Gamma(x,t)$ $(\rho = \overline{1, R_\Gamma})$ – from (3), (4), thus laying

$$Y(s) = Y^*(s); \; B(s) = (B_{11}(s), \quad B_{12}(s), \quad B_{13}(s)). \qquad (42)$$

Taking into account (42) by ratio (35) we define also value ε_2^2.

Note, that (as in solving problems 1.1 – 1.7) the calculating formulas are simplified in cases of unlimited spatial and temporal domains. In the absence of restrictions on the spatial domain in the calculated ratios will be absent modelling vector u_Γ and blocks $B_{3i}(s)$ $(i = \overline{1,3})$. In the absence of initial conditions, vector u_0 will be absent, which model them, and blocks $B_{2i}(s)$ $(i = \overline{1,3})$ in the expressions for the matrix functions $B(s)$.

As noted above, the problems' solutions 2.1 – 2.7 will be unique, if $\det P_3 \succ 0$.

III. CONCLUSIONS

A certain number of issues on the control of spatially distributed systems in given spatially distributed domains (restricted, partially restricted and unrestricted) is considered, provided that the mathematical model of their functioning is represented by a linear differential equation, supplied with the initial-boundary observation of their outward-dynamic condition. It is important that no restrictions are imposed on the quantity and quality (discretely, or continuously determined) of such observations, which makes the problems under consideration mathematically incorrect and insoluble by classical methods of analytical and computational mathematics. Due to this, the outward-dynamic control factors, i.e. distributed, boundary and initially-defined outward-dynamic effects on the system, are constructed in such a way, that the function of condition the considered system, satisfying the differential equation of the mathematical model of the system accurately, with the available initial-boundary observations for it agreed according to the root-mean-square criterion. Problems are solved both at a continuously defined desired condition, and at certain spatial-temporal points' condition. In each of the problems considered in the article, the accuracy of such agreement is estimated and the conditions for its unambiguity are introduced.

It is important that the problems of outputting the system's condition to the root-mean-square surrounding of a given one are solved for any combination of outward- dynamic control factors that have practical meaning. The computer implementation of solutions to these complex mathematical problems is based on simple and classically known methods of linear algebra and algorithms of numerical integration.

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