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Poisson Inverted Lomax Distribution: Properties and Applications

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ABSTRACT
In this paper, we have generated a new distribution by using the Poisson-G family with baseline distribution as inverted Lomax distribution named Poisson inverted Lomax distribution. Some distributional properties of the Poisson inverted Lomax distribution are presented. The model parameters of the proposed distribution are estimated using three well-known estimation methods namely maximum likelihood estimation (MLE), least-square estimation (LSE), and Cramer-Von-Mises estimation (CVME) methods. We have calculated the asymptotic confidence intervals based on maximum likelihood estimates. All the computations have been performed in R software. The application of the proposed model has been illustrated considering a real data set and investigated the goodness of fit attained by the observed model via different test statistics and graphical methods. We have found that the proposed distribution provided a better fit and more flexible in comparison with some other lifetime distributions.

***KEYWORDS**: Poisson-G family, Inverted Lomax distribution, Survival function.*

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I. INTRODUCTION

Last few years, it has been observed that the many life-time models have been generated but the real data sets related to engineering, life sciences, biology, hydrology, life testing, and risk analysis do not present a better fit to these distributions. So, the generation of new modified distributions appears to be necessary to deal with the problems in these fields. The generalized, extended, and modified distributions are created by inserting one or more parameters or making some transformation to the baseline distribution. Therefore, the new proposed distributions provide a better fit as compared to the existing models.

The Poisson-Weibull distribution has introduced by [4] this extended family can have increasing and decreasing failure rate functions, also exponential Poisson (EP) distribution has introduced by [13] by compounding exponential distribution with zero truncated Poisson distribution with decreasing failure rate. The CDF of exponential Poisson distribution is,

$$G\left(t;\beta,\lambda\right)=\frac{1}{\left(1-e^{-\lambda}\right)}\left[1-\exp\left\{-\lambda\left(1-e^{-\beta t}\right)\right\}\right] \quad ; t>0,\left(\beta,\lambda\right)>0$$

While [3] have generated generalized exponential Poisson distribution having the decreasing or increasing or upside-down bathtub shaped failure rate, which is the generalization of the distribution proposed by [13] adding a power parameter to this model.

Using the similar approach, [6] has presented a new distribution family also based on the exponential distribution with an increasing failure rate function known as Poisson exponential (PE) distribution. The CDF of PE distribution can be expressed as

$$G\left(t;\alpha,\theta\right)=\frac{e^{-\alpha}-\exp\left\{-\theta\left(1-e^{-\alpha t}\right)\right\}}{\left(1-e^{-\alpha}\right)} \quad ; t>0,\left(\alpha,\theta\right)>0$$

Using the same approach as used by [6], [15] has introduced a two-parameter Poisson-exponential with increasing failure rate under the Bayesian approach. Alkarni and Oraby [1] have presented a new lifetime class with a decreasing failure rate which was obtained by compounding truncated Poisson distribution and a lifetime distribution. The CDF of the Poisson family is given by,

$$M\left(t;\beta,\underline{\omega}\right)=\frac{1-\exp\left\{-\beta G\left(t;\underline{\omega}\right)\right\}}{\left(1-e^{-\beta}\right)}\quad;\ \beta>0 \tag{1.1}$$

And its corresponding PDF can be expressed as

$$m\left(t;\beta,\underline{\omega}\right)=\frac{\beta g\left(t;\underline{\omega}\right)\exp\left\{-\beta G\left(t;\underline{\omega}\right)\right\}}{\left(1-e^{-\beta}\right)}\quad;\ \beta>0 \tag{1.2}$$

Where $\underline{\omega}$ the parameter is space and $G\left(t;\underline{\omega}\right)$ and $g\left(t;\underline{\omega}\right)$ are the CDF and PDF of any distribution.

Employing same approach the Weibull power series class of distributions with Poisson has presented by [19]. Exponentiated Weibull–Poisson a new four-parameter distribution with increasing, decreasing, bathtub-shaped, and uni-modal failure rate have presented by [18] and it has generated by compounding exponentiated Weibull and Poisson distributions. Weibull–Poisson distribution is introduced by [16] having the shape of decreasing, increasing, upside-down bathtub-shaped, or uni-modal failure rate function. Further [10] have made an extensive study on Weibull–Poisson distribution through a reliability sampling plan. [14] has used the exponentiated exponential-Poisson as the software reliability model. [9] has presented Poisson exponential power distribution and used different estimation methods to estimate the model parameter.

In this paper we have taken inverse Lomax distribution as a baseline distribution. The inverse Lomax (IL) distribution is one of important life-time distribution. The inverse Lomax distribution was introduced by [11] and used it to get Lorenz ordering relationship among ordered statistics. [12] showed that the IL distribution can be used in economics and actuarial sciences. The IL distribution has a lot of applications in stochastic modeling of decreasing failure rate life components, and life testing. The CDF and PDF of two-parameter inverse Lomax can be expressed as

$$Z\left(t;\beta,\chi\right)=\left\{1+\left(\beta/t\right)\right\}^{-\chi};t\geq 0,\ \ \beta>0,\chi>0. \tag{1.3}$$

$$z(t;\beta,\chi)=\frac{\chi}{\beta^{\chi}}t^{\chi-1}\left\{1+\left(\beta/t\right)\right\}^{-\chi-1};t\geq 0,\ \ \beta>0,\chi>0 \tag{1.4}$$

The main objective of this paper is to present a more flexible model by adding just one extra parameter to attain a better fit to the life-time data sets. The different sections of this paper have organized as follows; in Section 2 we present the Poisson inverted Lomax distribution with its statistical and mathematical properties. We broadly discussed the three well-known estimation methods namely maximum likelihood estimation (MLE), least-square estimation (LSE), and Cramer-Von-Mises estimation (CVME) methods in Section 3. In Section 4 using a real dataset, we present the estimated values of the model parameters and their corresponding asymptotic confidence intervals and Fisher information matrix. Also, we have presented the different test criteria to assess the potentiality of the proposed model. Some concluding remarks are present in Section 5.

II. THE POISSON INVERSE LOMAX (PIL) DISTRIBUTION

We have generated the new distribution by using the Poisson-G family defined by [1]. The CDF and PDF of PIL distribution is obtained by using (1.1), (1.2), (1.3) and (1.4) and can be expressed as

$$F(x)=\frac{1}{(1-e^{-\lambda})}[1-\exp\{-\lambda(1+\beta/x)^{-\alpha}\}];x\geq 0,(\alpha,\beta,\lambda)>0 \tag{2.1}$$

$$f(x)=\frac{\alpha\beta\lambda}{(1-e^{-\lambda})}x^{-2}(1+\beta/x)^{-(\alpha+1)}\exp\left\{-\lambda(1+\beta/x)^{-\alpha}\right\};x\geq 0,(\alpha,\beta,\lambda)>0 \tag{2.2}$$

The **Survival /Reliability** function of PIL distribution is

$$S(x)=1-F(x)$$

$$=\frac{\exp\{-\lambda(1+\beta/x)^{-\alpha}\}-\mathrm{e}^{-x}}{(1-e^{-\lambda})};x\geq 0,(\alpha,\beta,\lambda)>0 \tag{2.3}$$

Hazard rate function of PIL distribution

Let x be life time of an item and we want the probability that it will not survive for an extra time *dx* then, hazard rate function is,

$$h(x) = \lim_{dx \to 0} \frac{pr(x \le X < x + dx)}{dx.R(x)} = \frac{f(x)}{R(x)} = \frac{f(x)}{1 - F(x)}$$

$$= \frac{\alpha\beta\lambda(1+\beta/x)^{-(\alpha+1)}\exp\{-\lambda(1+\beta/x)^{-\alpha}\}}{x^2[\exp\{-\lambda(1+\beta/x)^{-\alpha}\} - e^{-x}]}; x \ge 0, (\alpha, \beta, \lambda) > 0 \tag{2.4}$$

The curves of the probability density function and hazard function of PIL distribution are presented in Figure 1. It has been observed that the PDF of PIL distribution can have different variety of shapes. The hazard rate function (HRF) for the PIL distribution is also flexible due to its various shapes such as increasing, increasing–decreasing, reverse J-shaped for different values of parameters.

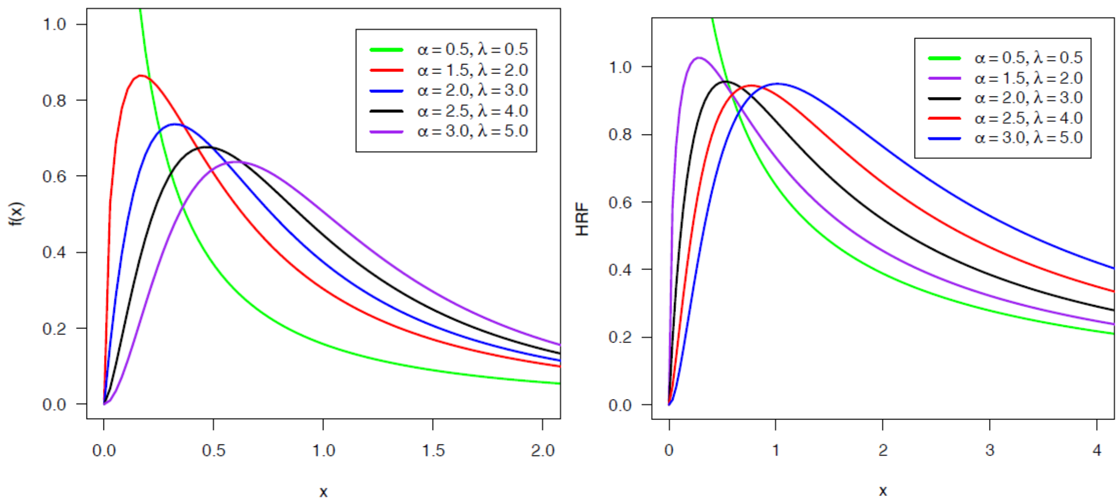

Figure 1. Graph of PDF (left panel) and hazard function (right panel) for β = 1 and different values of α and λ.

Quantile function of PIL distribution

The quantile function is the inverse cumulative distribution function.

$$Q(p) = F^{-1}(p)$$

The quantile function of PIL distribution is

$$x_p = \beta(z^{-1/\alpha} - 1)^{-1} \quad where \ z = -\frac{1}{\lambda}\ln\{1 - (1 - e^{-\lambda})p\} \ ; 0 < p < 1 \tag{2.5}$$

For the generation of the random numbers of the PIL distribution, we suppose simulating values of random variable X with the CDF (2.1). Let U denote a uniform random variable in (0, 1), then the simulated values of X are obtained by setting,

$$x = \beta(z^{-1/\alpha} - 1)^{-1} \quad where \ z = -\frac{1}{\lambda}\ln\{1 - (1 - e^{-\lambda})u\} \ ; 0 < u < 1 \tag{2.6}$$

Skewness and Kurtosis:

The Skewness and Kurtosis are used mostly in data analysis to study the shape of the probability distribution or data set, which can be calculated as follows,

$$S_k(B) = \frac{Q(0.75) + Q(0.25) - 2Q(0.5)}{Q(0.75) - Q(0.25)},$$ and

The coefficient of kurtosis based on octiles given by (Moors, 1988) is

$$K_u(Moors) = \frac{Q(0.875) - Q(0.625) + Q(0.375) - Q(0.125)}{Q(3/4) - Q(1/4)}$$

III. METHODS OF PARAMETER ESTIMATION

To estimate the unknown parameters of the PIL distribution we have used three well-known estimation methods, which are listed below

i. Maximum likelihood estimation methods
ii. Least square estimation methods

iii. Cramer-Von-Mises estimation methods

3.1. Maximum Likelihood Estimation (MLE) method

Let $\underline{x} = (x_1, \dots, x_n)$ be a random sample of size 'n' from PIL(α, β, λ) then the log likelihood function l(α, β, λ/ $\underline{x}$) can be written as,

$$l = n\ln\alpha + n\ln\beta + n\ln\lambda - n\ln(1-e^{-\lambda}) - 2\sum_{i=1}^{n}\ln x_i$$
$$-(\alpha+1)\sum_{i=1}^{n}\ln(1+\beta/x_i) - \lambda\sum_{i=1}^{n}\log(1+\beta/x_i)^{-\alpha} \qquad (3.1.1)$$

By differentiating (3.1.1) with respect to unknown parameters α, β and λ, we get

$$\frac{\partial l}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\log(1+\beta/x_i) - \lambda\sum_{i=1}^{n}(1+\beta/x_i)^{-\alpha}\ln(1+\beta/x_i)$$

$$\frac{\partial l}{\partial\beta} = \frac{n}{\beta} + \sum_{i=1}^{n}\frac{1}{x(1+\beta x_i)} + \lambda\alpha\sum_{i=1}^{n}\frac{1}{x(1+\beta x_i)^{\alpha+1}}$$

$$\frac{\partial l}{\partial\lambda} = \frac{n}{\lambda} + \frac{ne^{-\lambda}}{1-e^{-\lambda}} - \sum_{i=1}^{n}(1+\beta/x_i)$$

Equating to zero and solving these non-linear equations for the unknown parameters (α, β, λ) we will obtain the ML estimators of the PIL distribution. To solve them manually, one can used appropriate computer softwares like R, Mathmatica, Matlab etc. Let us denote the parameter vector by $\underline{\chi} = (\alpha, \beta, \lambda)$ and the corresponding MLE of $\underline{\chi}$ as $\hat{\underline{\chi}} = (\hat{\alpha}, \hat{\beta}, \hat{\lambda})$, then the asymptotic normality results in, $(\hat{\underline{\chi}} - \underline{\chi}) \to N_3\left[0, \left(B(\underline{\chi})\right)^{-1}\right]$ where $B(\underline{\chi})$ is the Fisher's information matrix given by,

$$B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$$

where $B_{11} = \frac{\partial^2 l}{\partial\alpha^2}$, $B_{12} = \frac{\partial^2 l}{\partial\alpha\partial\beta}$, $B_{13} = \frac{\partial^2 l}{\partial\alpha\partial\lambda}$

$$B_{21} = \frac{\partial^2 l}{\partial\beta\partial\alpha},\ B_{22} = \frac{\partial^2 l}{\partial\beta^2},\ B_{23} = \frac{\partial^2 l}{\partial\beta\partial\lambda}$$

$$B_{31} = \frac{\partial^2 l}{\partial\lambda\partial\alpha},\ B_{32} = \frac{\partial^2 l}{\partial\beta\partial\lambda},\ B_{33} = \frac{\partial^2 l}{\partial\lambda^2}$$

In practice, we don't know $\underline{\chi}$ hence it is useless that the MLE has an asymptotic variance $\left(B(\underline{\chi})\right)^{-1}$. Hence we approximate the asymptotic variance by plugging in the estimated value of the parameters. where $B(\underline{\chi})$ is the Fisher's information matrix. Using the Newton-Raphson algorithm to maximize the likelihood creates the observed information matrix and hence the variance-covariance matrix is obtained as,

$$\left(B(\underline{\chi})\right)^{-1} = \begin{pmatrix} \mathrm{var}(\hat{\alpha}) & \mathrm{cov}(\hat{\alpha},\hat{\beta}) & \mathrm{cov}(\hat{\alpha},\hat{\lambda}) \\ \mathrm{cov}(\hat{\alpha},\hat{\beta}) & \mathrm{var}(\hat{\beta}) & \mathrm{cov}(\hat{\beta},\hat{\lambda}) \\ \mathrm{cov}(\hat{\alpha},\hat{\lambda}) & \mathrm{cov}(\hat{\beta},\hat{\lambda}) & \mathrm{var}(\hat{\lambda}) \end{pmatrix}$$

(3.1.2)

Hence from the asymptotic normality of MLEs, approximate 100(1-α) % confidence intervals for α, β and λ can be constructed as,

$\hat{\alpha} \pm Z_{\alpha/2}SE(\hat{\alpha})$, $\hat{\beta} \pm Z_{\alpha/2}SE(\hat{\beta})$ and, $\hat{\lambda} \pm Z_{\alpha/2}SE(\hat{\lambda})$

3.2. Method of Least-Square Estimation (LSE)

The least-square estimators of the unknown parameters α, β and λ of PIL distribution can be obtained by minimizing

$$A(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}\left[F(X_i)-\frac{i}{n+1}\right]^2 \qquad (3.2.1)$$

with respect to unknown parameters α, β and λ.

Suppose $F(X_i)$ denotes the CDF of the ordered random variables $X_{(1)}<X_{(2)}<\dots<X_{(n)}$ where $\{X_1,X_2,\dots,X_n\}$ is a random sample of size n from a distribution function F(.). The least-square estimators of α, β and λ say $\hat{\alpha},\hat{\beta}, and\ \hat{\lambda}$ respectively, can be obtained by minimizing

$$A(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}\left[\frac{1}{(1-e^{-\lambda})}[1-\exp\{-\lambda(1+\beta/x_i)^{-\alpha}\}]-\frac{i}{n+1}\right]^2 \qquad (3.2.2)$$

with respect to α, β and λ.

Differentiating (3.2.2) with respect to α, β and λ we get,

$$\frac{\partial A}{\partial\alpha}=2\lambda\sum_{i=1}^{n}\left[\frac{1}{(1-e^{-\lambda})}[1-\exp\{-\lambda I(x_i)^{-\alpha}\}]-\frac{i}{n+1}\right]\exp\{-\lambda I(x_i)^{-\alpha}\}I(x_i)^{-\alpha}\ln I(x_i)$$

$$\frac{\partial A}{\partial\beta}=-2\alpha\lambda\sum_{i=1}^{n}\left[\frac{1}{(1-e^{-\lambda})}[1-\exp\{-\lambda I(x_i)^{-\alpha}\}]-\frac{i}{n+1}\right]x_i^{-1}\exp\{-\lambda I(x_i)^{-\alpha}\}I(x_i)^{-\alpha-1}$$

$$\frac{\partial A}{\partial\lambda}=2\sum_{i=1}^{n}\left[\frac{1}{(1-e^{-\lambda})}[1-\exp\{-\lambda I(x_i)^{-\alpha}\}]-\frac{i}{n+1}\right]$$
$$\left[\frac{1}{(1-e^{-\lambda})}\exp\{-\lambda I(x_i)^{-\alpha}\}I(x_i)^{-\alpha}-\frac{e^{-\lambda}}{(1-e^{-\lambda})^2}[1-\exp\{-\lambda I(x_i)^{-\alpha}\}]\right]$$

Where $I(x_i)=(1+\beta/x_i)$

Similarly the weighted least square estimators can be obtained by minimizing

$$D(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}w_i\left[F(X_{(i)})-\frac{i}{n+1}\right]$$

with respect to α, β and λ. The weights w_i are $w_i=\frac{1}{Var(X_{(i)})}=\frac{(n+1)^2(n+2)}{i(n-i+1)}$

Hence, the weighted least square estimators of α, β and λ respectively can be obtained by minimizing,

$$D(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[\frac{1}{(1-e^{-\lambda})}[1-\exp\{-\lambda(1+\beta/x_i)^{-\alpha}\}]-\frac{i}{n+1}\right]^2 \qquad (3.2.3)$$

with respect to α, β and λ.

3.3. Method of Cramer-Von-Mises estimation (CVME)

The Cramer-Von-Mises estimators of α, β and λ are obtained by minimizing the function

$$Z(X;\alpha,\beta,\lambda)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F(x_{i:n}\mid\alpha,\beta,\lambda)-\frac{2i-1}{2n}\right]^2$$

$$=\frac{1}{12n}+\sum_{i=1}^{n}\left[\frac{1}{(1-e^{-\lambda})}[1-\exp\{-\lambda(1+\beta/x_i)^{-\alpha}\}]-\frac{2i-1}{2n}\right]^2 \qquad (3.3.1)$$

Differentiating (3.3.1) with respect to α, β and λ we get,

$$\frac{\partial Z}{\partial \alpha} = 2\lambda \sum_{i=1}^{n} \left[\frac{1}{(1-e^{-\lambda})}[1-\exp\{-\lambda I(x_i)^{-\alpha}\}] - \frac{2i-1}{2n} \right] \exp\{-\lambda I(x_i)^{-\alpha}\} I(x_i)^{-\alpha} \ln I(x_i)$$

$$\frac{\partial Z}{\partial \beta} = -2\alpha\lambda \sum_{i=1}^{n} \left[\frac{1}{(1-e^{-\lambda})}[1-\exp\{-\lambda I(x_i)^{-\alpha}\}] - \frac{2i-1}{2n} \right] x_i^{-1} \exp\{-\lambda I(x_i)^{-\alpha}\} I(x_i)^{-\alpha-1}$$

$$\frac{\partial Z}{\partial \lambda} = 2 \sum_{i=1}^{n} \left[\frac{1}{(1-e^{-\lambda})}[1-\exp\{-\lambda I(x_i)^{-\alpha}\}] - \frac{2i-1}{2n} \right] \left[\frac{1}{(1-e^{-\lambda})} \exp\{-\lambda I(x_i)^{-\alpha}\} I(x_i)^{-\alpha} - \frac{e^{-\lambda}}{(1-e^{-\lambda})^2}[1-\exp\{-\lambda I(x_i)^{-\alpha}\}] \right]$$

Where $I(x_i) = (1 + \beta / x_i)$

By solving $\frac{\partial Z}{\partial \alpha} = 0, \frac{\partial Z}{\partial \beta} = 0 \text{ and } \frac{\partial Z}{\partial \lambda} = 0$ simultaneously we will get the CVM estimators.

IV. APPLICATION TO REAL DATASET

In this section, we demonstrate the applicability of the PIL distribution using a real dataset used by former researchers. We have taken 100 observations on breaking the stress of carbon fibers (in Gba) [21].
3.70, 2.74, 2.73, 2.50, 3.60, 3.11, 3.27, 2.87, 1.47, 3.11, 4.42, 2.41, 3.19, 3.22, 1.69, 3.28, 3.09, 1.87, 3.15, 4.90, 3.75, 2.43, 2.95, 2.97, 3.39, 2.96, 2.53, 2.67, 2.93, 3.22, 3.39, 2.81, 4.20, 3.33, 2.55, 3.31, 3.31, 2.85, 2.56, 3.56, 3.15, 2.35, 2.55, 2.59, 2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68, 2.97, 1.36, 0.98, 2.76, 4.91, 3.68, 1.84, 1.59, 3.19, 1.57, 0.81, 5.56, 1.73, 1.59, 2.00, 1.22, 1.12, 1.71, 2.17, 1.17, 5.08, 2.48, 1.18, 3.51, 2.17, 1.69, 1.25, 4.38, 1.84, 0.39, 3.68, 2.48, 0.85, 1.61, 2.79, 4.70, 2.03, 1.80, 1.57, 1.08, 2.03, 1.61, 2.12, 1.89, 2.88, 2.82, 2.05, 3.65

By employing the optim() function in R software [23] and [17], we have calculated the MLEs of PIL distribution by maximizing the likelihood function (3.1.1). We have obtained the value of Log-Likelihood is l = -141.7448. In Table 1 we have demonstrated the MLE's with their standard errors (SE) and 95% confidence interval for α, β, and λ.

Table 1
MLE and SE and 95% confidence interval for α, β, and λ of PIL

Parameter	MLE	SE	95% ACI
alpha	4.1507	0.6198	(2.9358, 5.3655)
beta	5.4091	1.3053	(2.8507, 7.9675)
theta	80.5762	2.7868	(75.1142, 86.0383)

The plots of profile log-likelihood function for the parameters α, β and λ have been displayed in Figure 2 and noticed that the ML estimates can be uniquely determined.

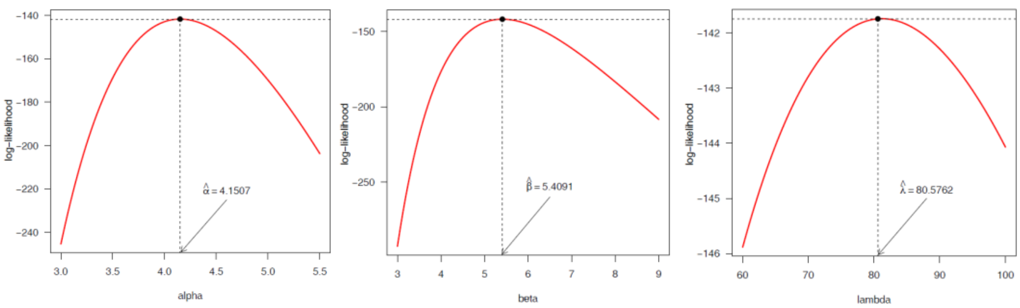

Figure 2. Graph of Profile log-likelihood function for the parameters α, β and λ.

In Figure 3 we have plotted the Q-Q plot and P-P plot and it is seen that the proposed distribution fits the data very well.

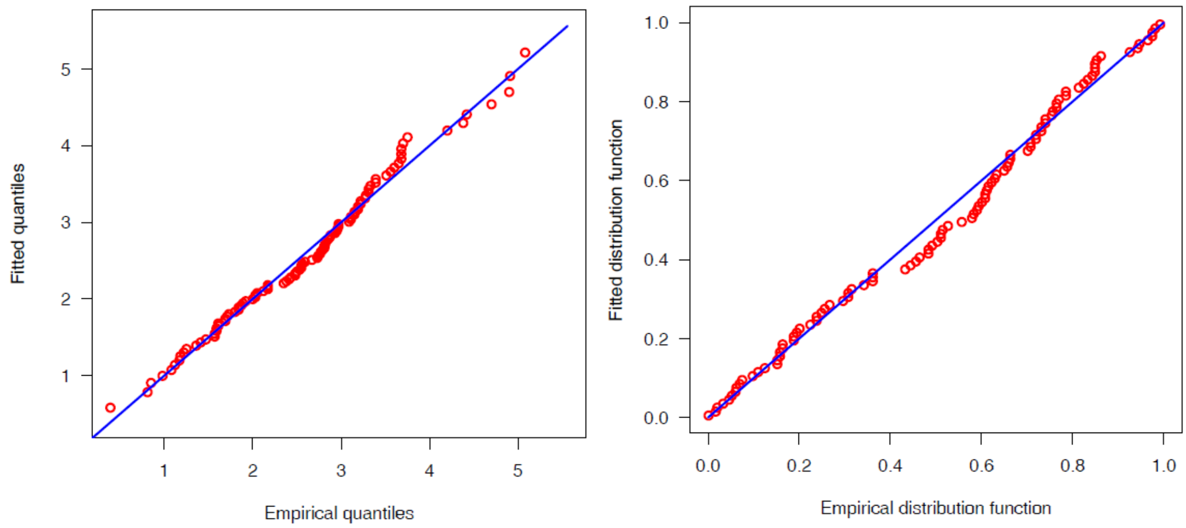

Figure 3. The Q-Q plot (left panel) and P-P plot (right panel) of the PIL distribution.

In Table 2 we have presented the estimated value of the parameters of PIL distribution using MLE, LSE and CVE method and their corresponding negative log-likelihood, and AIC criterion.

Table 2

Estimated parameters, log-likelihood, and AIC

Method of Estimation	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	**-LL**	**AIC**
MLE	4.1507	5.4091	80.5762	141.7448	289.4897
LSE	4.5765	4.5417	73.5387	141.9825	289.9650
CVE	4.7818	4.5785	92.7248	142.2249	290.4497

In Table 3 we have presented The KS, W and A^2 statistics with their corresponding p-value of MLE, LSE and CVE estimates.

Table 3

The KS, W and A^2 statistics with a p-value

Method of Estimation	***KS(p-value)***	***W(p-value)***	***A^2(p-value)***
MLE	0.0790(0.5608)	0.09718(0.6002)	0.5028(0.7435)
LSE	0.0642(0.8048)	0.0843(0.6683)	0.5323(0.7137)
CVE	0.0665(0.7677)	0.0899(0.6373)	0.6153(0.6332)

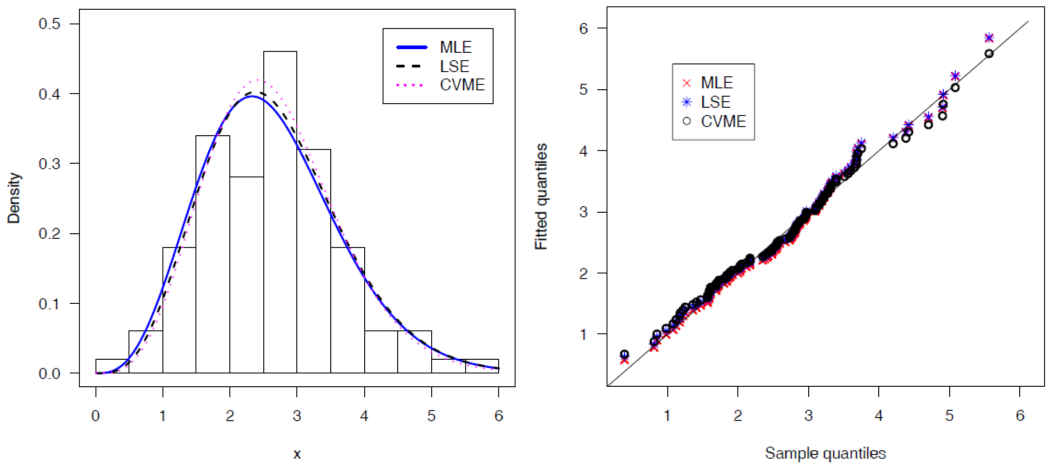

Figure 4. The Histogram and the density function of fitted distributions (left panel) and Q-Q plot (right panel) of estimation methods MLE, LSE and CVM.

In this section, we have presented the applicability of Poisson Gompertz distribution using a real dataset used by earlier researchers. To compare the potentiality of the proposed model, we have considered the following four distributions.

A. Exponential power (EP) distribution:

The probability density function Exponential power (EP) distribution [24] is

$$f_{EP}(x) = \alpha\, \lambda^{\alpha}\, x^{\alpha-1}\, e^{(\lambda x)^{\alpha}} \exp\left\{1 - e^{(\lambda x)^{\alpha}}\right\} \quad ; (\alpha, \lambda) > 0, \quad x \ge 0 .$$

where α and λ are the shape and scale parameters, respectively.

B. Lindley-Exponential (LE) distribution:

The probability density function of LE [5] can be expressed as

$$f_{LE}(x) = \lambda \left(\frac{\theta^{2}}{1+\theta}\right) e^{-\lambda x} \left(1 - e^{-\lambda x}\right)^{\theta-1} \left\{1 - \ln\left(1 - e^{-\lambda x}\right)\right\};\ \lambda, \theta > 0,\ x > 0$$

C. Gompertz distribution (GZ):

The probability density function of Gompertz distribution [20] with parameters α and θ is

$$f_{GZ}(x) = \theta\ e^{\alpha x}\ exp\left\{\frac{\theta}{\alpha}\left(1 - e^{\alpha x}\right)\right\} \quad ; x \ge 0, \theta > 0, -\infty < \alpha < \infty .$$

D. Generalized Exponential (GE) distribution

The probability density function of generalized exponential distribution [8]

$$f_{GE}(x;\alpha,\lambda) = \alpha\ \lambda\ e^{-\lambda x}\left\{1 - e^{-\lambda x}\right\}^{\alpha-1} ; (\alpha, \lambda) > 0, x > 0 .$$

For the assessment of the potentiality of the proposed model, we have calculated the Akaike information criterion (AIC), Bayesian information criterion (BIC), Corrected Akaike information criterion (CAIC), and Hannan-Quinn information criterion (HQIC) which are presented in Table 2.

Table 2

Log-likelihood (LL), AIC, BIC, CAIC and HQIC

Model	-LL	AIC	BIC	CAIC	HQIC
PIL	141.7448	289.4897	297.3052	289.7397	292.6528
LE	143.2473	290.4946	295.7049	290.6183	292.6033
EP	145.9589	295.9179	301.1282	296.0391	298.0266
GE	146.1823	296.3646	301.5749	296.4883	298.4733
GZ	149.1250	302.2500	307.4604	302.3737	304.3588

The Histogram and the density function of fitted distributions and Empirical distribution function with the estimated distribution function of PIL distribution and some selected distributions are presented in Figure 4.

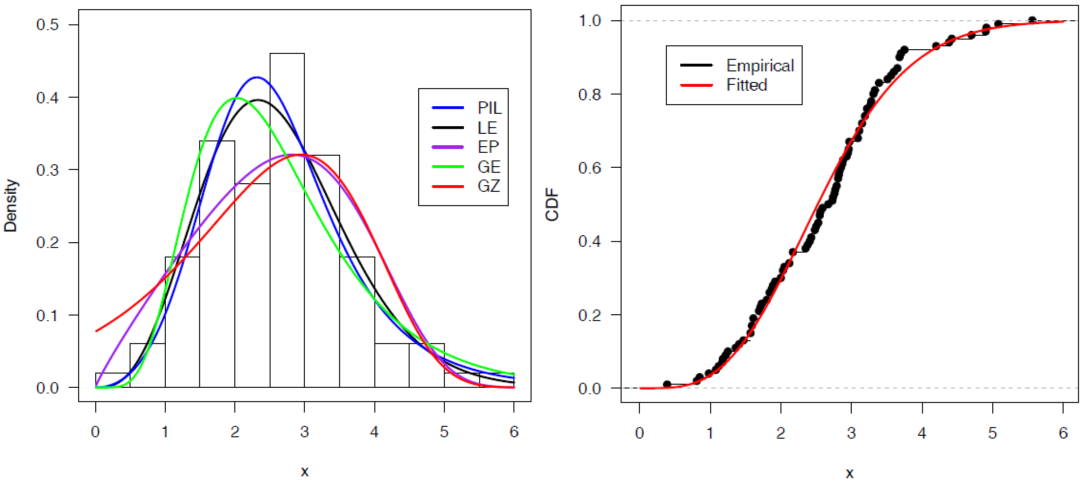

Figure 4. The Histogram and the density function of fitted distributions (left panel) and Empirical distribution function with estimated distribution function (right panel).

To compare the goodness-of-fit of the PIL distribution with other competing distributions, we have presented the value of Kolmogorov-Simnorov (KS), the Anderson-Darling (AD) and the Cramer-Von Mises (CVM) statistics in Table 3. It is observed that the PIL distribution has the minimum value of the test statistic and higher *p*-value thus we conclude that the PIL distribution gets quite better fit and more consistent and reliable results from others taken for comparison.

Table 3

The goodness-of-fit statistics and their corresponding p-value

Model	***KS(p-value)***	***AD(p-value)***	***CVM(p-value)***
PIL	0.0790(0.5608)	0.09718(0.6002)	0.5028(0.7435)
LE	0.0838(0.4836)	0.1225(0.4860)	0.7042(0.5549)
EP	0.0993(0.2771)	0.1861(0.2963)	1.3081(0.2297)
GE	0.1078(0.1959)	0.2293(0.2174)	1.2250(0.2581)
GZ	0.0962(0.3129)	0.2280(0.2193)	1.7537(0.1261)

V. CONCLUDING REMARKS

In this study, we have presented a new distribution called Poisson inverse Lomax distribution. A comprehensive study of some statistical and mathematical properties of the proposed distribution including the derivation of explicit expressions for its reliability function, survival function, hazard function, the quantile function and skewness and kurtosis. Three well-known estimation methods namely maximum likelihood estimation (MLE), least-square estimation (LSE), and Cramer-Von-Mises estimation (CVME) methods are used for the parameter estimation and we found that the MLEs are relatively good than LSE and CVM methods. The curves of the PDF of the proposed distribution have shown that its shape is increasing-decreasing and right skewed and flexible for modeling real-life data. Also, the graph of the hazard function is monotonically increasing or constant or reverse j-shaped according to the value of the model parameters. The applicability and suitability of the proposed distribution has been evaluated by considering a real-life dataset and the results exposed that the proposed distribution is much flexible as compared to some other fitted distributions.

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