

Comparing Metaheuristic Procedures to Solve the Single-Source Transportation Problems

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Abstract

In transport networks, it is generally required that all demand points must be supplied from a single source. The formulation of the model is presented and randomized versions of two well-known constructive heuristics are applied: On the one hand, the maximum regret method and on the other hand the maximum demand method. In addition, two other heuristic improvements are applied with local search techniques: “shift” and “swap” improvements. With all elements listed above, the GRASP method to solve single-source transportation problems is presented. However this work mainly provides simple tools for the optimization of the logistics (supply chains) of food and forestry products.

Keywords: Transportation Problems, Integer Programming, Heuristic Procedures, Combinatorial Optimization, Statistical Inference, Local Search.

Date of Submission: 23-06-2020

Date of acceptance: 11-07-2020

I. INTRODUCTION

In general, the transportation problem refers to the shipment of quantities, products, etc. from its origin to its destination. However, logistics must lead the way: First and foremost, to establish a transport model and also to be optimal. Consequently the modelling of the problem and its optimization are two essential subjects. The basic notions on necessary operational research can be found in [16].

In the formulation of a transportation model, m sources of a certain amount s_i of units supplies of a product, $i = 1, \dots, m$ are considered. In addition, we must consider the n destination points that have demands or orders of d_j units of that product. The cost of sending a unit of the product from origin i to destination j , what it is known as unit transport cost, is $c_{ij} > 0$, $i = 1, \dots, m$, $j = 1, \dots, n$.

It is necessary to limit transport costs, $j = 1, \dots, n$ and, obviously, the purpose is to minimize the total cost. It is normally assumed that the cost of transportation is directly proportional to the amount transported. Thus the problem can be handled as a linear programming problem. The data can be organized in a double entry table: supplies (s_i) vs. demands (d_j), the costs (c_{ij}) being the table coefficients.

Alternatively, a graph could be used since it allows not only a quick and intuitive but also qualitative visualization of transport routes, formulating it as a network optimization problem [3].

With regard to a transportation problem, a demand point could be supplied from a number of origins, but it might be quite expensive. Due to this reason, the condition that any demand point is supplied by a single origin can be imposed. We will call this condition the single-source constraint. Nevertheless, considering that it is necessary to analyze how much the total cost would increase. Therefore just knowing what the total cost is if the condition of single-source is required. With these purposes two formulations for the problem of single-source transportation (SSTP) are presented. In spite of its importance, this topic has not received a great deal of attention from the 70's ([1], [15] and [11]).

2 Model Formulation

2.1 First Model

As to demand point j , the variables that indicate how the demand is covered d_j are $z_{1j}; z_{2j}; \dots z_{mj}$, and the single-source constraint assumes that only one of these m variables can have a positive value in the optimal solution. This constitutes a special case of the logical constraint that we call "maximum number of variables", which is reduced to the fact that at most a single variable of m can be positive. Modelling this condition there are considered binary variables $y_{ij} \in \{0, 1\}$, $i = 1, \dots, m; j = 1, \dots, n$, $z_{ij} \leq u_{ij} \cdot y_{ij}$ and the logical constraint $\sum_{i=1}^m y_{ij} \leq 1, j = 1, \dots, n$. The upper bound can be taken as $u_{ij} = \min \{s_i, d_j\}$. The complete formulation is an example of **Mixed Integer Programming (MIP)**:

$$\begin{aligned}
 &\text{Minimize} && \sum_{i=1}^m \sum_{j=1}^n c_{ij} z_{ij} \\
 &\text{subject to} && \sum_{j=1}^n z_{ij} \leq s_i, \forall i = 1, \dots, m \\
 &&& \sum_{i=1}^m z_{ij} \geq d_j, \forall j = 1, \dots, n \\
 &&& z_{ij} \leq u_{ij} y_{ij}, \forall i = 1, \dots, m \forall j = 1, \dots, n \\
 &&& \sum_{i=1}^m y_{ij} \leq 1, \forall j = 1, \dots, n \\
 &&& z_{ij} \geq 0, \forall i = 1, \dots, m \forall j = 1, \dots, n \\
 &&& y_{ij} \in \{0, 1\}, \forall i = 1, \dots, m \forall j = 1, \dots, n
 \end{aligned}$$

2.2 Second Model

Variables are redefined as $y_{ij} = \frac{z_{ij}}{d_j}$. Those results in a transportation model equivalent to the original, however with variables $0 \leq y_{ij} \leq 1$ representing the fraction of the demand for point j covered or served from the source i . Formally, the model is as it follows:

$$\begin{aligned}
 &\text{Minimize} && \sum_{i=1}^m \sum_{j=1}^n c_{ij} d_j y_{ij} \\
 &\text{subject to} && \sum_{j=1}^n d_j y_{ij} \leq s_i, \forall i = 1, \dots, m \\
 &&& \sum_{i=1}^m y_{ij} = 1, \forall j = 1, \dots, n \\
 &&& 0 \leq y_{ij} \leq 1, \forall i = 1, \dots, m \forall j = 1, \dots, n
 \end{aligned}$$

Now the constraints $0 \leq y_{ij} \leq 1$ are replaced by $y_{ij} \in \{0, 1\}$ in agreement with the single-source constraint. Hence, the problem of single-source transportation can be modelled as a problem with all binary variables, providing an example of **Pure Binary Programming (PBP)**, which can be formulated this way:

$$\begin{aligned}
 &\text{Minimize} && \sum_{i=1}^m \sum_{j=1}^n c_{ij} d_j y_{ij} \\
 &\text{subject to} && \sum_{j=1}^n d_j y_{ij} \leq s_i, \forall i = 1, \dots, m \\
 &&& \sum_{i=1}^m y_{ij} = 1, \forall j = 1, \dots, n \\
 &&& y_{ij} \in \{0, 1\}, \forall i = 1, \dots, m \forall j = 1, \dots, n
 \end{aligned}$$

3 Bounds and Heuristic Procedures

As it is well known, these methods allow to solve complicated problems in an approximate way, providing feasible solutions that, although they do not optimize the objective function, it is sufficiently close to the optimum value by using a reasonable time.

While the original problem is difficult to solve, the relaxed problem may be easier to work out. The most common way to achieve a bound is the so-called linear relaxation where the $y_{ij} \in \{0, 1\}$ constraint is replaced by $0 \leq y_{ij} \leq 1$.

Another bound used in this work is the called Relax and Fix (see [18]). It consists in the relaxation of the objective function, previously establishing a tolerance (in our case $\text{tole} = 0.0001$ has been established) so that if $y_{ij} \geq 1 - \text{tole}$ then $y_{ij} = 1$ is fixed and it is enough to impose the condition that the lower bound of y_{ij} be 1 ($\text{setlb}(y_{ij}) := 1$). Similarly, if $y_{ij} \leq \text{tole}$ then set $y_{ij} = 0$, and it is enough to impose the condition that the upper bound of y_{ij} be 0 ($\text{setub}(y_{ij}) := 0$).

As for the heuristic procedures in the greedy method a feasible solution is built step by step. On the other hand, heuristic improvement methods start from a feasible solution to the problem and try to improve it. It is likely to combine both heuristics. For instance, a two-stage heuristic by using first a greedy method to obtain an initial solution, secondly try to improve it by a local search method.

Alternatively, the method can be both a single-step and multistart procedure. In the latter case, some type of randomization has to be used to obtain a different solution in each step. There are several ways to introduce randomness to the algorithm. One of the best known procedures use a RCL (Restricted Candidate List). This list consists of the elements candidates that offer the best values of the greedy criterion. The next candidate added to the solution is chosen randomly from the restricted list of candidates. Such a list may have a fixed number of elements (cardinality constraint).

In this work, randomized versions of two well-known constructive heuristics are applied: the maximum regret method and the maximum demand method. Both cover two interesting aspects of transport logistics with a single source, reinforcing the scope and applicability of our results to specific cases of interest to potential users. The greedy randomized code with cardinality-based RCL has been taken from [13] (Figure 2) and [14].

In the local search procedures Algorithm 1 and Algorithm 2, the environment or neighborhood of the feasible solution is explored through a basic operation called movement that, applied to the solution, provides the solutions of its environment. Concretely, the method known as VNS (Variable Neighborhood Search), can be found in [10]. In this work, two local search techniques were used: “shift” and “swap” improvements. In the first one, the movements consist of, given a solution, changing the assignment of a destination to a different origin. However in the second case the movements consist in the exchange of assignments to two destinations, Algorithm 3 and Algorithm 4.

In our case, only movements in the current solution environment are allowed to improve the value of the objective function. As a consequence, we have the so-called descent method that always ends at a local minimum; but a local optimum will not always be a global optimum. By starting with a given solution x , $k = 1$ is taken and repeated until $k = 2$, since two types of environment are considered. The environment is explored to find the best solution x' of the k -th environment $NK(x)$ and a decision is made to move or not. If the solution obtained x' is better than x , $x = x'$ and $k = 1$ are taken. Otherwise, $k = k + 1$ is taken. This scheme has been taken following [6].

In order to find better solutions than the ones we described before, the GRASP (Greedy Randomized Adaptive Search Procedures) were used where in each iteration of the multistart (randomized) method a local search procedure is applied to the greedy solution. This method incorporates the two main features that are required of a metaheuristic: diversification (by randomization and repetition) and intensification through local search. In each iteration one obtains a local minimum and, if randomization works correctly, multiple regions will be examined so that the best solution will be close to the global optimum. To be more precise, this paper presents hybrid algorithms that are based on constructive greedy heuristics with local search. First, the solution is built by a constructive heuristic. Second, the local search is used to improve that initial solution, and the last phase consists of updating the parameters that govern the process of construction in the first phase. The three phases are repeated until the stop criterion is reached.

The methodology described can be easily adapted to other combinatorial optimization problems, since it is in fact a particular case of the GAP (Generalized Assignment Problem) described in [8]. Also related problems are considered in [12]. Certainly the heuristics that have inspired this work has been taken from [7]. Moreover, other heuristics found in [2] [9] and [17] have been useful in this way.

4 Algorithm

This section includes main algorithms used, being M a sufficiently large quantity:

Algorithm 1 An algorithm for regret heuristic improvement using local search techniques

```

1: Initializations  $nasign=0$  ;  $zheur=0$  ;  $iter=0$  ;  $feas=1$ 
2: while ( $nasign < n$  and  $feas = 1$ ) do
3:    $iter=iter+1$ ;
4:   while ( $j$  in destinations /  $y(j)=0$ ) do
5:      $nk=0$ ;
6:     for ( $i$  in origins) do
7:       if ( $demand(j) \leq ofres(i)$ ) then
8:          $nk=nk+1$ ;
9:       end if
10:    end for
11:    if ( $nk = 0$ ) then
12:       $feas=0$ ;
13:    else
14:       $cmin1=M$ ;
15:      for ( $i$  in origins /  $demand(j) \leq ofres(i)$ ) do
16:        if ( $cost(i,j) < cmin1$ ) then
17:           $imin(j)=i$ ;  $cmin1=cost(i,j)$ ;
18:        end if
19:      end for
20:      if ( $nk = 1$ ) then

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21:     regret(j)=M;
22:   else
23:     cmin2=M;
24:     for (i in origins / demand(j)≤ ofres(i) and i≠imin(j)) do
25:       if (cost(i,j) < cmin2) then
26:         i2=i; cmin2=cost(i,j);
27:       end if
28:     end for
29:     regret(j)= cmin2 - cmin1;
30:   end if
31: end if
32: end while
33: if (feas =1) then
34:   nK=0;
35:   for (j in destinations / y(j)=0) do
36:     maxreg=-M;
37:     for (j in destinations / y(j)=0) do
38:       if (regret(j) > maxreg) then
39:         jmax=j; maxreg=regret(j);
40:       end if
41:     end for
42:     nK=nK+1; ind(nK)=jmax; mark(jmax)=1; y(jmax)=imin(jmax);
43:     zheur=zheur+cost(imin(jmax),jmax)*demand(jmax);
44:     ofres(imin(jmax))=ofres(imin(jmax))-demand(jmax); nasign=nasign+1;
45:   end for
46: end if
47: end while
48: if (feas = 1) then
49:   m1=swapimprovement; m2=shiftimprovement;
50: end if

```

Algorithm 2 An algorithm for maximum demand heuristic improvement using local search techniques

```

Initializations nasign=0 ;zheur=0 ;feas=1
2: while (nasign < n and feas = 1) do
   nK=0;
4:   dmax=-M;
   for (j in destinations/ y(j)=0) do
6:     if (demand(j) > dmax) then
       jmax=j ;
8:       dmax=demand(j) ;
     end if
10:   end for
   if (dmax > -M) then
12:     nK=nK+1;
     ind(nK)=jmax;
14:     mark(jmax)=1;
   end if
16:   cmin=M;
   nk=0;
18:   for (i in origins / demand(jmax)≤ ofres(i)) do
     if (cost(i,jmax)<cmin) then
20:       cmin=cost(i,jmax);
       omin=i;
22:       nk=nk+1;
     end if

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```

24: end for
    if (nk > 0) then
26:     y(jmax)=omin;
        zheur=zheur+cost(omin,jmax)*demand(jmax);
28:     ofres(omin)=ofres(omin)-demand(jmax);
        nasign=nasign+1;
30:     else
        feas=0;
32:     end if
    end while
34: if (feas = 1) then
    z1 = zheur;
36: m1=swapimprovement; m2=shiftimprovement;
    if (zheur < zimprove) then
38:     zimprove = zheur;
        yimprove = y;
40: end if
end if

```

Algorithm 3 An algorithm for shift function

```

zini=sum(j in destinations)demand(j)*cost(y(j),j);
final=0;
3: while (final=0) do
    maximprovement=-M;
    for (j in destinations) do
6:     i1=y(j);
        nk=0;
        for (i in origins / i ≠ i1 and demand(j)≤ ofres(i)) do
9:         nk=nk+1
            if (nk > 0) then
                cmin=M;
12:         for (i in origins / i ≠ i1 and demand(j)≤ ofres(i)) do
                    if (cost(i,j)<cmin) then
                        cmin=cost(i,j);
15:                     i2=i;
                    end if
                end for
18:         improvement=(cost(i1,j)-cost(i2,j))*demand(j);
                if (improvement > maximprovement) then
                    maximprovement=improvement;
21:                 jmax=j;
                    i1max=i1;
                    i2max=i2;
24:                 end if
            else
                improvement=-M;
27:             end if
        end for
    end for
30: if (maximprovement ≤ 0) then
    final=1;
    else
33:     y(jmax)=i2max;
        zheur=zheur+maximprovement;
        ofres(i1max)=ofres(i1max)+demand(jmax);
36:     ofres(i2max)=ofres(i2max)-demand(jmax);
        z3 = zheur;
    end if
39: end while
    if (zheur < zini) then
        returned=1;
42: else
        returned=0;
    end if

```

Algorithm 4 An algorithm for swap function

```

Initializations iter=0; final=0
zini=sum(j in destinations)demand(j)*cost(y(j),j);
while (final=0) do
4:   maximprovement=-M;
   iter=iter+1;
   for (j1,j2 in destinations / j1 ≠ j2) do
     i1=y(j1);i2=y(j2);
8:   if ofres(i1)-demand(j1)+demand(j2)≤0 and ofres(i2)-demand(j2)+demand(j1)≤0) then
     improvement=cost(i1,j1)+cost(i2,j2)-cost(i1,j2)-cost(i2,j1);
     if (improvement > maximprovement) then
       maximprovement=improvement;
12:    j1max=j1;
       j2max=j2;
     end if
   end if
16: end for
   if (maximprovement ≤ 0) then
     final=1;
     zheur=sum(j in destinations)demand(j)*cost(y(j),j);
20:    z2 = zheur;
   else
     i1=y(j1max);
     i2=y(j2max);
24:    y(j1max)=i2;
     y(j2max)=i1;
     ofres(i1)=ofres(i1)-demand(j1max)+demand(j2max);
     ofres(i2)=ofres(i2)-demand(j2max)+demand(j1max);
28:    zheur=sum(j in destinations)demand(j)*cost(y(j),j);
     z2 = zheur;
   end if
   end while
32: if (zheur < zini) then
     returned=1;
   else
     returned=0;
36: end if

```

5 Experiments and computational results

This section describes the computational experiments that were made so as to evaluate the proposed metaheuristics, the computational results and a comparison between the different methods.

For experiments, we have developed our own “dataset”, generating the following uniform probability distributions with the professional version of Xpress Mosel since the number of rows and columns exceeded the maximum allowed for the free license that is 5000. It is convenient to know the program interface and the basic optimization environment [5].

Uniform probability distributions have been considered for: Offers $\sim U(75, 95)$, Demands $\sim U(55, 85)$, Costs $\sim U(100, 100)$. The set of worked situations (different $m \times n$ scenarios) are collected in table 1 together with the integer optimal solution which was obtained for every one.

m x n	Optimal
40x40	295438
40x100	280369
40x150	273387
40x200	278303
60x100	430804
60x200	416723
100x100	708686
100x120	719392
100x200	719130

Table 1: Optimal solution for each scenario

The solution quality of each method was appraised by the percentage of variation with respect to the optimum, so we look for a small percentage gap to have a high quality solution. Also it is interesting to gather in Table 2 the percentage gap for bounds with all heuristic procedures.

m x n	Relaxed	Relax - Fix	VNS Regret	VNS Dem.	GRASP Regret	GRASP Dem.
40x40	1.17 %	1.4 %	1.8%	2.5 %	0.59 %	0.61 %
40x100	0.15 %	0.06 %	0.008%	0.11 %	0.0001 %	0.04 %
40x150	0.08 %	0.0001 %	0.0001%	0.16 %	0.0001 %	0.16 %
40x200	0.079 %	0.03 %	0.02%	0.05 %	0.005 %	0.0001 %
60x100	0.23 %	0.01 %	0.03%	0.33 %	0.03%	0.30 %
60x200	0.12%	0.02 %	0.001%	0.04 %	0.001 %	0.04 %
100x100	0.76 %	0.23 %	0.25%	1.09 %	0.26 %	0.74 %
100x120	0.21 %	0.07 %	0.057%	0.54 %	0.025 %	0.37 %
100x200	0.093%	0.058 %	0.02%	0.07 %	0.01 %	0.05 %
Average	0.321333%	0.208668 %	0.24289%	0.543333%	0.102336 %	0.256668 %

Table 2: Percentage gap for the different bounds and heuristic procedures

Moreover, computational time (in seconds) is reflected in Table 3 for each scenary and also its average values were calculated for the different methods that were used.

m x n	Relaxed	Relax - Fix	VNS Regret	VNS Dem.	GRASP Regret	GRASP Dem.
40x40	0.047	0.016	0.016	0.016	0.281	0.094
40x100	0.125	0.031	0.032	0.015	0.578	0.063
40x150	0.187	0.032	0.046	0.014	0.842	0.062
40x200	0.218	0.062	0.047	0.011	1.139	0.093
60x100	0.203	0.031	0.062	0.016	1.31	0.109
60x200	0.343	0.078	0.124	0.010	2.449	0.156
100x100	0.359	0.062	0.187	0.047	3.448	0.78
100x120	0.359	0.062	0.187	0.047	3.448	0.78
100x200	0.603	0.11	0.328	0.016	6.505	0.297
Average	0.278444	0.0555556	0.116111	0.0195556	2.27244	0.227111

Table 3: Computational times for the different bounds and heuristic procedures

The percentage gap versus the computational time is displayed in Figure 1. The average percentage gap versus the average computational time for bounds and heuristic procedures is displayed in Figure 2.

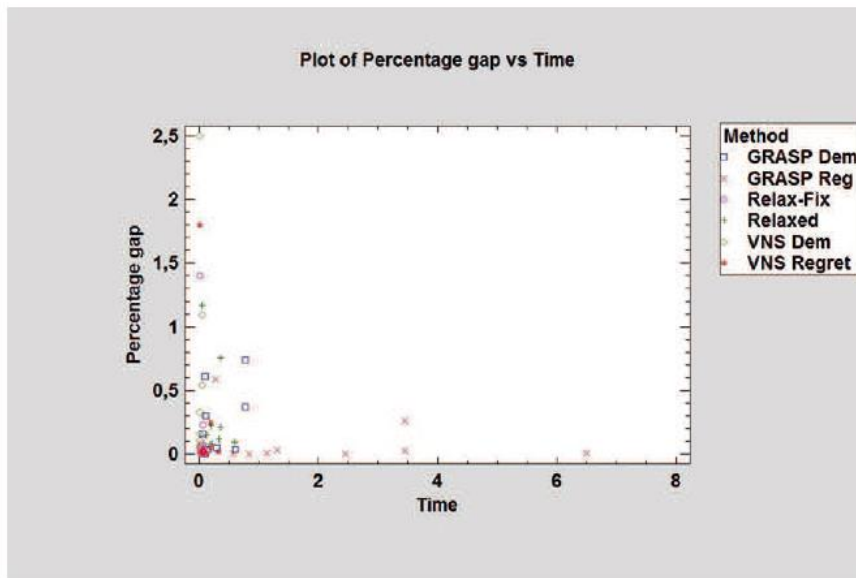


Figure 1: Percentage gap vs. Computational time

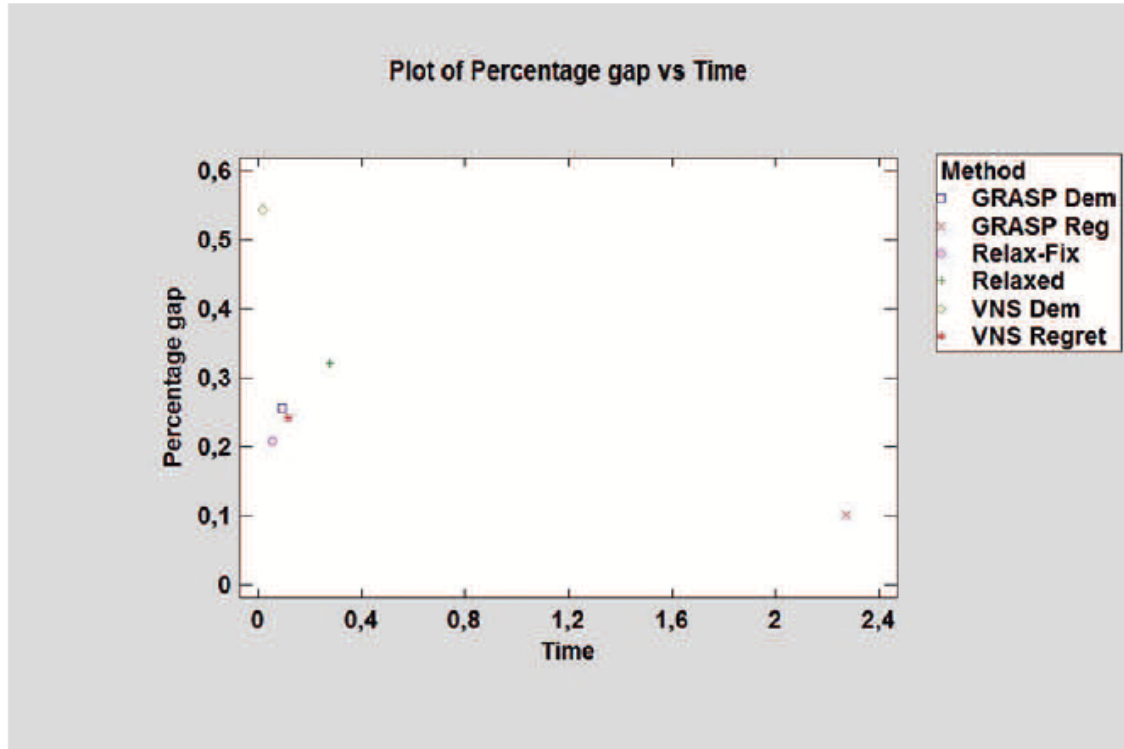


Figure 2: Average percentage gap vs. Average computational time

Besides, it is interesting to compare the average percentage gap versus the average computational time only for heuristic procedures as it is shown in Figure 3.

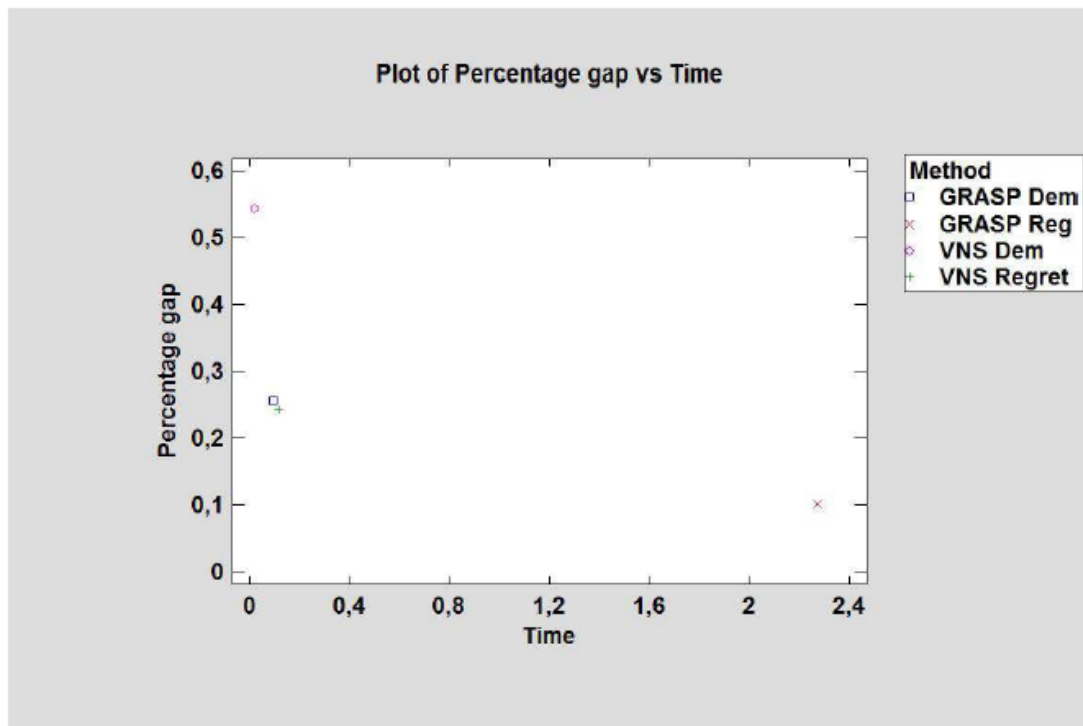


Figure 3: Heuristic procedures: Average percentage gap vs. Average computational time

From the statistical point of view, it appears to be relevant a comparative study of both percentage gap and time for the different methods. For that purpose the multifactor analysis of variance, ANOVA (4 Chapter 5) was used dealing with the data given in tables 2 and 3. In our case, the null hypothesis H_0 states that the average values of the six methods are the same and as an alternative hypothesis H_1 states that the average values of the six methods are different. In this analysis, we considered the confidence level at 95 % and therefore the nominal level $\alpha = 0.05$. Tables 4 and 5 summarize the ANOVAs of the variables quality and time, respectively.

	SS	Df	F	p-value
Method	0.9866	5	0.8082	0.5496
Residuals	11.7197	48		

Table 4: Analysis of Variance for percentage gap

Taking into account the result obtained (Table 4), the quality variable shows a p-value greater than the nominal level $\alpha = 0.05$, therefore the null hypothesis of equality between the six averages is not rejected and we cannot affirm that there is some difference between them. The six methods can be considered equivalent in terms of quality.

	SS	Df	F	p-value
Method	34.566	5	9.805	0.000001678 ***
Residuals	33.844	48		

Table 5: Analysis of Variance for time

The time p-value is small enough (as indicated by the lower significance coded with the asterisks in Table 5, the p-values are < 0.05) to reject the null hypothesis of equality between the six averages and there is some difference between different methods. The next step is to determine that difference. For this, the Tukey multiple comparison was used. The confidence level has also been established at 95 % and therefore the nominal level $\alpha = 0.05$.

The results which were obtained are given in Table 6.

Methods	Difference	Lower bound	Upper bound	p-value
GRASP Reg-GRASP Dem	2.04533333	0.8705436	3.2201231	0.0000643
Relaxed-GRASP Dem	0.05133333	-1.1234564	1.2261231	0.9999945
Relax Fix-GRASP Dem	-0.17155556	-1.3463453	1.0032342	0.9979388
VNS Dem-GRASP Dem	-0.20755556	-1.3823453	0.9672342	0.9949288
VNS Regret-GRASP Dem	-0.11100000	-1.2857897	1.0637897	0.9997503
Relaxed-GRASP Reg	-1.99400000	-3.1687897	-0.8192103	0.0000996
Relax Fix-GRASP Reg	-2.21688889	-3.3916786	-1.0420991	0.0000146
VNS Dem-GRASP Reg	-2.25288889	-3.4276786	-1.0780991	0.0000106
VNS Regret-GRASP Reg	-2.15633333	-3.3311231	-0.9815436	0.0000247
Relax Fix-Relaxed	-0.22288889	-1.3976786	0.9519009	0.9929369
VNS Dem-Relaxed	-0.25888889	-1.4336786	0.9159009	0.9860127
VNS Pen-Relaxed	-0.16233333	-1.3371231	1.0124564	0.9984177
VNS Dem-Relax Fix	-0.03600000	-1.2107897	1.1387897	0.9999991
VNS Reg-Relax Fix	0.06055556	-1.1142342	1.2353453	0.9999875
VNS Reg-VNS Dem	0.09655556	-1.0782342	1.2713453	0.9998742

Table 6: The Tukey multiple comparison test for pairwise time averages among the different methods

The Tukey multiple comparison of means allows us to state that the differences between the times of the GRASP method based on regret are clearly significatives with respect to the other methods since the p-values are the smallest ones.

6 Final conclusions

The main contribution of this work is the application of the GRASP heuristic for the single-source transport problem based on the maximum regret criterion or the maximum demand criterion. The computational experiment showed that *the GRASP method based on the maximum regret criterion obtains the best results*, even in problems considered as NP-hard within reasonable times. From the results in the comparison of the methods considered we can conclude that the worst option is the VNS method based on maximum demand in terms of quality, taking into account the percentage gap of variation with respect to the optimal value.

As for the bounds, the relaxed problem and the Relax-Fix get quick solutions, providing better solution in terms of quality the Relax-Fix. For quick solutions with heuristic methods, the VNS method based on maximum regret and the GRASP method based on the maximum demand criteria are also fast methods.

It is interesting to highlight the efficiency of the heuristic procedures that were used, both GRASP and VNS, based on maximum regret against the maximum demand criteria. The results favorably compare the GRASP method based on the maximum regret criteria in terms of cpu times and quality of the solution.

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