

Influence of the modified 60/70 bituminous concrete on the vehicle-pavement interaction with contact based on Lagrange multipliers

Jean Médard Ofamo Bajogueno¹; Madjadoumbaye Jeremie²; Moussa Sali³.

¹Department of Mechanical Engineering, Post Graduate School for Pure and Applied Science / University of Douala, Cameroon BP: 2701 Douala.

²Department of Civil Engineering, National Advanced School of Engineering / University of Yaoundé I, Cameroon 8390 Yaoundé-Cameroun.

³Department of Mechanical Engineering, Post Graduate School for Pure and Applied Science / University of Douala, Cameroon BP: 2701 Douala.

Abstract

The waste generation from used tires is growing and leading to environmental degradation with the increasing production of vehicle numbers especially in cities. The cost of bitumen has not been very affordable due to its extensive use in flexible pavement construction and the ongoing growth of its manufacturing businesses. Then, determining the possibility of partially substituting the standard 60 / 70 bitumen by using tire powder content can have a bright future if possible. However, it is essential to study to evaluate the modified bitumen concrete considering the vehicle-pavement interactions by taking into account the contact between the vehicle and the pavement. Therefore, this study evaluates the influence of the tire powder content on the vehicle-pavement system by introducing the localized Lagrange multiplier for describing the behavior of the contact points between the vehicle and the pavement. Experimental and numerical studies have operated. The results show the accuracy of the effectiveness of the proposed model. The riding comfort of the vehicle has a good performance score since the vehicle's vertical acceleration is lower by 20% and 25% of powder content than the standard 60 / 70 bituminous concrete. The absolute error between the standard 60/70 and 25% or 20% of the powder content is around 0.06 m/s^2 for the vertical displacement of the pavement.

Keywords: Standard 60 / 70 bitumen; tire powder content; vehicle-pavement system; localized Lagrange multiplier; contact points.

Date of Submission: 25-09-2026

Date of acceptance: 04-10-2026

I. INTRODUCTION

Around the world, bituminous materials are extensively utilized in various construction applications, such as asphalt for highways and roof waterproofing goods. The main component of all bituminous compounds is bitumen, which has a viscosity that ranges from liquid to solid and has strong adhesive qualities. Bitumen has been used as a binder for many years. Despite its effectiveness, bitumen may only provide a brief pavement lifespan because of fatigue loading, etc. [5], and it has also been expensive up until now. However, the disposal of used automobile tires brought on by increased traffic in emerging cities is a significant environmental problem. Several waste tires are disposed of in urban and rural areas, seriously harming the environment and the economy. Studies conducted in the last few decades have suggested using tire rubber to partially replace aggregates in flexible pavement as a sustainable way to improve the environment [6]. Since the quantity of used rubber tires was growing and a sizable fraction was being disposed of inappropriately, using them locally would have several advantages. Identified and run landfills, which have a negative influence on the environment and human health since they are not biodegradable. Therefore, this study proposes to investigate vehicle-pavement interaction based on the modified bitumen.

In comparison to 60/70 grade bitumen, the addition of waste plastic and waste rubber tires in flexible highway pavements may increase the mix's density and Marshall stability value [1]. Lo Presti [2] reviewed the research on modified bitumen made from recycled tire rubber for use in road asphalt mixtures; he disputed several of the benefits found, noted some technical difficulties, and made some recommendations on how to make it better. Magar [3] looked at the modification of bitumen by adding 15% by weight of different-sized crumb rubber. Incorporating recycled waste rubber and waste plastic into asphalt binder enhanced its mechanical and workability

qualities while also improving environmental protection [4]. However, the aforementioned studies focus on pavement analysis and neglect the vehicle-pavement interactions.

Existing research on pavement dynamics idealizes the vehicle as a moving force acting on the pavement and the pavement as a plate or beam resting on an elastic or viscoelastic basis [7], [8]. The two most used foundation models [9] are Winkler and Pasternak, in which the soil is modelled as closely spaced linear springs. The Pasternak foundation was discovered to be more mathematically straightforward and more in line with physical reality [10]. The vibration of Euler-Bernoulli beams sitting on foundations and subjected to concentrated movement loads has been extensively studied [11], [12]. Huang and Thambiratnam [8] examined the dynamic response of a thin plate on a Winkler foundation under a shifting load. They found that the foundation's frequency, velocity, and stiffness all substantially impacted the plate's dynamic response. In their boundary element technique for the nonlinear dynamic response of shear deformable beams resting on nonlinear foundations and Timoshenko beam columns partially supported on foundations, Sapountzakis and Kampitsis [18], [19] discovered that shear deformation has a substantial impact. However, the influence of vehicle vibration on the pavement response is rarely considered in pavement dynamics studies.

In vehicle dynamics research, the vehicle is frequently idealized as a multibody dynamics model (MDM) or lumped parameter model (LPM). The vehicle receives its only source of stimulation from the uneven pavement. The impact of tire characteristics and water on the pavement under dynamic loading is something that Saeed et al. [15] want to investigate. Krishnanunni and Rao [16] proposed a numerical method that discretizes the differential equation and calculates the dynamic response by combining the Galerkin, finite difference, and Newmark's methods. Ma and Wang [17] investigated how vehicle factors impacted tire pressure and road damage; there is an ideal damping ratio. Ignoring the tire damping will result in a design fault. Zhang et al. [20] used a half-vehicle model to examine the impact of random road roughness on vehicle dynamic load without considering pavement dynamics. A precise, cost-effective, and efficient approach for calculating vehicle characteristics and road roughness was established using field experiments' measured vehicle reactions. The papers above introduced the tire-pavement contact force but the contact procedure is not well understood. Moreover, the vehicle and the pavement are usually coupled through the contact area.

Two numerical methods arise for coupling the vehicle and the pavement named vehicle-pavement coupled method and Iterative algorithms. The first paradigm is based on time-step integration [13], [14]. In this scenario, the moving (tire-pavement) contact forces that function as (internal) loads in the global (coupled) vehicle-bridge system are the source of the time dependence. The global system matrices must therefore be factorized and updated at every time step, which requires a significant amount of computing power. The second paradigm rather uses an iterative process at each time step that ends when a convergence criterion is met, for as when a compatibility condition falls inside a given tolerance [21], [22], [23]. The localized version of the Lagrange multipliers method [24] is an alternate strategy that might combine the benefits of iterative methods [23] and coupled approaches [25], [26]. In contrast to the traditional Lagrange multiplier method, it assigns two sets of Lagrange multipliers and states two sets of kinematic constraints between the auxiliary point and the other contact points by introducing an extra artificial auxiliary point at the contact interface. This is what makes it novel in the vehicle-pavement interaction.

Despite these studies mentioned above, no one has given attention to the evaluation of the vehicle-pavement interactions on the pavement modified by adding tire powder and considering a localized Lagrange multiplier method. The present study proposes a numerical approach for solving such a complex model. Therefore, to achieve this goal, this study is partitioned into six modules summarized as: (1) section two presents the mathematical models of the vehicle, pavement, and vehicle-pavement interactions; (2) section three describes the proposed solution; (3) section four introduces the case study and the validation approach; (4) section five presents the results; (5) section six concludes the study.

II. MATHEMATICAL MODELS

2.1 Mathematical model of the vehicle

The numerical model of the vehicle can be described as a two-mass oscillator presenting one degree of freedom running with velocity v as illustrated in Fig. 1. The lower mass m_2 is connected with the ground over a contact point. The upper mass m_1 is attached to the lower mass m_2 through the spring k and dashpot C . The governing equation of motion describing the two-mass oscillator is expressed as follows:

$$\begin{cases} m_1 \frac{d^2 z_1}{dt^2} + k(z_1 - z_2) + c\left(\frac{dz_1}{dt} - \frac{dz_2}{dt}\right) = 0 \\ m_2 \frac{d^2 z_2}{dt^2} + k(z_2 - z_1) + c\left(\frac{dz_2}{dt} - \frac{dz_1}{dt}\right) = F_0 + m_1 g + m_2 g \end{cases} \quad (1)$$

in which F_0 denotes the vehicle-pavement interaction force, which is the link between the vehicle and pavement equation; t represents the time; z_2 can be expressed as follows:

$$z_2(t) = V_d(x, t)|_{x=vt} + z_p(x)|_{x=vt} \quad (2)$$

in which $V_d(x, t)|_{x=vt}$ and $z_p(x)|_{x=vt}$ defines the vertical displacement and the pavement roughness, respectively. Eq (1) can be rewritten as follows:

$$m_1 \frac{d^2 z_1}{dt^2} + c \frac{dz_1}{dt} + kz_1 = kz_2 + c \frac{dz_2}{dt} \quad (3)$$

$$F_0 = K_{eff}^v v^v - q^v \lambda^v \quad (4)$$

in which the effective stiffness of the vehicle can be expressed as follows:

$$K_{eff}^v = M^v \frac{\partial^2}{\partial t^2} + C^v \frac{\partial}{\partial t} + K^v \quad (5)$$

$$q^v = [q_N^v \ q_T^v], \lambda^v = [\lambda_N^v \ \lambda_T^v] \quad (6)$$

in which k_{eff}^v , q^v , v^v , and λ^v represent the effective stiffness of the vehicle, directional matrix, displacement vector of the vehicle, and contact force vector, respectively. The subscripts N and T describe the normal and tangential direction of the contact, respectively. Moreover, M^v , C^v , and K^v refer to the matrices associated with the mass, damping, and stiffness of the vehicle.

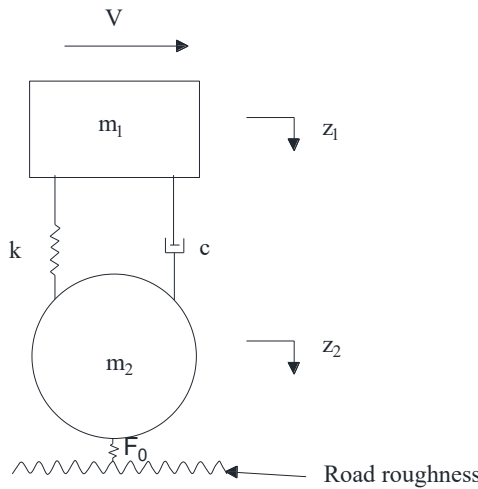


Figure 1. Vehicle-pavement interaction

2.2 Mathematical model of the pavement-foundation system

2.2.1 Pavement-foundation system

This section describes the pavement-foundation system through an Euler-Bernoulli beam carried by a Pasternak foundation. The governing equation of a single force running with a constant speed along the finite beam carried by a linear visco-elastic foundation is expressed as follows:

$$EI \frac{\partial^4 z_p}{\partial x^4} - k_p \frac{\partial^2 z_p}{\partial x^2} + m \frac{\partial^2 z_p}{\partial t^2} + c \frac{\partial z_p}{\partial t} = F \delta(x - x_t) \quad (7)$$

in which z_p denotes the transverse deflection of the beam corresponding to any position; EI defines the flexural rigidity of the beam; E refers to the Young's modulus of elasticity associated with the beam material; I denotes the second moment of area of the beam cross-section; m represents the mass per unit length of the beam; k_p denotes the shear layer parameter; F represents the total force acting on the pavement; $\delta(x - x_t)$ refers to the

Dirac-delta function introduced to manage the moving load; x_t defines the load position on the beam; k and c denote the soil stiffness and damping coefficients per unit length of the beam.

2.2.2 Galerkin method

Galerkin method, which is a kind of weighted residual approach, is introduced in this section to estimate the solution of Eq. (6). Hence, the solution corresponding to transverse deflection associated with a simply supported Euler-Bernoulli beam can be expressed as follows:

$$\tilde{z}_p(x, t) = \sum_{i=1}^n u_i(t) \Theta_i(x), \quad (i = 1, 2, \dots, n) \quad (8)$$

in which $u_i(t)$ represent a set of time-dependent coefficients that must be estimated; $\Theta_i(x)$ consists of admissible trial space functions, basic, in which all boundary conditions must be satisfied. $\Theta_i(x)$ can be expressed in the following form:

$$\Theta_i(x) = \sin\left(\frac{i\pi}{L}x\right) \quad (9)$$

The weighted residual approximation (Galerkin method) can be mathematically written in the following form:

$$\int_0^L R(x) w_j(x) dx = 0 \quad (10)$$

in which $R(x)$ and $w_j(x)$ denote the residual and weighting functions, respectively, expressed as follows:

$$R(x) = EI \frac{\partial^4 \tilde{z}_p}{\partial x^4} + k_p \frac{\partial^2 \tilde{z}_p}{\partial x^2} + m \frac{\partial^2 \tilde{z}_p}{\partial t^2} + c \frac{\partial \tilde{z}_p}{\partial t} + k \tilde{z}_p - F \delta(x - x_t) \quad (11)$$

$$w_j(x) = \Theta_j(x) = \sin\left(\frac{j\pi}{L}x\right), \quad (j = 1, 2, \dots, n) \quad (12)$$

Eq. (8) should be substituted into Eq. (10) and Eqs. (11) and (12) should be substituted into Eq. (10). The following form is obtained over integration by parts using appropriate boundary conditions:

$$\int_0^L \left[EI \frac{\partial^2 \Theta_j}{\partial x^2} \sum_{i=1}^n \frac{\partial^2 \Theta_i}{\partial x^2} u_i + k_p \frac{\partial \Theta_j}{\partial x} \sum_{i=1}^n \frac{\partial \Theta_i}{\partial x} u_i + m \Theta_j \sum_{i=1}^n \Theta_i \ddot{u}_i + c \Theta_j \sum_{i=1}^n \Theta_i \dot{u}_i + k \Theta_j \sum_{i=1}^n \Theta_i u_i - \delta(x - x_t) \Theta_j \right] dx = 0 \quad (13)$$

Assuming orthogonality of the selected trial function and by integrating Eq. (13), the ordinary differential equations related to pavement foundation can be expressed as follows:

$$\ddot{u}_i + 2\zeta_i \omega_i \dot{u}_i + \omega_i^2 u_i = F \varphi_i \quad (14)$$

$$2\zeta_i \omega_i = \frac{c}{m} \quad (15)$$

$$\omega_i^2 = \left[EI \left(\frac{i\pi}{L}\right)^4 + k_p \left(\frac{i\pi}{L}\right)^2 + k \right] \frac{1}{m} \quad (16)$$

$$\varphi_i(x_t) = \frac{2}{mL} \sin\left(\frac{i\pi}{L}x_t\right) \quad (17)$$

2.3 Road roughness

The existing literature on vehicle-pavement coupled systems assumes that the harmonic roughness profile is indicated [27], [28]. However, it must be realistically considered that the road roughness is the primary source of excitation for the coupled vehicle-pavement system. This paper randomly generates road roughness profiles for A-class and B-class roads using the power spectral density of road roughness defined by ISO 8608 [29]. Fig.2 illustrates the road roughness. The power spectral density for a road of length L can be written as follows:

$$Z_p(x) = \left(\sum_{i=1}^N \sqrt{2F(i\Delta n) \Delta n} \cos(2\pi i \Delta n x + \theta_i) \right) H_p(x) \quad (18)$$

in which $Z_p(x)$ represent the roughness profile; $H_p(x)$ can be defined as a slowly varying modulation function. The set of randomly generated phase angles with a uniform distribution between 0 and 2π is known as θ_i ; $F(i\Delta n) = G_d(n_0)(i\Delta n/n_0)^{-2}$; $\Delta n = 1/L$; N represents the number of harmonics; $G_d(n_0)$ is selected from ISO 8608 according to the road class; n_0 refers to the spatial frequency.; $H_p(x)$ can be expressed as follows:

$$H_p(x) = \begin{cases} 1, & 0 \leq x \leq L \\ 0, & \text{otherwise} \end{cases} \quad (19)$$

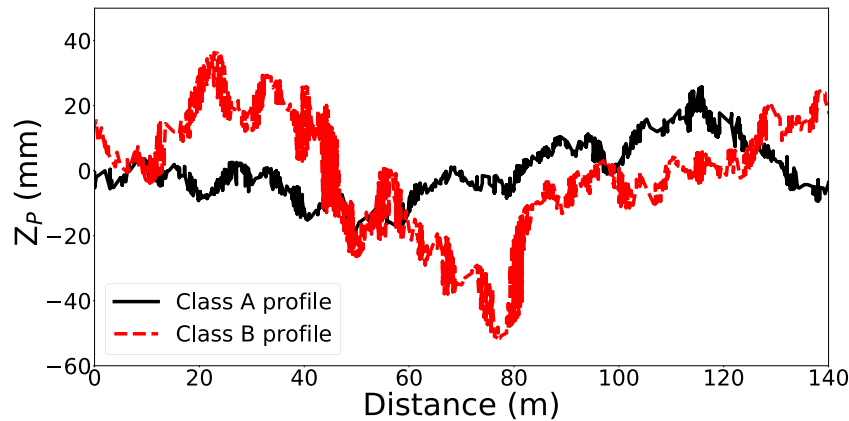


Figure 2. Road roughness

2.4 Modelling of the vehicle-pavement contact

2.4.1 Lagrange multiplier approximation

In this section, the vehicle-pavement interaction is solved by introducing the equation of motion of the vehicle running along the Euler-Bernoulli beam. This study defines the contact between the tire and the asphalt layer as rigid, i.e., the tire and asphalt layer are not deformed and remain in contact during the simulation. Auxiliary contact points between the tire and beam are introduced and considered to describe the sets of kinematic constraints and two sets of contact forces (Lagrange multiplier) λ^v and λ^b , respectively, assigned to the vehicle and the beam models. There is no separation between the tire and beam once the boundary displacements are equivalent to the global boundary displacement u^g . According to Fig. 2, this condition can be defined as a set of kinematic constraints as follows:

$$(q^v)^T u^v + \xi = u^g \quad (20)$$

$$(q^b)^T u^b = u^g \quad (21)$$

in which ξ denotes the vector related to the vertical and lateral irregularities.

$$\xi = [\xi_N(x) \quad \xi_T(x)]^T \quad (22)$$

The spectral representation is introduced to describe the irregularities. The contact between the tire and the beam remains closed. According to Eqs. (20) and (21), the following expression can be written as follows, considering the global boundary acceleration $\ddot{u}^g$, the vehicle and beam subsystems.

$$\frac{\partial^2}{\partial t^2} \left((q^v)^T u^v + \xi \right) = (q^v)^T \frac{\partial^2}{\partial t^2} u^v + \kappa^2 \xi'' = \ddot{u}^g \quad (23)$$

$$\frac{\partial^2}{\partial t^2} \left((q^b(x))^T u^b \right) = \left[(q^b)^T \frac{\partial^2}{\partial t^2} 2v(q'^b)^T \frac{\partial}{\partial t} + v^2 (q''^b)^T \right] u^b = \ddot{u}^g \quad (24)$$

in which $(\quad)'$ refers to the differentiation considering the location of tire x . The sets related to the kinematic constraints on the acceleration can be expressed as follows:

$$\left(q_{eff}^v\right)^T u^v + v \xi^v = E^v \ddot{u}^g \quad (25)$$

$$\left(q_{eff}^B\right)^T u^B = E^B \ddot{u}^g \quad (26)$$

Newton's third law is used to define and connect the two sets of contact forces (Lagrange multipliers) as follows:

$$\left(E^v\right)^T \lambda^v = \left(E^B\right)^T \lambda^B \quad (27)$$

2.4.2 Explicit time step integration

The Newmark-beta integration is introduced to solve the dynamic system. Hence, the velocity and the acceleration corresponding to the vehicle and the Euler-Bernoulli beam at the next time step $i+1$ can be expressed as follows:

$$\begin{cases} \frac{\partial}{\partial t} u^{(v/B)i+1} = \dot{u}^{(v/B)i+1} = a_1 u^{(v/B)i+1} - \bar{u}^{(v/B)i} \\ \frac{\partial^2}{\partial t^2} u^{(v/B)i+1} = \ddot{u}^{(v/B)i+1} = a_0 u^{(v/B)i+1} - \bar{\ddot{u}}^{(v/B)i} \end{cases} \quad (28)$$

in which $\bar{u}^{(V/B)i}$ and $\bar{\ddot{u}}^{(V/B)i}$ represent the equivalent velocity and acceleration, respectively. The subscript v refer to B the vehicle and bridge subsystem at the current time step i .

$$\begin{cases} \bar{u}^{(V/B)i} = a_1 u^{(V/B)i} + a_4 \dot{u}^{(V/B)i} + a_3 \ddot{u}^{(V/B)i} \\ \bar{\ddot{u}}^{(V/B)i} = a_0 u^{(V/B)i} + a_2 \dot{u}^{(V/B)i} + a_3 \ddot{u}^{(V/B)i} \end{cases} \quad (29)$$

in which the parameters to refer to the Newmark-beta coefficients [30]. a_0 a_5

$$\begin{cases} a_0 = \frac{1}{\beta \Delta t^2}, a_1 = \frac{\gamma}{\beta \Delta t}, a_2 = \frac{1}{\beta \Delta t}, \\ a_3 = \frac{1}{2\beta} - 1, a_4 = \frac{\gamma}{\beta} - 1, a_5 = \frac{\Delta t}{2} \left(\frac{\gamma}{\beta} - 2 \right) \end{cases} \quad (30)$$

In this simulation, the Newmark-beta parameters are $\gamma = 0.5$ and $\gamma = 0.25$. The equations containing the products $K_{eff}^v u^{(v)i+1}$ and $K_{eff}^B u^{(B)i+1}$ at time step $i+1$ can be written as follows:

$$\begin{cases} \left(M^v \frac{\partial^2}{\partial t^2} + C^v \frac{\partial}{\partial t} + K^v \right) u^{(v)i+1} = \bar{K}^v u^{(v)i+1} - C^v \bar{u}^{(v)i} - M^{(v)} \bar{\ddot{u}}^{(v)i} \\ \left(M^B \frac{\partial^2}{\partial t^2} + C^B \frac{\partial}{\partial t} + K^B \right) u^{(B)i+1} = \bar{K}^B u^{(B)i+1} - C^B \bar{u}^{(B)i} - M^{(B)} \bar{\ddot{u}}^{(B)i} \end{cases} \quad (31)$$

in which $\bar{K}^v$ and $\bar{K}^B$ denote the equivalent stiffness matrices all the integration long.

$$\begin{cases} \bar{K}^v = a_0 M^v + a_1 C^v + K^v \\ \bar{K}^B = a_0 M^B + a_1 C^B + K^B \end{cases} \quad (32)$$

in Eq. (31), the products $C^v \bar{u}^{(v)i}$ and $M^{(v)} \bar{\ddot{u}}^{(v)i}$, $C^B \bar{u}^{(B)i}$ and $M^{(B)} \bar{\ddot{u}}^{(B)i}$ consist of the additional force vector released during the integration. Since these products correspond to the current time step i , they can be known for the corresponding time step $i+1$. Therefore, the force vectors at the time step $i+1$ can be expressed as follows:

$$\begin{cases} \bar{F}^{(v)i+1} = F^v + C^v \bar{u}^{(v)i} + M^{(v)} \bar{\ddot{u}}^{(v)i} \\ \bar{F}^{(B)i+1} = F^B + C^B \bar{u}^{(B)i} + M^{(B)} \bar{\ddot{u}}^{(B)i} \end{cases} \quad (33)$$

$$\begin{bmatrix} u^v \\ u^B \end{bmatrix}^{i+1} = \begin{bmatrix} (\bar{K}^v)^{-1} & 0 \\ 0 & (\bar{K}^B)^{-1} \end{bmatrix} \left(\begin{bmatrix} \bar{F}^v \\ \bar{F}^B \end{bmatrix}^{i+1} - \begin{bmatrix} -q^v & 0 & 0 \\ 0 & q^{(B)i+1} & 0 \end{bmatrix} \begin{bmatrix} \lambda^v \\ \lambda^B \\ \ddot{u}^g \end{bmatrix} \right) \quad (34)$$

In addition, the products corresponding to $(q_{eff}^v)^T u^{(v)i+1}$ and $(q_{eff}^B)^T u^{(B)i+1}$ of Eqs. (25) and (26) at the time step $i+1$ can be written as follows:

$$\begin{cases} (q_{eff}^v)^T u^{(v)} = (\bar{q}^v)^T u^{(v)i+1} - (q^v)^T \ddot{u}^{(v)i} \\ (q_{eff}^B)^T u^{(B)} = (\bar{q}^B)^T u^{(B)i+1} - (q^B)^T \ddot{u}^{(B)i} - 2v(q'^B)^T \ddot{u}^{(B)i} \end{cases} \quad (35)$$

in which $\bar{q}^v$ and $\bar{q}^B$ represent the equivalent contact direction matrices all the entire integration length of the system. Their equations can be expressed as follows:

$$\begin{cases} \bar{q}^v = a_0 q^v \\ \bar{q}^B = a_0 q^B + a_1 2v q'^B + v^2 q''^B \end{cases} \quad (36)$$

Finally, the following expression can be written:

$$\begin{bmatrix} \lambda^v \\ \lambda^B \\ \ddot{u}^g \end{bmatrix} = \begin{bmatrix} (\bar{q}^v)^T (\bar{K}^v)^{-1} q^v & 0 & E^v \\ 0 & (\bar{q}^B)^T (\bar{K}^B)^{-1} q^B & E^B \\ (-E^v)^T & (-E^B)^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} (-\bar{q}^v)^T (\bar{K}^v)^{-1} \bar{q}^v + \bar{q}^v \\ (\bar{q}^B)^T (\bar{K}^B)^{-1} \bar{q}^B + \bar{q}^B \\ 0 \end{bmatrix} \quad (37)$$

III. PROPOSED SOLUTION

This study decomposes the vehicle-pavement interaction solution into three solvers: vehicle solver, pavement solver, and interaction solver. The proposed solution can be summarized as follows:

1. Input the vehicle and bridge parameters; the Euler-Bernoulli beam is modeled and meshed in ANSYS. M^B , C^B , and K^B matrices are extracted in ANSYS and imported into MATLAB.
2. Determine the tire's position at the next time step $x = vt^{i+1}$: then, estimate the contact direction matrices related to the beam, $w_N^B(x)$ and $w_T^B(x)$; determine the irregularities vector $\xi''(x)$.
3. Interaction solver: estimate the contact force vector associated with the vehicle and beam subsystems ($\lambda^{(V)i+1}$ and $\lambda^{(B)i+1}$) at the time step $i+1$ according to the responses of the current time step ($u^{(V)i}$, $\dot{u}^{(V)i}$, $\ddot{u}^{(V)i}$ and $u^{(B)i}$, $\dot{u}^{(B)i}$, $\ddot{u}^{(B)i}$).
4. Vehicle and beam solver: integrate the equation of motion, Eqs. (4) and (7) (independently), and update the vehicle and beam responses at the time step $i+1$ ($u^{(V)i+1}$, $\dot{u}^{(V)i+1}$, $\ddot{u}^{(V)i+1}$ and $u^{(B)i+1}$, $\dot{u}^{(B)i+1}$, $\ddot{u}^{(B)i+1}$).
5. Perform the time step integration until the current time step corresponds to the desired time step of the analysis.

IV. CASE STUDY AND VALIDATION

4.1 Case study

The modified bitumen with the tire powder model used in this study is the same as the bitumen employed by Bajogueno et al. [36]. The ELOUMDEM quarry, which is situated in the town of MBANKOMO division of the MEFOU and AKONO in the Centre Region of Cameroon, provides the natural gneissic aggregates utilized in this project. The , , and granular classes are employed. Table 1 below lists the typical physical properties of aggregates for asphalt concrete, bitumen ratios of , , and . The bitumen class chosen is 60/70 since it is more extensively used in the area and is more resistant to rutting in hot conditions. The key attributes of the bitumen employed in this investigation are displayed in Table 2. The MICHELIN brand LTX A/T2 old tires provided the used tire powder needed in this project. The process for collecting the tire powder is summarized in Fig. 3. 0 / 4 4 / 6 6 / 10 40 / 50 60 / 70 80 / 100



Figure 3. Framework of the process for collecting the tire powder

Table 1. Typical physical properties of aggregates

Tests	Sand 0/4	Gravel 4/6	Gravel 6/10	Specification
Water content	0.35	0.14	0.11	
Flatening coefficient		5.5	3.4	<20
Apparent density	1.54	1.46	1.45	
Los Angeles (%)		28		
Sand equivalent (%)	80.20			≥ 60

Table 2. Characteristics of the bitumen

Parameters	Bitumen 60/70		
	Standards	Obtained	Specification
Ball and ring softening point (°C)	NF T 66-008 [31]	50.5	46/54
Penetrability at 25 °C, 100g, 5s 1/10 mm	NFT 66-004 [33]	65	60/70
Relative density at 25 °C on pycnometer (g/cm³)	NF T 66-007 [32]	1.02	1.0-1.1

4.2 Validation

To demonstrate the validity of the proposed solution, the model consists of analyzing a single sprung-mass model running along a simply supported beam on an elastic foundation with a constant speed of . The main parameters of the model are listed in Table 3. The results are compared with the models Elnashar et al. [34] and Hayir [35] implemented. Fig. 4 (a) shows the vertical displacement of the beam at mid-span. Fig. 4 (b) illustrates the pavement displacement as a vehicle speed function. The results show that the models fit very well. Table 4 shows the performance evaluation of the proposed solution with the models proposed by Elnashar et al. [34] and Hayir [35]. The root mean square ($RMSE$), Mean absolute error (MAE), and R-squared (R^2) the proposed solution and Elnashar et al. [34] model are , , and , respectively. The , , and the proposed solution and Hayir [35] model are , , and , respectively. 20 m/s $RMSE$ MAE R^2 0.023 mm 0.035 mm 98 $RMSE$ MAE R^2 0.028 mm 0.041 mm 98

Table 3. Main parameters of the vehicle-pavement system [34].

Elements	Parameters	Values	Units
m_s	Sprung mass	4500	kg
m_u	Unsprung mass	650	kg
K_s	Suspension stiffness	1150×10^3	N/m
K_t	Tire stiffness	2500×10^3	N/m
C_s	Suspension damping	15×10^3	$N.s/m$
C_t	Tire damping	2×10^3	$N.s/m$
k_p	Pasternak parameter	66.687×10^4	N
S_r	Surface roughness	0.054	m
L	Pavement length	160	m
c	Soil damping	0.35×10^6	$N.s/m$
E	Elasticity modulus	6.223×10^9	Pa

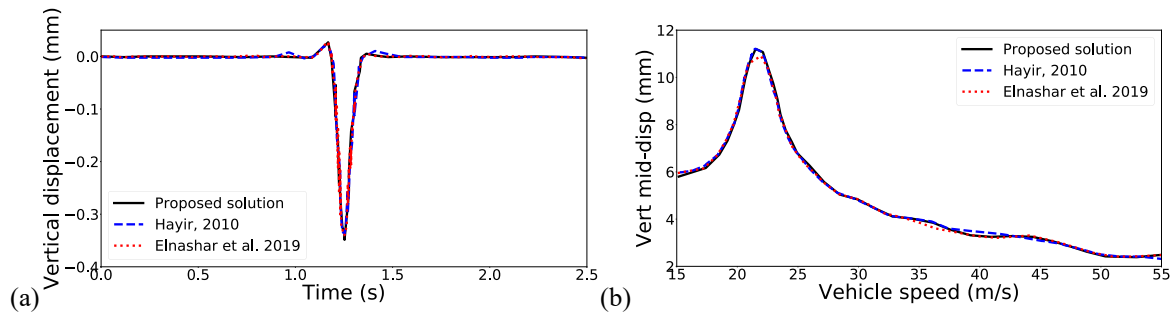


Figure 4. Validation (a) Vertical displacement (b) Vertical displacement with $k = 40.78e4 N / m / m$

Table 4. Performance evaluation of the proposed model

Models	RMSE(mm)	MAE(mm)	R^2
Elnashar et al. [34]	0.023	0.035	0.98
Hayir [35]	0.28	0.041	0.98

V. RESULTS

5.1 Effect of the tire powder on the vehicle response

In this section, the vehicle is running at , Class A profile is selected as road roughness. The standard bitumen NF T66-004 [33] listed in Table 2 is selected for the case study. Fig. 5 (a) shows the intensity of the contact force between the vehicle and the bridge over time. The simulation is performed by progressively adding , , and of the tire powder. It can be seen in the curves that the case where and of the tire powder are considered fit, and the intensity of the contact force is higher in the cases where the standard bituminous concrete is considered, and for of the tire powder. The absolute error between the standard bituminous concrete and the cases for and of the tire powder is around . The absolute error between the standard bituminous concrete and the case for of the tire powder is around . This result means that the riding comfort is better in the cases of and of the tire powder. Fig. 5 (b) shows the vertical acceleration of the vehicle. The same observation is made as for the contact force. The vertical displacement of the vehicle is slightly the same for the case where the bituminous concrete, and of the tire powder, are considered. The vertical acceleration is very high for the case where the standard bituminous concrete is considered. 20m/s 60/70 15% 20% 25% 25% 20% 60/70 15% 60/70 25% 20% 5kN 60/70 15% 1kN 20% 25% 25% 20% 60/70

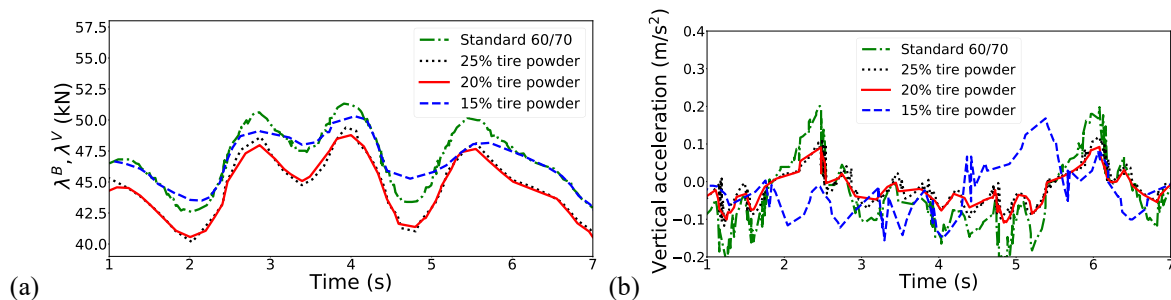
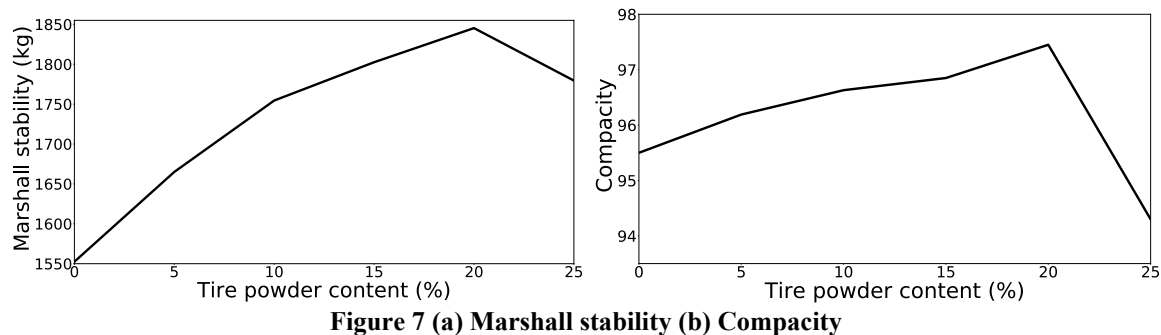
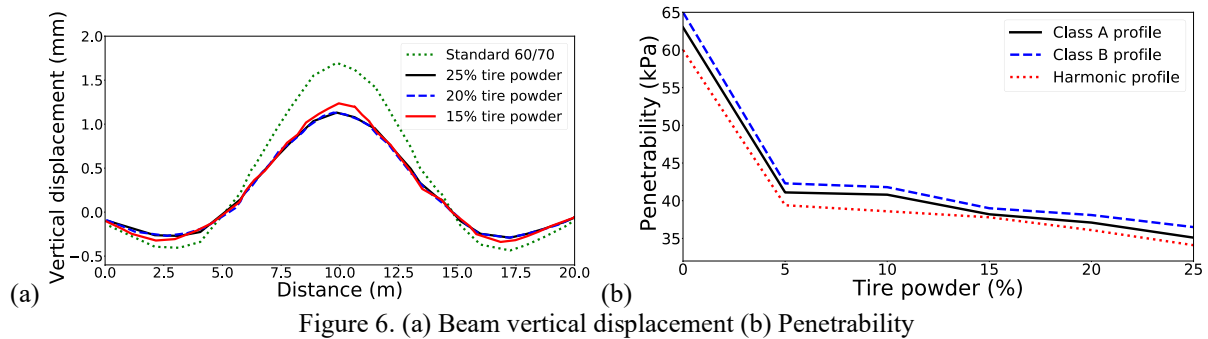


Figure 5. Vehicle response (a) Contact force (b) Vertical acceleration

5.2 Effect of the tire powder on the pavement response

In this section, $n_0 = 0.1 \text{ cycle}/m$. The sampling interval is selected as $0.006 m$ considering the Euler-Bernoulli beam model. $G_d(n_0)$ is set to $32 \times 10^{-6} m^3$ and $128 \times 10^{-6} m^3$ for Class A profile and Class B profile, respectively. The vehicle is running at $20 m/s$ along the beam. Fig. 6 (a) shows the vertical displacement of the beam with varying percentages of tire powder. It can be seen that the displacement curve for 25% and 20% of tire powder match very well with an absolute error equal to $0.002 mm$. The displacement is higher for the standard 60/70 bituminous concrete. The absolute error between the standard 60/70 and 25% or 20% of the powder content is around $0.06 m/s^2$. Fig. 6 (b) illustrates the penetrability of tire powder and the Class profile. It is observed that penetrability decreases while the tire powder increases. The penetrability of tire powder highly

decreases from 0 to 5% and progressively decreases from 5% to 25%. This result shows that the tire powder reinforces the strength of the pavement property. The type of Class profile can be selected randomly since it can provide a similar penetrability performance, whatever Class A, Class B, or harmonic profiles are selected. Fig. 7 shows the behavior of the modified bitumen in function of the Marshall stability and the compacity. The results demonstrate that the intensity of the Marshall stability progressively increases from 0 to 20% of the powder content then suddenly drop from 20% to 25% of powder content. The same observation can be made for the compacity. Its intensity progressively increases and suddenly drop. The result means that 20% of powder content is optimal than the other cases.



VI. CONCLUSION

This study proposes a numerical method for analyzing the influence of the modified 60 / 70 bituminous concrete on the vehicle-pavement interaction. The quarter-vehicle model running along a rough road is considered. The contact between the wheel and the rough road is described based on Lagrange multipliers. The material property of the standard 60 / 70 bituminous concrete has been modified by adding a tire powder content. The results obtained can be summarized below:

- (i) The accuracy of the effectiveness of the proposed vehicle-pavement interaction has been evaluated by comparing the proposed model with Elnashar et al. [34] and Hayir [35] models. The , , and the proposed solution and Elnashar et al. [34] model are , , and , respectively. The , , and the proposed solution and Hayir [35] model are , , and , respectively. $RMSE$ MAE R^2 $0.023mm$ $0.035mm$ 98 $RMSE$ MAE R^2 $0.028mm$ $0.041mm$ 98
- (ii) The results show that the riding comfort is good since the vehicle's vertical acceleration is lower for 20% and 25% of powder content than the standard 60/70 bituminous concrete. The absolute error of the vertical acceleration between the standard 60/70 bituminous concrete and the cases for 25% and 20% of tire powder is around $5kN$.
- (iii) The results demonstrated that adding the powder into the bituminous concrete increases the strength of the bituminous considered. The vertical displacement of the pavement is lower for 20% and 25% of powder content than for the case where the standard 60/70 bituminous concrete is considered. The absolute error between the standard 60/70 and 25% or 20% of the powder content is around $0.06m/s^2$.
- (iv) The observation shows that 20% of tire powder content is the optimal rate since the intensity of the Marshall stability and the compacity progressively increase from 0 to 20% of the powder content, then suddenly drop from 20% to 25% of powder content.

Data Availability

Statement Data are available on request from the authors.

Conflicts of Interest

The authors declare no conflicts of interest.

Author Contributions

Jean Medard Ofamo Bajogueno and Moussa Sali, supervised the data collection campaign. Jeremie Madjadoumbaye is the Head of the Research Unit. He supervised all the work.

Funding

No funding was received for this manuscript.

Acknowledgments

For the completion of this work, which is the first part of the Ph.D. thesis of the author number, we would particularly like to thank all those who contributed in any way to its completion.

REFERENCES

- [1] Rokade S (2012). Use of waste plastic and waste rubber tires in flexible highway pavements. International Conference on Future Environment and Energy. IPCBEE, 28, 105-108.
- [2] Lo Presti D. (2013). Recycled tire rubber modified bitumens for road asphalt mixtures: a literature review. Construction and Building Materials, 49: 863-881. <https://doi.org/10.1016/j.conbuildmat.2013.09.007>.
- [3] Magar N.R (2014). A study on the performance of crumb rubber bitumen product by varying the sizes of crumb rubber. International Journal of Engineering Trends and Technology, 14(2): 51-56. <https://doi.org/10.14445/22315381/IJETT-V14P211>.
- [4] Ling T., Lu Y., Zhang Z., Li C and Oeser M. (2019). Value-added application of waste rubber and waste plastic in asphalt binder as a multifunctional additive. Materials, 12(1280): 2-17. <https://doi.org/10.3390/ma12081280>.
- [5] Mbereyaho L., Manzi L., Kamanzi P., and Nizeyimana B (2021). Use and Influence of Tire Rubber Waste Powder in Bitumen Product. Journal of Engineering, Project, and Production Management, 11(2):82-88. <https://doi.org/10.2478/jeppm-2021-0009>.
- [6] Mashaaan N.S., Ali A.H., Karim M.R., and Abdelaziz M (2014). A Review on Using Crumb Rubber in Reinforcement of Asphalt Pavement. The Scientific World Journal. 2014(4):214612. <https://doi.org/10.1155/2014/214612>.
- [7] Ding H., Chen L., and Yang S (2012). Convergence of Galerkin truncation for dynamic response of finite beams on nonlinear foundations under a moving load. Journal of Sound and Vibration, 331(10):2426-42. <https://doi.org/10.1016/j.jsv.2011.12.036>.
- [8] Huang M and Thambiratnam D.P (2002). Dynamic response of plates on elastic foundation to moving loads. Journal of Engineering Mechanics, 128(9):1016-22. [https://doi.org/10.1061/\(ASCE\)0733-9399\(2002\)128:9\(1016\)](https://doi.org/10.1061/(ASCE)0733-9399(2002)128:9(1016)).
- [9] Winkler (1867). Die Lehre von der Elasticitaet und Festigkeit. Prague: Dominicus.
- [10] Kerr A.D (1964). Elastic and viscoelastic foundation models. Journal of Applied Mechanics, 31(3):491-8.
- [11] Abbas W., Bakr O.K., Nassar M.M., Abdeen M.A.M, Shabrawy (2021). Analysis of Tapered Timoshenko and Euler–Bernoulli Beams on an Elastic Foundation with Moving Loads. Journal of Mathematics. <https://doi.org/10.1155/2021/6616707>.
- [12] Zhang B., Kou Y., and Jin K (2025). Dynamic behavior of beam with an elastic foundation under the extended moving load for electromagnetic launch rail-structure. Journal of Sound and Vibration, 596(1):118756. <https://doi.org/10.1016/j.jsv.2024.118756>.
- [13] Sun L (2013). An overview of a unified theory of dynamics of vehicle–pavement interaction under moving and stochastic load. Journal of Modern Transportation, 21: 135–162. <https://doi.org/10.1007/s40534-013-0017-8>.
- [14] Wang H., Al-Qadi T.L, Stanculescu L (2013). Effect of Surface Friction on Tire–Pavement Contact Stresses during Vehicle Maneuvering. Journal of Engineering Mechanics, 140(4). [https://doi.org/10.1061/\(ASCE\)EM.1943-7889.0000691](https://doi.org/10.1061/(ASCE)EM.1943-7889.0000691).
- [15] Saeed F., Rahman M., and Chamberlin D (2018). Impact of Tire and Traffic Parameters on Water Pressure in Pavement. Journal of Transportation Engineering, Part B: Pavements, 144(4). <https://doi.org/10.1061/JPEODX.0000065>.
- [16] Krishnanunni C.G and Rao B.N (2019). Decoupled technique for dynamic response of vehicle-pavement systems. Engineering Structures, 191:264-279. <https://doi.org/10.1016/j.engstruct.2019.04.042>.
- [17] Ma J and Wang W (2021). Modeling and analytical calculation of road damage coefficient considering tire pressure and damping of vehicles. International Journal of Modeling, Simulation, and Scientific Computing, 12(66): 2150046. <https://doi.org/10.1142/S179396232150046X>.
- [18] Sapountzakis E and Kampitsis A (2010). Nonlinear dynamic analysis of timoshenko beamcolumns partially supported on tensionless winkler foundation. Computers & Structures, 88(21–22):1206–1219. <https://doi.org/10.1016/j.compstruc.2010.06.010>.
- [19] Sapountzakis E and Kampitsis A (2011). Nonlinear response of shear deformable beams on tensionless nonlinear viscoelastic foundation under moving loads. Journal of Sound and Vibration, 330(22):5410–5426. <https://doi.org/10.1016/j.jsv.2011.06.009>.
- [20] Zhang Q., Hou J., Hu X., Yuan L., Jankowski L., An X., and Duan Z (2022). Vehicle parameter identification and road roughness estimation using vehicle responses measured in field tests. Measurement, 199:111348. <https://doi.org/10.1016/j.measurement.2022.111348>.
- [21] Xia H., Zhang N., and Roeck De G (2003). Dynamic analysis of high speed railway bridge under articulated trains. Computers & Structures, 81(26):2467–78. [https://doi.org/10.1016/S0045-7949\(03\)00309-2](https://doi.org/10.1016/S0045-7949(03)00309-2).
- [22] Liu K., Reynders E., Roeck De G., and Lombaert G (2009). Experimental and numerical analysis of a composite bridge for high-speed trains. Journal of Sound and Vibration, 320(1):201–20. <https://doi.org/10.1016/j.jsv.2008.07.010>.
- [23] Zhang N and Xia H (2013). Dynamic analysis of coupled vehicle-bridge system based on intersystem iteration method. Computers & Structures, 114:26–34. <https://doi.org/10.1016/j.compstruc.2012.10.007>.
- [24] Park K., Felippa C.A., and Ohayon R (2001). Partitioned formulation of internal fluid-structure interaction problems by localized Lagrange multipliers. Computer Methods in Applied Mechanics and Engineering, 190(24):2989–3007. [https://doi.org/10.1016/S0045-7825\(00\)00378-9](https://doi.org/10.1016/S0045-7825(00)00378-9).
- [25] Dimitrakopoulos E.G and Zeng Q (2015). A three-dimensional dynamic analysis scheme for the interaction between trains and curved railway bridges. Computer & Structures. 149:43–60. <https://doi.org/10.1016/j.compstruc.2014.12.002>.
- [26] Zeng Q and Dimitrakopoulos E.G (2016). Seismic response analysis of an interacting curved bridge-train system under frequent earthquakes. Earthquake Engineering Structural Dynamics, 45(7):1129–48. <https://doi.org/10.1002/eqe.2699>.
- [27] Li S., Yang S., and Chen L (2013). A nonlinear vehicle-road coupled model for dynamics research. Journal of Computing and Nonlinear Dynamic, 8(2): 021001. <https://doi.org/10.1115/1.4006784>.
- [28] Ding H., Yang Y., Chen L., and Yang S (2014). Vibration of vehicle–pavement coupled system based on a timoshenko beam on a nonlinear foundation. Journal of Sound and Vibration, 333(24): 6623-6636. <https://doi.org/10.1016/j.jsv.2014.07.016>.

- [29] Agostinacchio M., Ciampa D., and Olita S (2014). The vibrations induced by surface irregularities in road pavements—a matlab approach. *European Transport Research Review*, 6(3):267. <https://doi.org/10.1007/s12544-013-0127-8>.
- [30] Newmark N.M (1959). A method of computation for structural dynamics. *Journal of the Engineering Mechanics Division*, 85:67–94. <https://doi.org/10.1061/JMCEA3.0000098>.
- [31] AFNOR (1979). Produits noirs - détermination du point de ramollissement des produits bitumineux. Méthode bille et anneau. <http://worldcat.org/issn/03353931>.
- [32] AFNOR (1969). Produits noirs : détermination de la densité relative des produits bitumineux : méthode du picnomètre : NF T 66-007.
- [33] Sofo H.A.I., Mohammed M.A., and Sidick B (2024). Recycling of Waste PET Plastic on Asphalt, by Using Application of Response Surface Methodology: Effect of Production Process Parameters. *American Journal of Applied Chemistry*, 12(2): 47-57. <https://doi.org/10.11648/j.ajac.20241202.12>.
- [34] Elnashar G., Bhat R.B and Sedaghati R (2019). Modeling and dynamic analysis of a vehicle-flexible pavement coupled system subjected to road surface excitation. *Journal of Mechanical Science and Technology*, 33: 3115–3125. <https://doi.org/10.1007/s12206-019-0606-5>.
- [35] Hayir A (2010). Dynamic behavior of an elastic beam on a winkler foundation under a moving load. *International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*. <https://scholarsmine.mst.edu/icrageesd/05icrageesd/session02/4>.
- [36] Bajogueno J.M.O., Madjadoumbaye J., Sali M., and Minane J.R (2024). Study Of The Mechanical Behaviour Of 60/70 Bitumen and Semi-Gritty Asphalt Concrete Modified With Used Tyre Powder. *IOSR Journal of Mechanical and Civil Engineering*, 21(3): 25-35. <https://doi.org/10.9790/1684-2103012535>
- Bajogueno J.M.O., Madjadoumbaye J., Sali M., and Minane J.R (2024). Study Of The Mechanical Behaviour Of 60/70 Bitumen and Semi-Gritty Asphalt Concrete Modified With Used Tyre Powder. *IOSR Journal of Mechanical and Civil Engineering*, 21(3): 25-35. <https://doi.org/10.9790/1684-2103012535>