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A solution to the Riemann Hypothesis concerning an aspect of prime numbers: A formula for a binary matrix

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Abstract
The purpose of this article is showing that the number of prime numbers less than n (PP<n), in an attempt to solve the Riemann Hypothesis concerning that aspect of primes, is given by the summation of n-1 column products minus 1 (Σ(CP)n-1 - 1) in a binary matrix built through specific rules and conditions, thus yielding the formula Σ((CP)n-1)ABM - 1 = PP<n.
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I. INTRODUCTION

Solving the Riemann Hypothesis has been a major goal among mathematicians for a long time. The purpose of this article is attempting to solve the Riemann Hypothesis, concerning finding the number of prime numbers less than *n*. This solution is given within what we call the Alvarezian Binary Matrix, a mathematical system able to yield precise, exhaustive, endless and orderly information about the primality of all natural numbers. The information about primes this binary matrix can yield, has been efficiently worked on in previous research. The special contribution of this article, is a formula that yields the information we have been looking for about prime numbers concerning the Riemann hypothesis, namely Σ((CPn-1)ABM - 1 = PP<n. Next sections will further develop what we aim at presenting in this publication.

II. THEORETICAL FRAMEWORK

2.1 Riemann Hypothesis

The Riemann Hypothesis is a mathematical problem which has not been solved so far since its inception in 1859 by German mathematician Bernhard Riemann. It tells us about the deviation from the average distribution of prime numbers. Formulated in Riemann's paper, it asserts that all the 'non-obvious' zeros of the zeta function are complex numbers with real part 1/2 (Clay Mathematics Institute, n.d.; Riemann, 1859).

2.2 Alvarezian Binary Matrix

The binary matrix we have worked on in the last years, has proven a useful tool in understanding, orderly generating and looking for the number of prime numbers within a given limit (Alvarez, 2024, 2025). For the sake of algebraic specificity, we identify it from now on as the Alvarezian Binary Matrix.

2.3 Prime number

A prime number can basically be defined as a number greater than 1, only divisible by 1 and itself (Lehmer, n.d.). Beyond this elementary and still fundamental definition, we can interpret or aim at re-defining this notion, in terms of the Alvarezian Binary Matrix mentioned in previous section, as we will later see throughout this publication.

III. DISCUSSION

In this section, we will go through the general description, presentation and validity demonstration of a specific binary matrix, that is able to provide precise, key and endless information about finding the number of prime numbers less than a given number. After that, we will go through the progressive transformations of a formula derived from the binary matrix, to then provide a demonstration of the actual primality of the prime numbers found in it, as an addition to the validity demonstration previously shown. Finally, we will test whether the formula works, using some small fragments of the matrix which are equivalent to specific numbers, for that purpose. The information that a system of this nature can yield, solves the Riemann Hypothesis with a reasonable degree of optimality.

As we have analyzed in previous research we can build an infinite binary matrix, based on a simple generation mechanism. This mechanism is composed of a few rules: 1) Setting the base of the matrix through an infinite row of 1s. 2) Setting an infinite diagonal semi-perimeter. 3) Filling with 1s everything in the matrix outside the inner zone. 4) Filling the inner zone with sequences of 1s and 0s, following the logic of multiples. Once we have the constructed matrix, we can multiply the 1s and 0s from each column, to get a product per column that can be 1 or 0. If that product is 1, the number corresponding to the column is prime. If the product is 0, the number corresponding to the column is not prime. This visually shows in the following fragment of the matrix from 1 to 15 but as we have said, it extends to infinity (Alvarez, 2024, 2025).

1	1	1	1	1	1	1	1	1	1	1	1	1	1	**1**
1	1	1	1	1	1	1	1	1	1	1	1	1	**1**	1
1	1	1	1	1	1	1	1	1	1	1	1	**1**	1	1
1	1	1	1	1	1	1	1	1	1	1	**1**	1	1	1
1	1	1	1	1	1	1	1	1	1	**1**	1	1	1	1
1	1	1	1	1	1	1	1	1	**1**	1	1	1	1	1
1	1	1	1	1	1	1	1	**1**	1	1	1	1	1	1
1	1	1	1	1	1	1	**1**	1	1	1	1	1	1	1
1	1	1	1	1	1	**1**	1	1	1	1	1	1	0	1
1	1	1	1	1	**1**	1	1	1	1	1	0	1	1	1
1	1	1	1	**1**	1	1	1	1	0	1	1	1	1	0
1	1	1	**1**	1	1	1	0	1	1	1	0	1	1	1
1	1	**1**	1	1	0	1	1	0	1	1	0	1	1	0
1	**1**	1	0	1	0	1	0	1	0	1	0	1	0	1
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1	**1**	**1**	**0**	**1**	**0**	**1**	**0**	**0**	**0**	**1**	**0**	**1**	**0**	**0**
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

As we can see, the row at the pre-bottom of the binary matrix shows the primality of numbers from 1 to 15 (1 is the exception but we will solve this later), orderly and in the form of 1 or 0 but as we have said, this extends to infinity. This is easily demonstrated by the fact the binary matrix can be filled exactly the same way, by deriving that from a much simpler matrix, also infinite, which provides precise, exhaustive and endless information about the exact divisibility of any number by any other. However, this does not affect the autonomous generativity of the rules that yield the entire binary matrix. We will now show a fragment of the second matrix we mentioned, also from 1 to 15, as a proof of the binary matrix validity.

1:15	2:15	3:15	4:15	5:15	6:15	7:15	8:15	9:15	10:15	11:15	12:15	13:15	14:15	**15:15**
1:14	2:14	3:14	4:14	5:14	6:14	7:14	8:14	9:14	10:14	11:14	12:14	13:14	**14:14**	**15:14**
1:13	2:13	3:13	4:13	5:13	6:13	7:13	8:13	9:13	10:13	11:13	12:13	**13:13**	**14:13**	**15:13**
1:12	2:12	3:12	4:12	5:12	6:12	7:12	8:12	9:12	10:12	11:12	**12:12**	**13:12**	**14:12**	**15:12**
1:11	2:11	3:11	4:11	5:11	6:11	7:11	8:11	9:11	10:11	**11:11**	**12:11**	**13:11**	**14:11**	**15:11**
1:10	2:10	3:10	4:10	5:10	6:10	7:10	8:10	9:10	**10:10**	**11:10**	**12:10**	**13:10**	**14:10**	**15:10**
1:9	2:9	3:9	4:9	5:9	6:9	7:9	8:9	**9:9**	**10:9**	**11:9**	**12:9**	**13:9**	**14:9**	**15:9**
1:8	2:8	3:8	4:8	5:8	6:8	7:8	**8:8**	**9:8**	**10:8**	**11:8**	**12:8**	**13:8**	**14:8**	**15:8**

1:7	2:7	3:7	4:7	5:7	6:7	**7:7**	**8:7**	**9:7**	**10:7**	**11:7**	**12:7**	**13:7**	14:7	**15:7**
1:6	2:6	3:6	4:6	5:6	**6:6**	**7:6**	**8:6**	**9:6**	**10:6**	**11:6**	12:6	**13:6**	**14:6**	**15:6**
1:5	2:5	3:5	4:5	**5:5**	**6:5**	**7:5**	**8:5**	**9:5**	10:5	**11:5**	**12:5**	**13:5**	**14:5**	15:5
1:4	2:4	3:4	**4:4**	**5:4**	**6:4**	**7:4**	8:4	**9:4**	**10:4**	**11:4**	12:4	**13:4**	**14:4**	**15:4**
1:3	2:3	**3:3**	**4:3**	**5:3**	6:3	**7:3**	**8:3**	9:3	**10:3**	**11:3**	12:3	**13:3**	**14:3**	15:3
1:2	**2:2**	**3:2**	4:2	**5:2**	6:2	**7:2**	8:2	**9:2**	10:2	**11:2**	12:2	**13:2**	14:2	**15:2**
1:1	**2:1**	**3:1**	**4:1**	**5:1**	**6:1**	**7:1**	**8:1**	**9:1**	**10:1**	**11:1**	**12:1**	**13:1**	**14:1**	**15:1**
1	**1**	**1**	**0**	**1**	**0**	**1**	**0**	**0**	**0**	**1**	**0**	**1**	**0**	**0**
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

As we can see here, this fragment of the second matrix we were mentioning, contains the possible divisions from 1 to 15. The divisions in bold within the semi-perimeter are those yielding decimal outputs, and everything in bold fits what is also in bold in our binary matrix. What makes primes primes and non-primes non-primes, is fully and independently based on this second matrix, which is an obvious fact based on basic mathematics.

In order for the generation rule calculations to perform, we provisionally need to consider number 1 at the beginning of the matrix, whose column is to be filled only with 1s because of generation rule 3. The specific contribution of this publication, is a summation formula that yields the result of a specific mathematical pursuit, meaning finding the number of prime numbers less than *n*, in an attempt to solve the Riemann Hypothesis concerning primes. Before that however, we can consider the initial version of the formula, where P stands for *number of prime numbers*, CP for *column product,* n for the number under which we want to find P and ABM for Alvarezian Binary Matrix (Alvarez, 2024, 2025), since it is this particular binary matrix and no other, the one holding the mathematical space we need for our discussion. This shows as follows:

$$\Sigma(P_{n-1})ABM = ((CP)_{n-1} + (CP)_{n-2} + (CP)_{n-3} + \ldots + (CP)_{2} + (CP)_{1})ABM$$

As we have implied, what this initial version of the formula shows, is the summation of each and all matrix column products, less than any number we want to select. In reverse and for the sake of order, we can adjust the formula to make the columns show from left to right. This is presented in the following modification of the formula:

$$\Sigma(P_{n-1})ABM = ((CP)_{1} + (CP)_{2} + \ldots + (CP)_{n-3} + (CP)_{n-2} + (CP)_{n-1})ABM$$

As we do not need column 1 in the binary matrix, we can adjust the formula without deleting that column from the matrix. For this we will need to add a substraction after the summation on both sides of the formula. This will reflect in the updated formula as follows:

$$\Sigma(P_{n-1})ABM - 1 = ((CP)_{1} + (CP)_{2} + \ldots + (CP)_{n-3} + (CP)_{n-2} + (CP)_{n-1})ABM - 1$$

We can synthesize what we have been discussing so far, through the following update of the formula as follows:

$$\Sigma((CP)_{n-1})ABM - 1 = P_{P<n}$$

This formula in an alternative solution to the question of how many prime numbers there are less than a given number. For that matter, it is an improvement to the primality consequences of the Riemann Hypothesis, given the primality of prime numbers found through the matrix being discussed, is by definition their corresponding column being only composed by 1s, a statement far from circularity. Next step is going through the proof of this definition.

The proof we will present through the following lines, is partially based on some of the generation rules that yield the binary matrix being analyzed. To begin with a prime number is that only divisible by one and itself. In the binary matrix we work with, these two divisibility options are already covered by 1s in their corresponding squares, because of generation rules 1 and 2. Now what additionally makes a prime number actually a prime number is firstly, filling the inner zone column of that specific prime number only with 1s (which could be interpreted as a sub-rule of rule 4 but that has not been worked on yet); secondly, rule 3, which fills the outside zone only with 1s, will therefore fill the rest of each column only with 1s as well. Therefore, a column filled only by 1s because of the generation rules, will be by definition a prime number. This proves the semantic reciprocity between a prime number definition based on the mathematical construct being analyzed and the most basic prime number definition, which the Riemann Hypothesis lacks (Alvarez, 2024, 2025; Clay Mathematics Institute, n.d.; Lehmer, n.d.; Riemann, 1859).

Let us test the found formula with a few numbers now. For this we will need to show the matrix in its visual form. Let us pick numbers 4, 6 and 16 for that purpose. If we pick number 4 first, then $n = 4$. Let us develop this process in a step-by-step fashion:

$$n = 4$$
$$\therefore$$
$$n - 1 = 3$$

1	1	1
1	1	1
1	1	1
1	**1**	**1**
1	2	3

$$\therefore$$
$$\Sigma((CP)3)ABM - 1 = ((CP)1 + (CP)2 + (CP)3)ABM - 1 \quad \Sigma((CP)3)ABM - 1 = (1 + 1 + 1)ABM - 1$$
$$\Sigma((CP)3)ABM - 1 = 2$$

Additionally, and by deduction:

$$\Sigma((CP)3)ABM - 1 = PP<4$$

Therefore,

$$PP<4 = 2$$

, meaning the number of prime numbers less than 4 is 2. In this way, we have successfully proven the found formula applied to $n = 4$. Let us now apply this formula to $n = 6$ as mentioned earlier. If we pick number 6, then $n = 6$. Let us develop this process step-by-step as well:

$$n = 6$$
$$\therefore$$
$$n - 1 = 5$$

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	0	1
1	**1**	**1**	**0**	**1**
1	2	3	4	5

$$\therefore$$
$$\Sigma((CP)5)ABM - 1 = ((CP)1 + (CP)2 + (CP)3 + (CP)4 + (CP)5)ABM - 1 \quad \Sigma((CP)5)ABM - 1 = (1 + 1 + 1 + 0 + 1)ABM - 1$$
$$\Sigma((CP)5)ABM - 1 = 3$$

Additionally, and by deduction:

$$\Sigma((CP)5)ABM - 1 = PP<6$$

Therefore,

$$PP<6 = 3$$

, meaning the number of prime numbers less than 6 is 3. In this way, we have successfully proven the found formula applied to n = 6. Let us now apply this formula to n = 16 as mentioned earlier. If we pick number 16, then n = 16. Let us develop this process step-by-step as well:

$$n = 16$$
$$\therefore$$
$$n - 1 = 15$$

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
1	1	1	1	1	1	1	1	1	0	1	1	1	1	0
1	1	1	1	1	1	1	0	1	1	1	0	1	1	1
1	1	1	1	1	0	1	1	0	1	1	0	1	1	0
1	1	1	0	1	0	1	0	1	0	1	0	1	0	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	**1**	**1**	**0**	**1**	**0**	**1**	**0**	**0**	**0**	**1**	**0**	**1**	**0**	**0**
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

$$\therefore$$

$\Sigma((CP)15)ABM - 1 = ((CP)1 + (CP)2 + (CP)3 + (CP)4 + (CP)5 + (CP)6 + (CP)7 + (CP)8 + (CP)9 + (CP)10 + (CP)11 + (CP)12 + (CP)13 + (CP)14 + (CP)15)ABM - 1$ $\Sigma((CP)15)ABM - 1 = (1 + 1 + 1 + 0 + 1 + 0 + 1 + 0 + 0 + 0 + 1 + 0 + 1 + 0 + 0)ABM - 1$

$$\Sigma((CP)15)ABM - 1 = 6$$

Additionally, and by deduction:

Therefore,

$\Sigma((CP)15)ABM - 1 = PP<16$

$PP<16 = 6$

, meaning the number of prime numbers less than 16 is 6. In this way, we have successfully proven the found formula applied to $n = 16$. These three demonstrations together with the presented explanations, should be enough to infer the formula works for any natural number. In any case, the invitation is open to test the presented formula to any number, by any reliable tool or technological device.

IV. CONCLUSION

Through this publication, we have solved an aspect of prime numbers in the Riemann Hypothesis, by means of a mathematical system outside the mathematics used by Riemann in his original publication, which we call the Alvarezian Binary Matrix. We have accomplished this objective with a formula that simplifies the calculations in this matrix to its simplest and most precise possible form, providing in this way exact information about the number of primes less than n. This information has finally taken the form of the formula $\Sigma((CP)n\text{-}1)ABM - 1 = PP<n$, in which the summation of n-1 column products in this specific matrix minus 1, yields the exact number of primes less than n, in all possible cases and in an internally logical way. Therefore, this kind of analysis adds semantic reprocity between the most basic prime number definition and the prime number definition given by the system we analyzed in this publication, something research on primes did not have so far.

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