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spm*-irresolute mappings

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Abstract
In this paper, we apply the concepts of semipreopen sets and m-open sets intopological spaces to define and study spm*-irresolute mappings.*

Keywords: *Semipreopen sets, m*-open sets, m*-irresolute map, m*- neighbourhood, semipre-irresolute, semipre-continuous.*

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I. Introduction

In 1986, D. Andrajevic[1] had defined and studied the concepts of semipreopen sets. In 2002, Navalagi[3] has defined and studied the concepts of continuous functions, open functions, irresolute maps in between topological spaces.
In 2019[4], introduced a new class of sets namely, m*-closed sets ,m*-open sets. And explored various continuous functions, open functions and irresolute maps using these sets. The purpose of this paper is to define and study the notions of spm*-irresolute mappings using semipreopen sets and m*-open sets.

II. Preliminaries

In this paper, X, Y, Z always means a topological spaces on which no separation axioms are assumed. Unless otherwise mentioned.

For a subset A of X, Cl(A) and Int(A) represents the closure of A and interior of A respectively.

The following definitions and results are useful in the sequel:

Definition 2.1: Let A be a subset of a space X. Then the union of all semipreopen sets contained in A is called semipre-interior[1] and is denoted by spInt(A).

Definition 2.2: Let X be a topological space. A subset A is called semipreopen[1] set if $A \subset Cl(Int(Cl(A)))$.

Definition 2.2: Let X be a topological space. A subset A is called γ-open[2] set if $A \subset Cl(Int(A)) \cup Int(Cl(A))$.
The complement of γ-open set is γ-closed[2] set.

Definition 2.3: A subset A of a space X is termed as m^*-open[5] set if $F \subset spInt(A)$ whenever $F \subset A$ and F is γ-closed set in X.

The family of all semipreopen sets and m^*-open sets of a space X is denoted by SPO(X) and $M^*O(X)$ respectively.

Definition 2.4: Let $x \in X$. A set $U \subset X$ is called semipre- neighbourhood[3] of a point x , if their exists $A \in SPO(X)$ such that $A \subset U$.

Definition 2.5: A set $U \subset X$ is said to be m^*-neighbourhood[5] (in brief, m^*-nbd) of a point $x \in X$ if and only if there exists $A \in M^*O(X)$ such that $A \subset U$.

Definition 2.6: A mapping $i : X \to Y$ is said to be semipre-continuous[4], if the inverse image of each open set of Y is semipreopen set in X.

Definition 2.7: A mapping $i: X \to Y$ is said to be m^*-continuous[5] if the inverse image of each open set of Y is m^*-open set in X.

Definition 2.8: A mapping $i : X \to Y$ is said to be semipre-irresolute[4], if the inverse image of each semipreopen set of Y is semipreopen set in X.

Definition 2.9: A mapping $i: X \to Y$ is said to be m^*-irresolute[5] if the inverse image of each m^*-open set of Y is m^*-open set in X.

3. On spm*-irresolute mappings

We define the following:

Definition 3.1: A mapping $i:X \to Y$ is termed as spm*-irresolute, if the inverse image of each semipreopen set of Y is m^*-open set in X.

Theorem 3.2 : Every m^*-irresolute map is (sp, m^*)-irresolute.

Proof: Let H be any semipreopen subset of Y. Since every semipreopen set is m^*-open set, hence H be m^*-open set in Y and given that, i is m^*-irresolute. Therefore $i^{-1}(H)$ is m^*-open set in X. This shows that i is spm*-irresolute map.
Characterization of (sp, m^*)-irresolute mapping:

Theorem 3.3 : The following statements are equivalent for a mapping $i:X \to Y$.

(i) i is spm*-irresolute.

(ii) for each point x of X and each semipre-neighbourhood H of i(x), there exist a m^*- neighbourhood G of x such that $i(G) \subseteq H$.

(iii) for each x in X and each $H \in SPO(i(x))$, there exist $G \in M^*O(X)$ such that $i(G) \subseteq H$.

Proof: (i)$\Rightarrow$(ii): Assume $x\in X$ and H is semipreopen set in Y containing i(x). Since, i is spm*-irresolute and let $K= i^{-1}(H)$ be a m*-open set in X containing x and hence $i(K)\subset i(i^{-1}(H))\subset H$.

(ii)$\Rightarrow$(iii): Assume $H\subset Y$ is a semipreopen set containing i(x). Then by (ii), there exist a m*-open set G such that $x\in G\subset i^{-1}(H)$. Therefor $x\in i^{-1}(H)\subset Cl(i^{-1}(H))$. This shows that $Cl(i^{-1}(H))$ is a m*- neighbourhood of x.

(iii)$\Rightarrow$(i): Let H be a semipreopen set in Y, then $Cl(i^{-1}(H))$ is a m*- neighbourhood of each $x\in i^{-1}(H)$. Thus, for each x is a m*-interior point of $Cl(i^{-1}(H))$ which implies $i^{-1}(H)\subset IntCl(i^{-1}(H))\subset ClIntCl(i^{-1}(H))$. Therefore, $i^{-1}(H)$ is a m*-open set in X and hence i is a spm*-irresolute mapping.

Theorem 3.4 : Let $i:X\rightarrow Y$ and $j:Y\rightarrow Z$ be two mappings. The following statements are valid:

(i) If i is spm*-irresolute and j is semipre-irresolute, then $j\circ i: X\rightarrow Z$ is spm*-irresolute mapping.

(ii) If i is m*-irresolute and j is spm*-irresolute, then $j\circ j: X\rightarrow Z$ is spm*-irresolute mapping.

(iii) If i is spm*-irresolute and j is semipre-continuous, then $j\circ i: X\rightarrow Z$ is m*-continuous mapping.

Proof: (i) Let H be an arbitrary semipreopen set in Z, then $j^{-1}(H)$ is semipreopen set in Y, since j is semipre-irresolute and $j^{-1}(H)$ is semipreopen set in Y and i is spm*-irresolute implies $i^{-1}(j^{-1}(H))$ is m*-open set in X. But $i^{-1}(j^{-1}(H))=(j\circ i)^{-1}(H)$. Therefore for each semipreopen subset H in Z, $(j\circ i)^{-1}(H)$ is m*-open set in X. Thus $j\circ i$ is spm*-irresolute mapping.

(ii) Let $K\subset Z$ be an arbitrary semipreopen set, then $j^{-1}(K)$ is m*-open set in Y. Again i is m*-irresolute and $j^{-1}(K)$ is m*-open set in Y implies $i^{-1}(j^{-1}(K))$ is m*-open set in X. But $i^{-1}(j^{-1}(K))=(j\circ i)^{-1}(K)$. Therefore for each semipreopen set in Z, $(j\circ i)^{-1}(K)$ is m*-open set in X. Hence $j\circ i$ is spm*-irresolute mapping.

(iii) Let B be an arbitrary open set in Z. Since j is semipre-continuous, $j^{-1}(B)$ is semipreopen set in Y. Again, we have i is spm^*-irresolute and $j^{-1}(B)$ is semipreopen set in Y, then $i^{-1}(j^{-1}(B))=(j\circ i)^{-1}(B)$ is m^*-open set in X. Hence $j\circ i$ is m^*-continuous mapping.

Theorem 3.5: Let $i:X\rightarrow Y$ and $j:Y\rightarrow Z$ be mappings such that $j\circ i: X\rightarrow Z$ is spm^*-irresolute, then

(i) i is spm^*-irresolute then j is semipre-irresolute.

(ii) i is m^*-irresolute then j is spm^*-irresolute.

Proof: (i) Let $V\subset Z$ be an arbitrary semipreopen set and $j\circ i$ is spm^*-irresolute, then $(j\circ i)^{-1}(V)$ is m^*-open set in X. But $(j\circ i)^{-1}(V)=i^{-1}(j^{-1}(V))$. Therefore $i^{-1}(j^{-1}(V))$ is m^*-open set in X and i is spm^*-irresolute mapping, therefore $j^{-1}(V)$ is semipreopen set in Y. So, for semipreopen set $V\subset Z$ and $j^{-1}(V)$ is semipreopen set in Y, hence j is semipre-irresolute.

(ii) Let B be an arbitrary semipreopen set in Z and $j\circ i$ is spm^*-irresolute, then $(j\circ i)^{-1}(B)$ is m^*-open set in X. But $(j\circ i)^{-1}(B)= i^{-1}(j^{-1}(B))$. Therefore $i^{-1}(j^{-1}(B))$ is m^*-open set in X and p is m^*-irresolute, therefore $j^{-1}(B)$ is m^*-open set in Y. As B is semipreopen set in Z, $j^{-1}(B)$ is m^*-open set in Y, hence j is spm^*-irresolute mapping.

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