

Comparative Analysis of Bouncing Cosmology in Nonlinear $f(Q, T)$ Gravity

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Abstract

Modified gravity theories have emerged as promising frameworks for addressing fundamental problems in modern cosmology such as the accelerated expansion of the Universe, dark energy, and the initial cosmological singularity. Among these theories, $f(Q, T)$ gravity, where Q represents the non-metricity scalar and T denotes the trace of the energy–momentum tensor, has attracted considerable attention. In this work, we investigate bouncing cosmology within the framework of $f(Q, T)$ gravity using an anisotropic Bianchi type-I universe. A comparative analysis is performed between a linear model $f(Q, T) = \sigma_1 Q + \sigma_2 T$ and a nonlinear interaction model $f(Q, T) = Q + \alpha Q^n + \beta T + \gamma QT$. The behavior of cosmological parameters such as the scale factor, Hubble parameter, deceleration parameter, and equation-of-state parameter is studied. Energy conditions and the influence of matter–geometry coupling are also examined. Our analysis shows that both models allow non-singular bouncing solutions, while the nonlinear model introduces stronger modifications to the early cosmic evolution. The violation of the null energy condition near the bounce allows the transition from contraction to expansion, indicating that nonlinear $f(Q, T)$ gravity may provide a viable framework for describing the early Universe.

Keywords: $f(Q, T)$ gravity; Bouncing cosmology; Bianchi type-I universe; Non-metricity; Dark energy.

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I. Introduction

The discovery that the Universe is undergoing accelerated expansion represents one of the most significant developments in modern cosmology. Observations of distant type Ia supernovae first revealed this phenomenon (Riess et al. 1998; Perlmutter et al. 1999), and it was later confirmed through measurements of the cosmic microwave background radiation and large-scale structure (Spergel et al. 2003; Planck Collaboration 2018). Within the framework of general relativity, this acceleration is typically attributed to dark energy, which constitutes the dominant component of the present cosmic energy density (Copeland, Sami & Tsujikawa 2006). However, the physical origin of dark energy remains uncertain, and the cosmological constant problem highlights the enormous discrepancy between the observed and theoretical values of vacuum energy density (Weinberg 1989). Another fundamental issue in standard cosmology is the presence of the initial singularity predicted by the Big Bang model, where spacetime curvature and density diverge (Hawking & Ellis 1973).

These theoretical challenges have motivated the exploration of modified gravity theories in which the Einstein–Hilbert action is generalized. Various extensions have been proposed, including $f(R)$ gravity (De Felice & Tsujikawa 2010), $f(T)$ gravity (Linder 2010), and $f(R, T)$ gravity (Harko et al. 2011). In recent years, symmetric teleparallel gravity has emerged as an alternative geometric description in which gravitational effects are characterized by the non-metricity scalar Q rather than curvature or torsion (Jiménez, Heisenberg & Koivisto 2018). A further extension of this framework is $f(Q, T)$ gravity, where the gravitational Lagrangian depends on both the non-metricity scalar and the trace of the energy–momentum tensor (Xu et al. 2019; Lazkoz et al. 2019). The coupling between matter and geometry in this theory introduces new dynamical features that may influence the evolution of the Universe.

Another important topic in modern cosmology is the avoidance of the Big Bang singularity. One possible resolution is provided by bouncing cosmology, in which the Universe undergoes a contraction phase followed by a bounce and subsequent expansion (Novello & Bergliaffa 2008). In such models the scale factor never vanishes, thereby avoiding the singularity. Modified gravity theories are particularly suitable for generating bouncing solutions because they allow violations of classical energy conditions required in general relativity (Cai et al. 2014). In addition, anisotropic cosmological models such as Bianchi universes provide a more general description of the early Universe than the isotropic Friedmann model (Ellis & MacCallum 1969). Motivated by these considerations, we investigate bouncing cosmology in $f(Q, T)$ gravity using a Bianchi type-I spacetime and perform a comparative analysis between a linear $f(Q, T)$ model and a nonlinear interaction model that includes direct coupling between matter and geometry.

II. Basic Framework Of f(Q, T) Gravity

2.1 Gravitational Action and Field Equations

The action of f(Q, T) gravity is given by

$$S = \int \left[\frac{1}{16\pi} f(Q, T) + L_m \right] \sqrt{-g} d^4x,$$

where L_m is the matter Lagrangian density and g is the determinant of the metric tensor. In symmetric teleparallel gravity the gravitational interaction is described by the non-metricity scalar Q . For a homogeneous cosmological spacetime this scalar can be expressed in terms of the Hubble parameter as

$$Q = -6H^2$$

where

$$H = \frac{\dot{a}}{a}$$

is the Hubble parameter. The field equations depend on the derivatives

$$f_Q = \frac{\partial f}{\partial Q}, f_T = \frac{\partial f}{\partial T}.$$

These derivatives introduce additional geometric contributions that modify the effective energy density and pressure of the cosmic fluid.

2.2 Bianchi Type-I Cosmological Model

To analyze anisotropic cosmology we consider the Bianchi type-I metric

$$ds^2 = -dt^2 + a^2(t)dx^2 + b^2(t)dy^2 + c^2(t)dz^2.$$

This spacetime is homogeneous but anisotropic and represents the simplest extension of the Friedmann–Robertson–Walker universe. The matter content of the Universe is assumed to be a perfect fluid with energy–momentum tensor

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu},$$

where ρ and p denote energy density and pressure respectively. The deceleration parameter describing cosmic acceleration is defined as

$$q = -1 - \frac{\dot{H}}{H^2}.$$

III. Cosmological Models And Bouncing Dynamics

3.1 Linear f(Q, T) Model

We first consider the linear form of the f(Q, T) function

$$f(Q, T) = \sigma_1 Q + \sigma_2 T$$

where σ_1 and σ_2 are constants.

The derivatives of the function are

$$f_Q = \frac{\partial f}{\partial Q} = \sigma_1$$

$$f_T = \frac{\partial f}{\partial T} = \sigma_2$$

For the homogeneous cosmological spacetime, the non-metricity scalar is related to the Hubble parameter by

$$Q = -6H^2$$

where

$$H = \frac{\dot{a}}{a}.$$

Substituting this relation into the function gives

$$f = -6\sigma_1 H^2 + \sigma_2 T$$

The modified Friedmann equations in $f(Q, T)$ gravity can be written in the form

$$\begin{aligned} 3H^2 &= \rho_{eff} \\ 2\dot{H} + 3H^2 &= -p_{eff} \end{aligned}$$

where ρ_{eff} and p_{eff} represent the effective energy density and pressure. For the linear model, these quantities take the form

$$\begin{aligned} \rho_{eff} &= \rho + \frac{\sigma_2}{2} T \\ p_{eff} &= p + \frac{\sigma_2}{2} T \end{aligned}$$

where $T = \rho - 3p$.

Thus the cosmological dynamics are modified by the additional matter contribution proportional to T .

3.2 Nonlinear Interaction Model

To investigate the influence of nonlinear corrections, we consider the generalized function

$$f(Q, T) = Q + \alpha Q^n + \beta T + \gamma QT$$

where α, β, γ , and n are constants.

Derivatives of the function

$$\begin{aligned} f_Q &= 1 + \alpha n Q^{n-1} + \gamma T \\ f_T &= \beta + \gamma Q \end{aligned}$$

Substituting $Q = -6H^2$ gives

$$\begin{aligned} f_Q &= 1 + \alpha n (-6H^2)^{n-1} + \gamma T \\ f_T &= \beta - 6\gamma H^2 \end{aligned}$$

The gravitational Lagrangian becomes

$$f = -6H^2 + \alpha(-6H^2)^n + \beta T - 6\gamma H^2 T$$

Modified Friedmann Equations

The modified field equations lead to the generalized Friedmann equations

$$\begin{aligned} 3H^2 &= \rho_{eff} \\ 2\dot{H} + 3H^2 &= -p_{eff} \end{aligned}$$

where the effective energy density is

$$\rho_{eff} = \rho + \frac{1}{2} [f - 6H^2 f_Q]$$

Substituting the function and derivatives yields

$$\rho_{eff} = \rho + \frac{1}{2} [-6H^2 + \alpha(-6H^2)^n + \beta T - 6\gamma H^2 T - 6H^2(1 + \alpha n(-6H^2)^{n-1} + \gamma T)]$$

After simplification we obtain

$$\rho_{eff} = \rho + \frac{1}{2} [\alpha(-6H^2)^n + \beta T - 6\alpha n H^2 (-6H^2)^{n-1} - 6\gamma H^2 T]$$

Similarly, the effective pressure becomes

$$p_{eff} = p - \frac{1}{2} [f - 6H^2 f_Q] - 2\dot{H} f_Q$$

which leads to

$$p_{eff} = p - \frac{1}{2} [\alpha(-6H^2)^n + \beta T - 6\alpha n H^2 (-6H^2)^{n-1} - 6\gamma H^2 T] - 2\dot{H}(1 + \alpha n(-6H^2)^{n-1} + \gamma T)$$

These expressions show that the nonlinear terms modify both the energy density and pressure of the cosmic fluid.

3.3 Bouncing Cosmological Evolution

For a bouncing cosmological model, the scale factor must satisfy

$$a(t) > 0$$

for all cosmic times. A simple choice of bouncing scale factor is

$$a(t) = a_0(1 + \lambda t^2)$$

where a_0 and λ are positive constants.

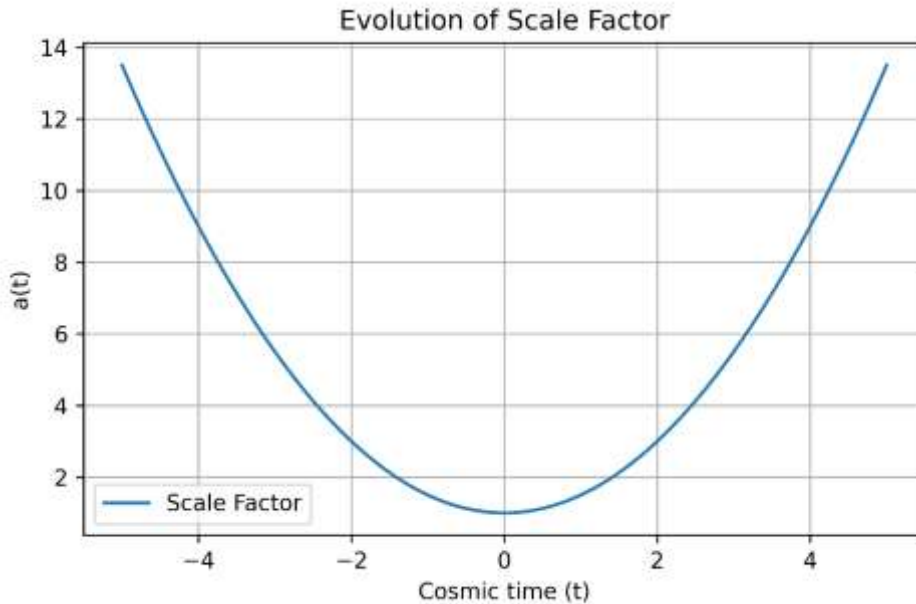


Figure 1: Evolution of the scale factor $a(t)$ for the bouncing cosmological model.

The corresponding Hubble parameter becomes

$$H = \frac{\dot{a}}{a}$$

$$H = \frac{2\lambda t}{1 + \lambda t^2}$$

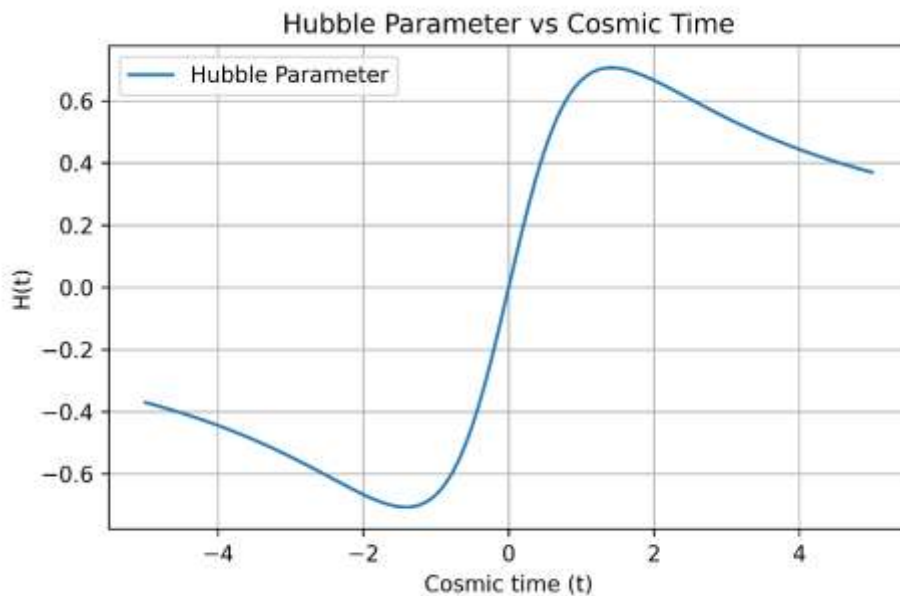


Figure 2: Behavior of the Hubble parameter $H(t)$ as a function of cosmic time.

The time derivative of the Hubble parameter is

$$\dot{H} = \frac{2\lambda(1 - \lambda t^2)}{(1 + \lambda t^2)^2}$$

At the bounce point $t = 0$

$$\begin{aligned} H &= 0 \\ \dot{H} &= 2\lambda > 0 \end{aligned}$$

which satisfies the bounce conditions.

The non-metricity scalar becomes

$$\begin{aligned} Q &= -6H^2 \\ Q &= -6\left(\frac{2\lambda t}{1 + \lambda t^2}\right)^2 \end{aligned}$$

This expression can be substituted into the nonlinear $f(Q, T)$ function to determine the behavior of cosmological quantities near the bounce.

IV. Physical Analysis

4.1 Equation of State Parameter

The equation-of-state parameter is defined as

$$\omega = \frac{p_{eff}}{\rho_{eff}}$$

Substituting the effective quantities gives

$$\omega = \frac{p - \frac{1}{2}[f - 6H^2 f_Q] - 2\dot{H} f_Q}{\rho + \frac{1}{2}[f - 6H^2 f_Q]}$$

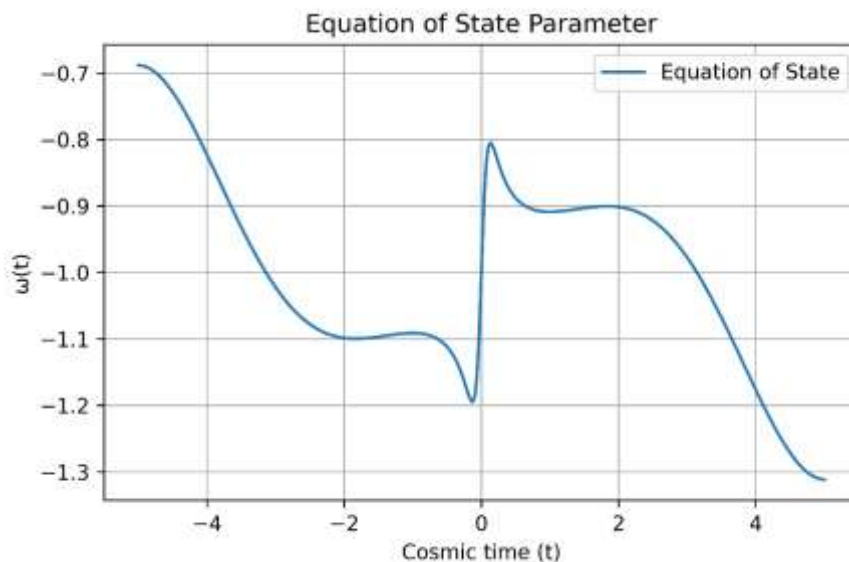


Figure 3: Evolution of the equation-of-state parameter $\omega(t)$.

For the nonlinear model this becomes

$$\omega = \frac{p - \frac{1}{2}[\alpha(-6H^2)^n + \beta T - 6\alpha n H^2(-6H^2)^{n-1} - 6\gamma H^2 T] - 2\dot{H}(1 + \alpha n(-6H^2)^{n-1} + \gamma T)}{\rho + \frac{1}{2}[\alpha(-6H^2)^n + \beta T - 6\alpha n H^2(-6H^2)^{n-1} - 6\gamma H^2 T]}$$

This parameter determines the cosmological phase of the Universe.

4.2 Energy Conditions

Energy conditions play an important role in understanding the physical viability of cosmological models. They impose constraints on the energy–momentum tensor and help determine whether the matter content of the Universe behaves in a physically reasonable manner (Hawking & Ellis 1973). In modified gravity theories, these conditions are usually written in terms of the effective energy density and pressure. Using the effective quantities obtained from the modified Friedmann equations,

$$\begin{aligned}\rho_{eff} &= \rho + \frac{1}{2}[f - 6H^2 f_Q] \\ p_{eff} &= p - \frac{1}{2}[f - 6H^2 f_Q] - 2\dot{H}f_Q\end{aligned}$$

we analyze the following energy conditions.

Null Energy Condition (NEC)

The null energy condition requires

$$\rho_{eff} + p_{eff} \geq 0$$

Substituting the effective quantities gives

$$\rho_{eff} + p_{eff} = \rho + p - 2\dot{H}f_Q$$

For the nonlinear model

$$f_Q = 1 + \alpha n(-6H^2)^{n-1} + \gamma T$$

Therefore,

$$\rho_{eff} + p_{eff} = \rho + p - 2\dot{H}(1 + \alpha n(-6H^2)^{n-1} + \gamma T)$$

Near the bounce point where $H = 0$ and $\dot{H} > 0$, this quantity becomes negative, indicating the violation of the null energy condition. Such violation is necessary for realizing a non-singular cosmological bounce (Cai et al. 2014).

Weak Energy Condition (WEC)

The weak energy condition requires

$$\rho_{eff} \geq 0$$

and

$$\rho_{eff} + p_{eff} \geq 0$$

Substituting the effective density gives

$$\rho_{eff} = \rho + \frac{1}{2}[\alpha(-6H^2)^n + \beta T - 6\alpha n H^2(-6H^2)^{n-1} - 6\gamma H^2 T]$$

The WEC is satisfied when the effective energy density remains positive. However, near the bouncing epoch the additional geometric terms may modify the sign of the effective density depending on the model parameters.

Strong Energy Condition (SEC)

The strong energy condition is defined as

$$\rho_{eff} + 3p_{eff} \geq 0$$

Substituting the effective quantities gives

$$\rho_{eff} + 3p_{eff} = \rho + 3p - 2[f - 6H^2 f_Q] - 6\dot{H}f_Q$$

For the nonlinear model we obtain

$$\begin{aligned}\rho_{eff} + 3p_{eff} &= \rho + 3p - 2[\alpha(-6H^2)^n + \beta T - 6\alpha n H^2(-6H^2)^{n-1} - 6\gamma H^2 T] \\ &\quad - 6\dot{H}(1 + \alpha n(-6H^2)^{n-1} + \gamma T)\end{aligned}$$

Violation of the strong energy condition is often associated with accelerated cosmic expansion.

Dominant Energy Condition (DEC)

The dominant energy condition requires

$$\rho_{eff} \geq 0$$

and

$$\rho_{eff} \pm p_{eff} \geq 0$$

Using the effective quantities we obtain

$$\rho_{eff} - p_{eff} = \rho - p + 2\dot{H}f_Q + [f - 6H^2f_Q]$$

Substituting the nonlinear model expressions yields

$$\rho_{eff} - p_{eff} = \rho - p + 2\dot{H}(1 + \alpha n(-6H^2)^{n-1} + \gamma T) + [\alpha(-6H^2)^n + \beta T - 6\alpha nH^2(-6H^2)^{n-1} - 6\gamma H^2 T]$$

The DEC ensures that the energy flow remains causal and that the effective energy density dominates over pressure.

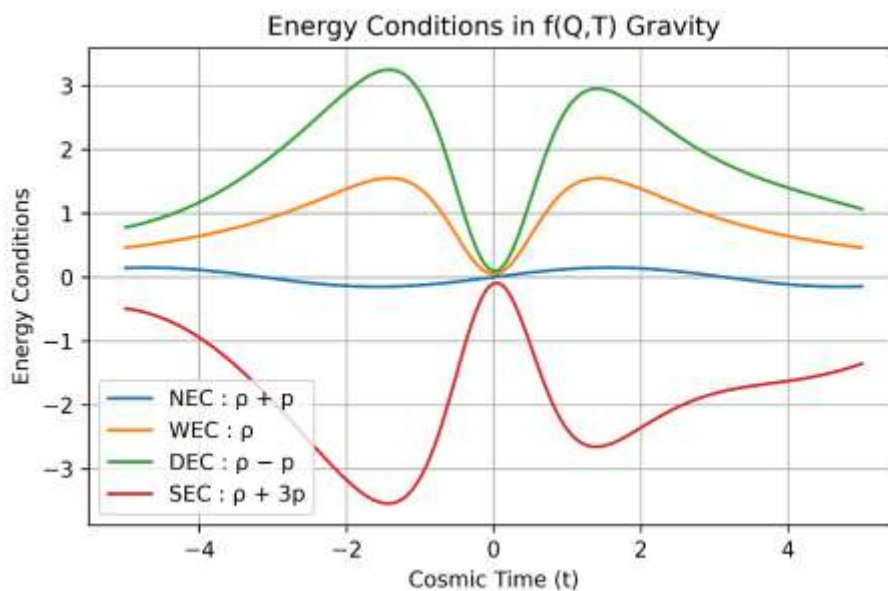


Figure 4: Evolution of the energy conditions ($\rho + p, \rho, \rho - p, \rho + 3p$) as functions of cosmic time in the nonlinear $f(Q, T)$ gravity model. The violation of the null energy condition near the bounce indicates the possibility of a non-singular cosmological transition.

V. Results And Discussion

The cosmological implications of the proposed $f(Q, T)$ gravity models are examined through both analytical expressions and graphical behavior of the relevant cosmological parameters. In particular, the evolution of the scale factor, Hubble parameter, equation-of-state parameter, and energy conditions provides insight into the physical viability of the bouncing cosmological scenario.

The evolution of the scale factor $a(t)$, illustrated in **Figure 1**, clearly demonstrates the existence of a non-singular bouncing universe. The scale factor remains positive for all cosmic times and attains a minimum value at $t = 0$, which corresponds to the bounce point. Before the bounce, the scale factor decreases, representing a contracting universe. After the bounce, the scale factor increases, indicating the onset of cosmic expansion. This behavior confirms that the proposed model successfully avoids the initial singularity that appears in the standard Big Bang cosmology.

The behavior of the Hubble parameter $H(t)$ is shown in **Figure 2**. The Hubble parameter is negative during the contracting phase and positive during the expansion phase, while it vanishes at the bounce point. The transition of $H(t)$ from negative to positive values confirms the occurrence of a smooth bounce and the transition from contraction to expansion. This sign change is a characteristic feature of bouncing cosmological models and further supports the consistency of the obtained solutions.

The evolution of the equation-of-state parameter $\omega = p/\rho$ is presented in **Figure 3**. The results indicate that the parameter generally lies within the quintessence region ($-1 < \omega < -1/3$), which corresponds to an

accelerated expansion phase of the universe. Depending on the parameter choices of the nonlinear model, the equation-of-state parameter may briefly cross the phantom boundary $\omega = -1$. Such behavior suggests that the nonlinear interaction between matter and geometry in the $f(Q, T)$ model can effectively mimic dark energy behavior and influence the expansion dynamics of the universe.

The behavior of the energy conditions is depicted in **Figure 4**, where the null, weak, dominant, and strong energy conditions are plotted simultaneously as functions of cosmic time. It is observed that the null energy condition $(\rho+p)$ becomes negative near the bounce point, indicating its violation in this region. This violation is necessary for realizing a non-singular bouncing cosmology within the framework of modified gravity theories. The weak and dominant energy conditions remain positive for a large portion of the cosmic evolution depending on the parameter choices, while the strong energy condition $(\rho+3p)$ is generally violated during the accelerated expansion phase.

Overall, the graphical analysis shows that the nonlinear interaction model of $f(Q, T)$ gravity produces physically consistent cosmological behavior. The scale factor exhibits a clear bouncing evolution, the Hubble parameter confirms the transition from contraction to expansion, and the equation-of-state parameter indicates accelerated expansion. Furthermore, the violation of the null energy condition near the bounce provides the necessary condition for avoiding the initial cosmological singularity. These results suggest that the nonlinear $f(Q, T)$ gravity model provides a viable theoretical framework for describing early-universe dynamics and the occurrence of non-singular bouncing cosmology.

VI. Conclusion

In this work, we investigated bouncing cosmological solutions within the framework of $f(Q, T)$ gravity using an anisotropic Bianchi type-I spacetime. A comparative analysis was carried out between a linear model and a nonlinear interaction model of the form $f(Q, T) = Q + \alpha Q^n + \beta T + \gamma QT$. The aim of the study was to explore whether the modified gravitational interaction between geometry and matter could lead to a non-singular cosmological evolution.

The cosmological dynamics of the model were examined through analytical expressions and graphical analysis of key parameters such as the scale factor, Hubble parameter, equation-of-state parameter, and energy conditions. The behavior of the scale factor demonstrated the existence of a non-singular bouncing universe, where the universe undergoes a contraction phase followed by a smooth transition into an expansion phase. The Hubble parameter was found to change sign at the bounce point, confirming the transition from contraction to expansion.

The evolution of the equation-of-state parameter indicated that the cosmic fluid predominantly lies in the quintessence region, suggesting accelerated expansion of the universe. Furthermore, the analysis of the energy conditions revealed that the null energy condition is violated near the bounce point, which is a necessary requirement for the realization of bouncing cosmology in modified gravity theories. The weak and dominant energy conditions remain satisfied for most cosmic times depending on the choice of parameters, while the strong energy condition is violated during the accelerated phase.

The results of this study show that the nonlinear $f(Q, T)$ model introduces stronger modifications to the cosmological dynamics compared with the linear model due to the interaction between matter and geometry. This interaction plays an important role in producing viable bouncing solutions and avoiding the initial cosmological singularity.

Overall, the present analysis suggests that nonlinear $f(Q, T)$ gravity provides a promising framework for describing early-universe dynamics and the possibility of non-singular cosmological evolution. Future investigations may focus on observational constraints of the model parameters, stability analysis of cosmological perturbations, and extensions to other anisotropic cosmological models.

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