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Fuzzy Analysis of a Forced Nonlinear Telegraph Equation for Transmission Line Design

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Abstract
Transmission line models predict voltage and current from distributed electrical parameters, yet these parameters may be known only through nominal values and tolerances. This study develops an uncertainty analysis for a forced telegraph equation derived from a uniform line with passive cubic shunt leakage. The classical linear model provides a distortionless reference, while an energy balance establishes the dissipative character of the nonlinear model. A centered spatial difference scheme with fourth order Runge–Kutta time integration is checked against an exact manufactured solution. Grid refinement yields approximately second order convergence for voltage and its time derivative. Triangular fuzzy inputs describe uncertainty in shunt conductance, nonlinear leakage, and forcing amplitude. Their alpha cuts are propagated through deterministic simulations to estimate intervals for midpoint voltage and peak response. A separate design calculation minimizes the worst mismatch between the series and shunt loss rates. In the normalized example, the selected resistance reduces that mismatch from 0.12 to 0.04 and reduces sampled phase deviation over the frequency band. These improvements come with greater attenuation. The resulting workflow distinguishes numerical error, parameter uncertainty, and design tradeoffs. The examples establish computational feasibility and indicate what measurements are needed to assess the model for a physical communication link.
Keywords: *nonlinear telegraph equation, fuzzy numbers, alpha cuts, transmission lines, numerical verification, robust design*

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I. Introduction

A communication cable changes the electrical waveform it carries. Even when the cable is uniform, resistance dissipates current, dielectric leakage draws current from the conductors, and the distributed capacitance and inductance determine how quickly changes at one location influence another. For a narrowband signal, it may be enough to know the loss near one frequency. A short pulse spans a wider band, so differences in attenuation and phase across frequencies can alter its shape. A model intended to guide cable design must therefore distinguish a reduction in amplitude from a change in the timing or shape of the signal. That distinction becomes especially important when the circuit operates near a receiver threshold.

Classical transmission line theory describes these effects through a pair of coupled equations for voltage and current (Paul, 2008; Pozar, 2012). In the constant coefficient linear model there is a useful special case. When the normalized series and shunt loss rates coincide, a forward wave on a matched line is delayed and attenuated without a change in shape. The criterion is straightforward to check for a line with known parameters. Manufacturing tolerances, temperature changes, and uncertainty in fitted loss parameters complicate the design decision. In particular, a designer may be able to choose a nominal series resistance while having only approximate bounds on the shunt conductance. A design balanced at one parameter value can then give a different response at the edges of the plausible range.

A second complication arises when a shunt element draws current in a way that depends nonlinearly on voltage. The same line may respond differently to two copies of a pulse with different amplitudes. A cubic term is a useful first model for a symmetric voltage-dependent leakage mechanism over a limited operating range. It preserves the direction of dissipation for either polarity and gives a direct way to study what remains of the linear design criterion once the waveform is large enough to excite nonlinear effects. This choice addresses a different mechanism from nonlinear reactance, which can change propagation speed and steepen a waveform (Landauer, 1960). The mathematical consequences of the two mechanisms should be distinguished when a circuit model is selected.

This study asks three questions. Can the forced nonlinear voltage equation be derived from a distributed circuit while retaining a transparent energy interpretation? Can a numerical solution resolve changes due to uncertain parameters at a scale larger than the discretization error? Finally, how much can a single choice of series resistance reduce the worst departure from the classical balance condition when shunt conductance is

imprecisely known? The first question is addressed analytically, the second through verification and parameter propagation, and the third through a minimax calculation supported by a frequency response comparison.

The analysis uses fuzzy inputs to describe graded compatibility with a nominal specification. A narrower set of values represents operating conditions close to the nominal description, while the support retains the full assumed tolerance range. This information can be used even when repeated measurements are insufficient for a probability model. The paper first establishes the governing equations and the linear reference. It then verifies a deterministic solver, propagates fuzzy inputs through a forced nonlinear example, and applies a separate loss-balance calculation to a reflection-free linear link. The two examples answer different questions and are interpreted in their own boundary and forcing settings.

Theoretical Background

Transmission line models represent a short segment of cable by series resistance and inductance and by shunt conductance and capacitance. Applying Kirchhoff's laws and taking the limit as segment length tends to zero gives the distributed equations. The formulation remains appropriate when the wavelength and geometry support the transmission line approximation (Paul, 2008). The governing variables are local voltage and current, and their product is the signed power flowing through a cross section. This interpretation motivates checking a voltage-only equation against the energy balance of the original coupled system.

The literature on nonlinear lines includes examples in which a nonlinear capacitance generates harmonics and produces rapid changes in wavefront shape. Landauer (1960) analyzed shock formation in nonlinear transmission lines and its implications for parametric amplification. In the present problem the storage parameters are constant and the nonlinearity lies in the leakage current. A local sinusoidal voltage nevertheless produces a third harmonic in the shunt current because the cube of a cosine contains both the fundamental and its third harmonic. The possibility of harmonic generation is therefore present even when the principal propagation speed in the differential equation stays fixed.

The telegraph equation also has a substantial numerical literature. For example, Mohanty (2004) constructed an implicit difference scheme for a one-dimensional linear hyperbolic equation with Dirichlet data. For the smooth semilinear example considered here, centered differences in space and explicit time integration make the implementation easy to reproduce. The advantages are a direct stencil and simple handling of the nonlinear terms. The main cost is a time step restriction that becomes more severe as the mesh is refined or the nonlinear damping grows. LeVeque (2007) provides the finite difference and stability framework used to interpret this tradeoff.

Zadeh (1965) formulated fuzzy sets through grades of membership. Dubois and Prade (1980) developed the representation of fuzzy quantities and their arithmetic. The extension principle transfers input membership through a deterministic map, and the conditions for carrying this out at the level of cuts were studied by Nguyen (1978). In a transmission line calculation the deterministic map can be an entire PDE solve followed by an output calculation, such as finding the maximum voltage over a specified time interval. Hanss (2002) demonstrated the role of cut-based simulation when uncertain parameters recur within the same model. That recurrence matters here because shunt conductance appears in both damping and the zero-order voltage term. Both appearances must use the same value during any one solve.

Mathematical Model

Let x denote distance along a uniform line of length ℓ, and let t denote time. The per-unit-length parameters are series resistance R, series inductance L, linear shunt conductance G, and shunt capacitance C, with L and C positive. Write V and I for voltage and current, and let S denote a prescribed distributed shunt current injection per unit length. The shunt leakage law is $q(V) = GV + \kappa V^3$ with $\kappa \geq 0$. In dimensional variables κ has units of amperes per metre per volt cubed. The current injection is positive when it adds energy to a region where voltage is positive. With subscripts denoting derivatives, the equations are

$$V_x = -RI - LI_t, \qquad I_x = S - CV_t - q(V). \tag{1}$$

Differentiate the series equation with respect to x and substitute the shunt equation for I_x and its time derivative for I_{xt}. Dividing by LC and collecting terms yields the forced voltage equation

$$V_{tt} - c^2 V_{xx} + \left(r + \frac{q'(V)}{C}\right) V_t + \frac{rq(V)}{C} = \frac{S_t + rS}{C}, \tag{2}$$

where $c = 1/\sqrt{LC}$, r = R/L, and $q'(V) = G + 3\kappa V^2$. The differential operator is semilinear because its highest derivatives have coefficients independent of the solution. The cubic leakage affects both the coefficient of V_t and the zero-order term. Retaining only a cubic restoring term would omit the amplitude dependence in the damping and would describe a different circuit. The forcing in (2) contains the time derivative of the physical injection. Conversely, a smooth scalar forcing F in (2) can be realized by solving $S_t + rS = CF$ with a specified initial injection at each x.

The scalar problem prescribes V(x, 0) and $V_t(x,0)$ as well as voltage boundary values. If the initial current is supplied instead of $V_t(x,0)$, the second equation in (1) determines the compatible voltage derivative. For the scalar tests below, voltage is fixed at zero at both endpoints. The initial current can be reconstructed by integrating $I_x(x,0) = S(x,0) - CV_t(x,0) - q\big(V(x,0)\big)$ and choosing one reference value of current. The first equation then advances current through a linear ordinary differential equation at each position. Substitution shows that the residual of the second equation evolves according to a homogeneous equation with decay rate r. If the residual starts at zero, it stays zero. This establishes compatibility of the voltage formulation with the coupled equations for smooth data.

The sign of the nonlinear term can be checked without solving the PDE. Multiply the series equation by I and the shunt equation by V, then add. The sum of the two spatial terms is the spatial derivative of VI. Integration along the line gives the balance

$$\frac{dE}{dt} = -(VI)_{x=\ell} + (VI)_{x=0} - \int_0^\ell (RI^2 + GV^2 + \kappa V^4)\, dx + \int_0^\ell V\, S\, dx, \tag{3}$$

where $E = 1/2 \int_0^\ell (LI^2 + CV^2)\, dx$. The boundary term accounts for power entering or leaving through the ends. Under homogeneous voltage boundary conditions, it vanishes. With zero distributed injection, the remaining terms make E nonincreasing for nonnegative R, G, and κ. The quartic term shows why the fitted leakage coefficient has a greater effect at high voltage. The energy identity also supplies a useful diagnostic for implementations of the coupled model, since the signs of boundary power and distributed dissipation can be checked directly.

For the linear reference, take κ = 0 and S = 0. Use harmonic time dependence $\exp(i\omega t)$ and select the forward propagation constant with nonnegative real part and positive imaginary part for $\omega > 0$. With g = G/C it satisfies

$$\gamma(\omega) = \sqrt{(R + i\omega L)(G + i\omega C)} = c^{-1}\sqrt{(r + i\omega)(g + i\omega)}. \tag{4}$$

When r = g, this reduces to $\gamma = (r + i\omega)/c$. The forward transfer over length ℓ is $\exp(-r\ell/c)\exp(-i\omega\ell/c)$. The first factor represents uniform amplitude loss and the second gives a fixed transit delay. This is the classical distortionless condition. Terminations matter because reflected waves can still alter the received waveform even when the forward transfer has this form. The numerical frequency example therefore assumes a forward wave without reflections.

The cubic model cannot inherit this conclusion simply by setting r = g. If a linear solution V is inserted into the nonlinear equation with the same source, the extra residual is $\kappa(3V^2V_t + rV^3)/C$. For a generic time-dependent waveform this residual is nonzero. The same point can be seen in the shunt current when $V = a\cos(\omega t)$. The current contains a third harmonic of amplitude $\kappa a^3/4$, in addition to the change in its fundamental component. The linear balance condition remains a meaningful design target near zero amplitude, where the cubic term becomes small. Its use at finite amplitude requires a separate assessment of waveform quality.

Numerical and Fuzzy Methodology

The numerical experiments use normalized variables and a unit interval in space and time. Inductance and capacitance are set to one so that c = 1. On a uniform grid $x_j = jh$, h = 1/N, let U_j approximate voltage and W_j approximate its time derivative. Homogeneous endpoint voltages are imposed at all time stages. For interior nodes, the method of lines uses

$$\dot{U}_j = W_j,$$
$$\dot{W}_j = \frac{U_{j-1} - 2U_j + U_{j+1}}{h^2} - \big(R + G + 3\kappa U_j^2\big)W_j - R\big(GU_j + \kappa U_j^3\big) + F_j(t). \tag{5}$$

Classical fourth order Runge–Kutta advances this system with time step $\Delta t = h/4$. The forcing and damping are evaluated at each stage using the stage time and voltage. For the undamped wave operator, the largest semidiscrete frequency is below 2/h, so Δt times that frequency is below 0.5 and lies inside the Runge–Kutta stability interval on the imaginary axis. This calculation motivates the step size for the tested grids. It is a wave-part check. Large nonlinear damping can introduce real eigenvalues that require a separate stability analysis. The observed convergence rates in the manufactured solution test assess the combined implementation under the stated settings.

The manufactured solution is $V^*(x,t) = \sin(\pi x)e^{-t}\cos(2t)$. Take R = G = 0.20 and κ = 0.40. Define F by substituting V^* and its derivatives into (2) in normalized form. In particular, $V^*_{xx} = -\pi^2 V^*$, and $V^*_{tt} = \sin(\pi x)e^{-t}[-3\cos(2t) + 4\sin(2t)]$. The initial voltage is $\sin(\pi x)$, and the initial derivative is $-\sin(\pi x)$. These functions satisfy the imposed endpoints. Computing F from the continuum derivatives preserves the spatial truncation error in the test. A source formed from the discrete stencil could hide that error by making the sampled reference satisfy the discrete equation directly. The method of manufactured solutions uses this distinction to test the implementation against a known answer (Roache, 2002; Salari & Knupp, 2000).

The refinement study compares maximum nodal errors in U and W at t = 1 for N = 20, 40, 80, and 160. The observed order is calculated as $\log_2(E_n/E_{2n})$, where E_n is the error at resolution N. This comparison is a verification exercise. The ability to reproduce an exact solution for constructed forcing checks the numerical treatment of the specified model. Physical validation calls for independently obtained observations of a line or a circuit with the modeled leakage mechanism (Oberkampf & Roy, 2010).

The second experiment starts from zero voltage and zero voltage derivative, retains R = 0.20, and uses $F(x,t) = A\sin(\pi x)\sin(2\pi t)$. Its forcing amplitude A is noninteractive with G and κ under the chosen membership rule. Let a triangular fuzzy number be described by its lower support endpoint a, modal value m, and upper support endpoint b. At level $\alpha \in (0,1]$, its cut is $[a + \alpha(m-a), b - \alpha(b-m)]$. The zero cut is the closure of the positive membership support. The three input triples are (0.16, 0.20, 0.24) for G, (0.20, 0.40, 0.60) for κ, and (0.80, 1.00, 1.20) for A. These illustrative values describe the spread around the normalized nominal problem. They are model inputs, with no estimated frequencies of occurrence attached to them.

Joint membership is defined by the minimum of the marginal memberships. This choice makes every cut a Cartesian box. In a deterministic run the selected G, κ, and A are kept fixed throughout the simulation. For a continuous scalar response J and compact connected input box Θ^α, the image $J(\Theta^\alpha)$ is a compact interval with endpoints obtained by minimizing and maximizing J over the box. Continuity ensures the extrema are attained, and connectedness ensures that the interval between them is filled. The finite-dimensional numerical problem has continuous dependence on its parameters for the calculations considered here. For the peak output, taking the maximum of finitely many continuous time-sample outputs preserves continuity. The corresponding continuous-time peak has the same property if the solution depends continuously on parameters uniformly in time.

Two responses are reported. The first is signed voltage at x = 0.5 and t = 1. The second is the maximum absolute midpoint voltage over the numerical time nodes from t = 0 to 1. The signed response records the voltage after one cycle of forcing, while the peak indicates the largest excursion seen at the observation point. A peak must be evaluated separately for each parameter realization before taking its range. Otherwise it is easy to confuse the minimum achievable peak with the maximum over time of a pointwise minimum. Those two operations can give different answers when the timing of the peak varies with the inputs.

At each of five levels, α = 0, 0.25, 0.50, 0.75, and 1, the procedure constructs the box, samples each coordinate on an equally spaced grid including its endpoints, and integrates (5) at every combination. A five-point grid per coordinate is run at N = 40 and 80 to compare spatial and temporal resolution. A nine-point grid at N = 80 tests sensitivity to the sampling of the parameter box. Duplicate coordinates at α = 1 represent the same modal run. The smallest and largest sampled values estimate the cut endpoints. These are ranges of computed samples. Certified endpoints would require monotonicity proofs, verified global optimization, or validated enclosures. Nestedness is a property of the exact cuts and is also checked in the reported results.

A separate design step chooses r to minimize the maximum absolute difference $|r-g|$ at a specified membership level. When g belongs to $[g_-, g_+]$, the maximum is the larger distance from r to either endpoint. For any r, the sum of the two endpoint distances is at least $g_+ - g_-$, so the larger distance is at least half the interval width. The midpoint attains that bound because the distances agree. Consequently, the unconstrained minimizer is $r^* = (g_- + g_+)/2$ and the minimum mismatch is $(g_+ - g_-)/2$. If hardware bounds restrict r to an interval, the optimum is the midpoint projected onto that feasible interval. This argument optimizes the stated loss-rate objective exactly.

The connection between this objective and frequency response follows from (4). Define m = (r + g)/2 and $d = (r-g)/2$. Then $c^2\gamma^2 = (m+i\omega)^2 - d^2$. When $|d/(m+i\omega)|$ is small, $c\gamma$ is approximately $m + i\omega - d^2/[2(m+i\omega)]$. The leading correction is quadratic in the mismatch and depends on frequency. Reducing the mismatch targets this correction, while changing r also changes m and hence the mean attenuation. The frequency comparison uses the exact complex square root. It reports the maximum change in attenuation constant between the band edges, the maximum departure of the phase constant from ω, and the range of forward amplitude transmission. These measures refer to a unit length line with c = 1.

II. Results

Verification against the Manufactured Solution

Table 1 gives the maximum errors at the end of the test. Doubling N reduces the voltage and derivative errors by approximately a factor of four. The measured orders are close to two, consistent with the spatial stencil dominating the error under the chosen time step scaling. W has a larger absolute error than U in this example. Since the solution and its derivative have different magnitudes at the reporting time, the two error columns should be read against their respective reference quantities. Both columns exhibit the expected refinement trend over the full range tested.

Table 1
Grid refinement for the manufactured solution

N	Voltage error	Order	Time derivative error	Order
20	1.28135×10^{-3}	—	4.41833×10^{-3}	—
40	3.19664×10^{-4}	2.003	1.10573×10^{-3}	1.998
80	7.98742×10^{-5}	2.001	2.76506×10^{-4}	2.000
160	1.99659×10^{-5}	2.000	6.91311×10^{-5}	2.000

The manufactured field is smooth and contains both a decaying envelope and an oscillation. This makes the derivative terms active throughout much of the integration, while the cubic terms remain significant early in the run. The result provides evidence for the discretization and implementation in this smooth regime. Spatial and temporal errors were refined together. Separating them would require an additional study in which one resolution is held fixed while the other changes. For a computation dominated by rapidly varying forcing or nonsmooth boundary data, the measured order could differ from the order in Table 1.

Fuzzy Voltage and Peak Response

Table 2 gives the estimated output cuts on the finer spatial grid and the nine-point parameter grid. At the support level the final midpoint voltage spans approximately -0.006791 to -0.003765. The peak spans approximately 0.063759 to 0.097302. At α = 0.50, the widths of both intervals are close to half their values at the support level. At α = 1 both responses reduce to their nominal values. The negative terminal response and positive peak summarize different features of the same motion. The terminal observation is taken after one cycle of forcing and its absolute value is small compared with the peak attained earlier in the interval.

Table 2
Sampled output cuts for midpoint voltage and peak magnitude

α	Final voltage lower	Final voltage upper	Peak lower	Peak upper
0.00	-0.006791	-0.003765	0.063759	0.097302
0.25	-0.006374	-0.004104	0.067888	0.093043
0.50	-0.005968	-0.004454	0.072034	0.088804
0.75	-0.005573	-0.004816	0.076198	0.084583
1.00	-0.005189	-0.005189	0.080381	0.080381

At fixed N = 80, expanding the parameter grid from five to nine points per coordinate leaves the reported endpoints unchanged to the displayed precision. Across all levels, refining N from 40 to 80 at the five-point parameter resolution changes a terminal endpoint by at most 4.10×10^{-5} and a peak endpoint by at most 1.42×10^{-5}. At the support level these changes are small relative to the output spreads, approximately 0.00303 and 0.03354, respectively. The sampled peak moves both because the underlying solution changes and because refining the time grid permits observations closer to its continuous-time maximum. The study reports the aggregate change attributable to the numerical refinement.

For the signed terminal voltage, the smallest sampled value occurs at the combination of upper endpoints of G, κ, and A at each nondegenerate cut. The largest occurs at the corresponding lower endpoints. This pattern helps explain why additional interior samples leave the reported terminal values unchanged. Its usefulness is local to the stated experiment. A different observation time may place the output on the opposite side of a zero crossing, and a different choice of forcing frequency can change which parameter combinations yield extreme voltages. A numerical search is particularly important when an output is sensitive to shifts in phase or peak time.

To assess the amplitude dependence separately from the fuzzy cuts, an additional sweep keeps G = R = 0.20 and compares κ = 0 with κ = 0.40 at forcing amplitudes 1, 5, and 10. On the N = 80 grid the linear peak divided by A is 0.080408 in all three cases, as expected from linear scaling. For the cubic model the corresponding ratios are 0.080381, 0.079747, and 0.077904. The reduction at A = 10 is approximately 3.1% relative to the linear peak per unit input. The endpoint voltage at that amplitude is -0.060814 with leakage and -0.051790 in the linear case. A lower peak can coexist with a larger magnitude at the terminal observation because the nonlinear loss changes the evolution of the full response. Amplitudes 5 and 10 lie outside the fuzzy amplitude support and serve only as a sensitivity experiment for the mathematical model.

Loss Balance and Forward Transfer

For the design comparison, g ranges from 0.16 to 0.24 because C = 1. The minimax selection is R = 0.20 since L = 1. Against the illustrative reference R = 0.12, the maximum loss-rate mismatch falls from 0.12 to 0.04, a reduction of two thirds. The forward-wave quantities in Table 3 were evaluated on a uniform grid of 401

conductance values and 701 angular frequencies on [0.5, 4.0]. The phase deviation is measured from the ideal phase constant ω in the normalized variables. The attenuation tilt is the maximum over sampled conductances of the absolute difference in attenuation constant between the two band edges. The sampled transmission range includes variation across both conductance and frequency.

Table 3

Forward wave comparison for a unit length linear link

Measure	Reference R = 0.12	Balanced R = 0.20
Worst loss-rate mismatch	0.120000	0.040000
Maximum sampled attenuation tilt	0.001118	0.000145
Maximum sampled phase deviation	0.003181	0.000354
Sampled amplitude transmission range	0.8353 to 0.8694	0.8025 to 0.8354

The smaller deviations are consistent with the quadratic correction in (4). The increase in series resistance raises the mean loss rate, producing a lower transmission range over the chosen band. This tradeoff matters because a system that is limited by received voltage may value a small increase in amplitude more than a reduction in phase error. At G = 0.20, the balanced design has exactly linear phase and constant attenuation under the forward-wave model. Elsewhere in the support it gives approximate balance. The mismatch objective is evaluated exactly, whereas the frequency response summaries describe the stated grid and band.

III. Discussion

The analysis separates three sources of variation that can otherwise be mistaken for one another. Mesh refinement addresses the error introduced by solving a differential equation on a finite grid. The fuzzy cuts describe the response to a specified family of parameter values. A comparison of resistance choices changes the design itself. Reporting a single fuzzy voltage curve would make it difficult to tell whether its width reflected genuine sensitivity to the inputs or numerical inaccuracy. Here the computed spread can be compared directly with the refinement changes, and the resistance calculation has a separate objective with a closed form solution.

A classical formula is particularly valuable when it gives a reference against which the nonlinear model can be interpreted. Equation (4) identifies the ideal transit delay and the condition for uniform attenuation. Equation (3) establishes the direction of energy transfer for the chosen nonlinear leakage. Neither formula gives a general closed form solution of the forced nonlinear boundary problem. The numerical method fills that gap by solving the same model repeatedly. Fuzzy propagation then changes the question from finding one trajectory to characterizing the range of a response across a cut. It adds information when the inputs are imprecise, while the accuracy of each deterministic trajectory still depends on the solver and the model.

The use of fixed parameter realizations preserves dependence within the model. An instructive example is the uncertain G in the damping term and in the zero-order term. An interval calculation that treated these occurrences as independent could include trajectories that no single conductance produces. The joint input model is equally important. A rectangular cut allows all combinations of the listed conductance, leakage coefficient, and amplitude. If G and κ are extracted together from the same current–voltage fit, their plausible values may lie along a much narrower region. The image-of-a-cut procedure still applies when a joint membership function is available, but the deterministic simulations must use samples from its actual cuts.

The results also depend on the observation being summarized. Peak magnitude is a practical check when a load has an upper voltage limit. The signed terminal observation answers a timing-specific question and may be relevant when a receiver samples at a prescribed instant. A pulse communication system would normally require additional outputs such as delay variation, receiver threshold crossings, and residual energy after the main pulse. Each can be computed for a complete trajectory and then propagated through the same cuts. If a design depends on simultaneous satisfaction of several requirements, the requirements should be checked on the same realization. Comparing separately optimized marginal bounds can hide whether the extremes occur together.

Application to a Communication Link

Consider a one metre laboratory line or a distributed ladder used to carry a short electrical pulse. Suppose its nominal impedance scale is 50 Ω and its transit time is 5 ns. These scales give a wave speed of 2×10^8 m/s, L = 250 nH/m, and C = 100 pF/m in the linear model. Multiplying a dimensional loss rate by the transit time gives its normalized value. The example support for G then corresponds to 3.2 to 4.8 mS/m, and the reference and balanced values of R correspond to 6 and 10 Ω/m. The normalized angular frequency band corresponds to approximately 15.9 to 127.3 MHz. These conversions give the design example physical units and describe an illustrative loaded link.

The practical implementation would start by measuring the propagation characteristics at small amplitude. A forward transfer measurement across the working band can be used to estimate attenuation and phase, with fixture and termination effects accounted for. A separate set of measurements at several amplitudes would identify whether a positive cubic leakage coefficient is compatible with observed changes in the shunt current. It would also establish the voltage range over which the cubic law is adequate. The dimensional value of κ requires a voltage scale or direct calibration. The normalized κ values in the simulations therefore remain illustrative.

The fitted model can then be used to compare feasible resistance values under the intended source and receiver conditions. For a pulse launched through one terminal, the boundary relations should incorporate the source and load impedances rather than impose zero voltage at both ends. The resulting simulation can track the received waveform and its sensitivity to the fitted parameter cuts. A nominally balanced design is useful as a candidate if the measured received amplitude satisfies the link budget and pulse distortion is a concern. If frequency-dependent losses or reflections dominate, the constant parameter balance is only one part of the comparison. Testing with pulses separate from those used for calibration would show whether the modeled uncertainty range is informative for the intended application.

The resistance comparison also illustrates how a tolerance specification translates into an operating decision. With the symmetric conductance cuts used here, changing the membership level changes the worst mismatch at the optimal design but leaves the selected midpoint fixed. At level α its value is $0.04(1-\alpha)$, whereas the reference value is $0.08 + 0.04(1-\alpha)$. This gives a direct reading of the sensitivity to the tolerance specification. If a measured conductance distribution is skewed and the left and right membership slopes differ, the midpoint can depend on the selected level. Selecting a design against the support then gives priority to the full plausible range. Selecting a higher level gives priority to the more compatible region and needs a separate check at the support.

For the illustrative one metre link, the lower ends of the sampled transmission ranges also give a simple way to read the attenuation tradeoff. Their ratio is about 0.961, equivalent to an additional loss of about 0.35 dB when the two worst sampled magnitudes are compared. This difference is small in a link with ample receiver margin and can be material in a system already operating near its sensitivity threshold. The figures describe forward transmission over the selected band. A pulse with significant spectral content outside the band would require a wider frequency comparison, and a pulse that drives the leakage into its nonlinear regime would also need a time-domain calculation with the actual load.

Limitations

The uncertainty examples use triangular inputs and smooth forcing on a uniform line. The reported cuts are based on deterministic samples and provide no enclosure guarantee between sampled inputs. They depend on the assumed support and membership profiles, which would need to be elicited or estimated for each physical application. The cubic law represents a local approximation to a leakage mechanism. Real lines may have frequency-dependent parameters, spatially varying characteristics, or reactive nonlinearities that change the leading wave dynamics. The minimax calculation assumes that R can be adjusted independently of G, L, and C. A design that changes conductor geometry or introduces distributed loading may move several of these parameters together.

The benchmark uses one manufactured field and one family of forcing profiles. A second verification case with a higher spatial mode would help separate implementation errors that are difficult to detect in a smooth fundamental-mode test. Likewise, a denser time evaluation around the midpoint peak would be needed if the required quantity were a continuous-time maximum to a prescribed accuracy. The numerical tables are interpreted at the reporting precision of the present experiments.

IV. Conclusion

A forced semilinear telegraph equation follows from a uniform transmission line with a passive cubic shunt current. Its energy balance identifies a nonnegative nonlinear loss, and its small-amplitude limit recovers the classical condition for distortionless forward propagation. An exact manufactured field verifies the numerical procedure in the smooth test problem. Propagating triangular cuts through that procedure gives nested estimates for terminal and peak voltages, with changes under mesh refinement much smaller than the support-level spreads. Balancing the nominal series rate at the midpoint of the plausible shunt rates reduces the worst mismatch from 0.12 to 0.04. The corresponding forward transfer has less sampled phase deviation and higher loss. The analytical criterion, the numerical verification, and the amplitude sweep provide complementary evidence for interpreting these results when a transmission line must meet both timing and amplitude requirements.

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