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A Comparative Regression Analysis of the Growth Structure of the Fibonacci Sequence

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Abstract
This study examines the growth structure of the Fibonacci sequence using the first 50 Fibonacci numbers and compares alternative regression models to determine the most appropriate functional form. Linear, logarithmic, quadratic, cubic, and exponential regression models are estimated and evaluated based on their explanatory power and goodness-of-fit statistics.
The results clearly indicate that the exponential regression model significantly outperforms all other specifications, explaining nearly all variation in the data ($R^2 \approx 0.9999$). This finding is fully consistent with the theoretical properties of the Fibonacci sequence, particularly Binet's formula, which describes Fibonacci numbers as an exponential function of their index.
The study demonstrates that classical mathematical sequences can be effectively analyzed and validated using statistical regression techniques, highlighting the importance of functional form selection in empirical modeling.
Keywords: *Fibonacci sequence, exponential growth, regression analysis, nonlinear models*

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I. Introduction

The Fibonacci sequence is one of the most well-known numerical sequences in mathematics and has played a fundamental role in number theory and growth modeling. Defined recursively such that each term is the sum of the two preceding ones, the sequence exhibits remarkably rich structural properties despite its simple formulation. As a result, Fibonacci numbers have attracted sustained interest from both theoretical and applied perspectives.

From a number-theoretic standpoint, the Fibonacci sequence has been extensively studied, particularly through its closed-form representation known as Binet's formula, which highlights the intrinsic connection between the sequence and exponential growth. Numerous studies have investigated generalizations of the Fibonacci sequence, including Lucas sequences and bi-periodic or r-Fibonacci sequences, revealing deeper algebraic and structural properties (Aıt-Amrane & Belbachir, 2022).

In recent years, research attention has increasingly shifted toward the statistical and analytical properties of Fibonacci numbers. Several studies have explored concepts such as convergence, summability, and statistical behavior of Fibonacci-based sequences in both real and complex domains. These works have introduced novel notions such as Fibonacci statistical convergence and summability methods, demonstrating how deterministic recursive sequences can be embedded within statistical frameworks (MDPI, 2024).

Beyond pure mathematics, Fibonacci numbers have also been employed in a wide range of applied contexts. For instance, Fibonacci-based models have been used in population dynamics, where the recursive growth structure of the sequence provides insight into biological reproduction processes (Supriatna et al., 2019). Additionally, Fibonacci-derived polynomials and functional forms have been incorporated into predictive modeling and regression-based applications, showing promising performance in empirical data analysis (Liu, 2023).

Since its introduction in medieval mathematics, the Fibonacci sequence has attracted considerable attention due to its mathematical properties and diverse applications across disciplines. The sequence appears not only in number theory but also in computer science, biology, and the natural sciences, often in connection with the golden ratio (Koshy, 2001; Vlahos, 2018).

Theoretical studies have largely focused on the derivation of closed-form expressions such as Binet's formula and the asymptotic characteristics of the sequence (Hardy & Wright, 2008).

In contrast, there is comparatively less literature on the statistical analysis of the Fibonacci sequence, particularly regarding regression-based modeling of its growth structure. While nonlinear regression and exponential growth modeling have been widely used in other applied contexts (Montgomery, Peck, & Vining, 2012; Gujarati & Porter, 2009), studies applying these techniques directly to Fibonacci numbers remain limited.

Despite this extensive body of literature, comparative regression modeling of the Fibonacci sequence itself remains relatively limited. Most existing studies focus either on theoretical properties or on indirect applications of Fibonacci numbers, while fewer studies empirically compare linear and nonlinear regression models to capture the growth dynamics of the sequence. The present study addresses this gap by conducting a comparative regression analysis of the first 50 Fibonacci numbers, evaluating linear, polynomial, logarithmic, and exponential models to empirically confirm the exponential growth behavior predicted by theory.

This study fills this gap by systematically comparing alternative regression specifications to empirically validate the exponential nature of the Fibonacci sequence.

II. Materials and Methods

Materials

The dataset used in this study consists of the first 50 terms of the Fibonacci sequence. The term index (n) is defined as the independent variable, while the corresponding Fibonacci numbers (F_n) are used as the dependent variable.

To examine the growth structure of the Fibonacci sequence from different perspectives, five regression models were estimated: linear, logarithmic, quadratic, cubic, and exponential regression models. The exponential regression model was estimated in a log-linear form by taking the natural logarithm of the dependent variable.

Model comparisons were based on measures of explanatory power (R^2 and adjusted R^2), prediction accuracy (standard error of the estimate), and overall model significance (F-statistic). All statistical analyses were conducted using SPSS software, and the significance level was set at 5%.

III. Method

Fibonacci numbers

Let $(F_n)_{n\geq 0}$ be the Fibonacci sequences, respectively, represented by $F_0 = 0$, $F_1 = 1$, $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$ (Altassan and Alan, 2022).

Starting from 0 and 1, these series are created by adding the final two digits to win the third (Orhani, 2022):

0 1 is how the series begins, so it is now 0 1 1, then 0 1 1 2, and finally 0 1 1 2 3.

This is how the series would go on: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987,...

With the exception of F(6)=8 and F(12)=144, every Fibonacci number greater than 1 includes at least one simple factor that is not a factor in any Fibonacci predecessor. This endless sequence is known as the Fibonacci sequence. Each number in the Fibonacci series or sequence is represented as. The Fibonacci formula can be used to get the sequence's Fibonacci numbers. Any specific Fibonacci number in the series, given its position, can be calculated using the formula based on the relationship between the subsequent number and the two preceding numbers. The formula to determine the $(n+1)_{th}$ number in the Fibonacci number sequence is (Dasdan, 2018),

$$F_n = F_{n-1} + F_{n-2}$$

where n>1

F_{n-1}: n_{th} Fibonacci number

F_{n-2}: $(n-1)_{th}$ Fibonacci number.

The Golden Ratio and some Comparative Cases of Two Fibonacci Numbers

Golden ratio, denoted by ϕ. Two real numbers, a > b > 0, are in the golden ratio if their ratio equals the ratio of their total to the bigger of the two amounts. That is,

$$\frac{a+b}{b} = \frac{a}{b} = \phi,$$

The Greek symbol "phi" (ϕ) represents the golden ratio. Using the above identity, we get the quadratic equation.

$$x^2 - x - 1 = 0$$

By assuming, $x = \frac{a}{b}$,

Thus,

$$\phi = \frac{1+\sqrt{5}}{2} \ and \ \psi = \frac{1-\sqrt{5}}{2}$$

ϕ and ψ are algebraic numbers, as they are the roots of a quadratic equation with integer coefficients.

Clearly, the constant ϕ satisfies the quadratic equation $\phi^2 = \phi + 1$, and is an irrational number with a value of

$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618033988749 \ldots$$

and

$$\psi = \frac{1-\sqrt{5}}{2} \approx -0.618033 \ldots$$

ϕ is inevitably the positive root since it is a ratio between positive numbers. The negative root is actually the negative inverse $-\frac{1}{\phi}$, which has

$$\psi = \frac{1-\sqrt{5}}{2} = 1 - \phi = -\frac{1}{\phi} \approx -0.618033 \ldots$$

and shares many characteristics with the golden ratio (Mehdi-Nezhad and Badawi, 2024).

Fibonacci Sequence and Exponential Growth

The Fibonacci sequence is defined recursively as

$$F_n = F_{n-1} + F_{n-2},\ F_0 = 0,\ F_1 = 1.$$

A closed-form expression for the Fibonacci numbers is given by Binet's formula:

$$F_n = \frac{1}{\sqrt{5}}(\varphi^n - (1-\varphi)^n),$$

where $\varphi = \frac{1+\sqrt{5}}{2}$ is the golden ratio.

As n increases, the second term converges to zero, implying

$$F_n \approx \frac{1}{\sqrt{5}}\varphi^n.$$

Taking natural logarithms yields a linear relationship:

$$\ln(F_n) = \ln\left(\frac{1}{\sqrt{5}}\right) + n\ln(\varphi),$$

which provides the theoretical foundation for the exponential regression model employed in this study (Koshy, 2001; Hardy and Wright, 2008).

In order to identify the functional form governing the relationship between the dependent and independent variables, a comparative regression modeling approach was employed. Following standard econometric practice, both linear and nonlinear regression specifications were estimated to assess whether a simple linear model adequately captures the underlying structure of the data (Gujarati and Porter, 2009; Wooldridge, 2016).

The analysis includes linear, logarithmic, polynomial (quadratic and cubic), and exponential regression models. Polynomial regression models are commonly used to approximate nonlinear relationships within a parametric framework (Montgomery et al., 2012). The exponential regression model was estimated in log-linear form by taking the natural logarithm of the dependent variable, which is a standard approach for modeling exponential growth processes (Gujarati and Porter, 2009).

The regression models used in the analysis are defined as follows (Gujarati & Porter, 2009; Wooldridge, 2016):

Linear model:

$$F_n = \beta_0 + \beta_1 n + \varepsilon_n$$

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

Logarithmic model:

$$F_n = \beta_0 + \beta_1 \ln(n) + \varepsilon_n$$

$$y_i = \beta_0 + \beta_1 \ln(x_i) + \varepsilon_i$$

Quadratic model:

$$F_n = \beta_0 + \beta_1 n + \beta_2 n^2 + \varepsilon_n$$

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \varepsilon_i$$

Cubic model:

$$F_n = \beta_0 + \beta_1 n + \beta_2 n^2 + \beta_3 n^3 + \varepsilon_n$$

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + \varepsilon_i$$

Exponential model:

$$\ln(F_n) = \beta_0 + \beta_1 n + \varepsilon_n$$

$$\ln(y_i) = \beta_0 + \beta_1 x_i + \varepsilon_i$$

and equivalently:

$$y_i = e^{\beta_0} \cdot e^{\beta_1 x_i} \cdot e^{\varepsilon_i}$$

Here, F_n denotes the *n*th term of the Fibonacci sequence, n represents the term index, β_i are the model parameters, and ε_n denotes the error term.
Model performance was evaluated using goodness-of-fit measures, including R^2, adjusted R^2, the standard error of the estimate, and the F-statistic. The final model selection was based on both explanatory power and the principle of parsimony, favoring models that achieve superior fit with fewer parameters (Burnham and Anderson, 2002).

IV. Results

The analysis of the linear, logarithmic, quadratic, cubic, and exponential regression models used in the study is given in Table 1.

Table 1. Comparative Table for Best Model Selection

Model	**Functional Form**	**R²**	**Adj. R²**	**Std. Error**	**F**	**p**	**Evaluation**
Linear	$y = \beta_0 + \beta_1 x$	0.233	0.217	1.94×10^9	14.56	<0.001	Weak fit
Logarithmic	$y = \beta_0 + \beta_1 \ln(x)$	0.099	0.080	2.10×10^9	5.25	0.026	Weakest model
Quadratic	$y = \beta_0 + \beta_1 x + \beta_2 x^2$	0.506	0.484	1.57×10^9	24.02	<0.001	Moderate fit
Cubic	$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$	0.732	0.714	1.17×10^9	41.78	<0.001	Strong but complex
Exponential	$\ln(y)=\beta_0 + \beta_1 x$	0.999946	0.999945	0.052	890 684.7	<0.001	Best model

As shown in Table 1, the exponential regression model is statistically significant (F = 890 684.7; $p < 0.001$). The explanatory power of the model is remarkably high ($R^2 = 0.999946$), indicating that almost all of the variation in the dependent variable is explained by the independent variable. Moreover, the low standard error of the estimate (Std. Error = 0.052) suggests a high level of predictive accuracy.
According to the coefficient estimates, the coefficient of x is positive and statistically significant ($\beta = 0.481$; t = 943.761; $p < 0.001$). This result indicates that a one-unit increase in x leads to an exponential increase in y.
The estimated exponential regression model is given by:

$$\ln(y) = 0.455 + 0.481x$$

or equivalently,

$$y = e^{0.455} \cdot e^{0.481x}$$
$$y = e^{0.455+0.481x}$$

Using the first 50 terms of the Fibonacci sequence, the comparative analysis of different regression models provides results that are fully consistent with theoretical expectations regarding the growth structure of Fibonacci numbers.
According to Binet's formula, which represents the closed-form solution of the Fibonacci sequence, the *n*th term can be approximated as follows:

$$F_n \approx \frac{1}{\sqrt{5}} \varphi^n$$

where

$$\varphi \approx 1.618$$

denotes the golden ratio.
This formulation clearly indicates that Fibonacci numbers follow an exponential growth pattern. Therefore, the exceptionally strong performance of the exponential regression model is not a statistical coincidence, but rather a theoretically expected outcome.
The results of the comparative regression analysis for selecting the best model in a mathematical context with data consisting of Fibonacci numbers are presented in Table 2.

Table 2. Selection of the Best Model in Mathematical Context

Model	**Functional Form**	**R²**	**Adj. R²**	**Evaluation**
Linear	$F_n = \beta_0 + \beta_1 n$	0.233	0.217	Cannot capture linear growth
Logarithmic	$F_n = \beta_0 + \beta_1 \ln(n)$	0.099	0.080	Weakest fit

Quadratic	$F_n = \beta_0 + \beta_1 n + \beta_2 n^2$	0.506	0.484	Short interval approximation
Cubic	$F_n = \beta_0 + \beta_1 n + \beta_2 n^2 + \beta_3 n^3$	0.732	0.714	Numerically strong but not theoretical
Exponential	$\ln(F_n) = \beta_0 + \beta_1 n$	0.999946	0.999945	Consistent with Binet's formula

The results presented in Table 2 indicate that the regression analyses conducted using the first 50 terms of the Fibonacci sequence clearly demonstrate that the growth structure of the sequence is not linear or polynomial, but rather exponential in nature. The exponential regression model is statistically highly significant ($F = 890{,}684.7$; $p < 0.001$) and explains almost all of the variation in the dependent variable ($R^2 \approx 0.9999$).
When the term index (n) is used as the independent variable, the positive and statistically significant coefficient ($\beta = 0.481$; $p < 0.001$) confirms that Fibonacci numbers follow an increasing-rate growth process.
Although all estimated models are generally statistically significant, substantial differences are observed in their explanatory power. Linear and logarithmic models fail to adequately capture the growth dynamics of the Fibonacci sequence. Quadratic and cubic models provide higher goodness-of-fit measures; however, their increased complexity limits their theoretical consistency.
In contrast, the exponential regression model clearly outperforms all other models, exhibiting an exceptionally high explanatory power ($R^2 \approx 0.9999$) and a very low prediction error. These findings provide strong evidence that the growth structure of the Fibonacci sequence is fundamentally exponential.
The linear regression model fails to capture the exponential growth pattern of the Fibonacci sequence and therefore provides the weakest fit among all estimated models.
The logarithmic model offers only a limited approximation over a short range and does not adequately represent the true growth dynamics of Fibonacci numbers.
Polynomial models (quadratic and cubic) achieve relatively high numerical goodness-of-fit measures due to their functional flexibility. However, despite their strong numerical performance, these models lack a solid theoretical foundation.
The exponential regression model, in contrast, is fully consistent with Binet's formula, making it the most appropriate model both statistically and theoretically.
The empirical success of the exponential regression model directly reflects the theoretical exponential structure of the Fibonacci sequence, rather than being a consequence of model overfitting or sample-specific behavior.

V. Discussion

The Fibonacci sequence is a fundamental mathematical structure that is well known in the literature to exhibit exponential growth through its closed-form solution, known as Binet's formula. The findings of this study demonstrate that this theoretical property is strongly confirmed within a statistical regression framework.

Although polynomial models, particularly cubic regression, yield high levels of explanatory power (R^2), this does not imply that they accurately capture the true growth structure of the sequence. Rather, such models provide numerical approximation over a limited range of the data, while failing to explain the underlying growth mechanism of the Fibonacci sequence. The ability of polynomial models to achieve high R^2 values primarily stems from their functional flexibility and should not be interpreted as evidence of theoretical adequacy.

In contrast, the exponential regression model, owing to its parsimonious structure and its direct consistency with Binet's formula, provides the most appropriate representation of the growth dynamics of the Fibonacci sequence. Consequently, the superior statistical performance of the exponential regression model should be regarded not merely as a numerical outcome, but as a theoretically expected and mathematically well-grounded result.
The comparison of alternative functional forms allows the researcher to distinguish between numerical goodness-of-fit and structural consistency of the model.

VI. Conclusion

This study examines the growth structure of the first 50 terms of the Fibonacci sequence using various regression models. The findings indicate that the sequence cannot be adequately represented by linear or polynomial growth models, whereas the exponential regression model provides the most appropriate approach from both theoretical and empirical perspectives.

Although polynomial models yield high explanatory power, only the exponential regression model is theoretically consistent with Binet's formula. Therefore, the dominance of the exponential model reflects not a statistical coincidence, but a theoretically expected result grounded in the mathematical structure of the Fibonacci sequence.

The near-perfect fit of the exponential model is fully consistent with the exponential nature of the Fibonacci sequence as expressed by Binet's formula. In this regard, the study demonstrates that a classical mathematical sequence can be robustly validated through statistical modeling techniques.

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