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Asymptotic attenuation of oscillations in a two-stage vibration isolation system

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Abstract *The aim of this work is to develop a methodology for selecting parameters of a two-stage vibration isolation system to achieve an asymptotically stable oscillatory process with minimal possible amplitudes. The methodology is based on V.S. Voronov's algebraic criteria and makes it possible to find optimal coefficient values that meet the requirements of dynamic stability. In the course of the study, the following tasks were accomplished: two vibration isolation schemes were examined; a parameter selection method for the proposed two-stage vibration isolation model was developed; the vibration damping criteria for stability were verified on a specific example using V.S. Voronov's conditions; graphical illustrations were provided and corresponding conclusions were drawn. The most effective vibration isolation schemes were found to be those with viscoelastic modules connected in series.*

Keywords: *vibration isolation, vibration damping, hydraulic support, inertial mass, stability criteria, damping coefficient, mathematical model, dynamic model, dynamic diagram, vibration suppression, configuration, attenuation, two-stage vibration isolation, three-mass mechanical system, kinematic excitation, vibration displacement, vibration velocity, vibration acceleration, performance indices*.

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I. INTRODUCTION

In the modern engineering and scientific world, vibrations are regarded as harmful phenomena that must be completely eliminated. They occur in mechanical, electrical, hydraulic, and other technical systems, and their control plays a critical role in ensuring the reliability, safety, and operational efficiency of machinery. Left unaddressed, vibrations can lead to structural failures, reduced equipment performance, and even severe consequences for service personnel. For instance, in the aerospace industry, vibrations may induce material fatigue, ultimately resulting in catastrophic outcomes. In civil engineering, vibrations from industrial equipment or transportation systems can adversely affect the stability of buildings and structures.

One of the methods to combat the aforementioned phenomena is the use of vibration damping systems. Hydraulic vibration dampers have also found wide application, and improving them remains a pressing task [1]. This work is devoted to investigating vibration damping using a two-stage hydraulic support system with an inertial mass.

This work focuses on investigating vibration suppression in mechanical systems using a rigorously defined configuration. The research examines a schematic comprising a combination of springs, dampers, and masses subjected to external random harmonic excitation. The primary objective is to determine optimal parameters for a two-stage vibration isolation system in mechanical structures through original parameter selection methodologies developed by the authors. The achieved results demonstrate asymptotically stable oscillatory regimes with minimal vibration amplitudes, ensuring both qualitative stability and stability margins based on V.S. Voronov's criteria [2]. Particular emphasis is placed on analyzing asymptotic system stability and developing control methodologies. The investigation employs mathematical modeling techniques, along with analytical and numerical methods.

The expected results of the work include the development of new theoretical approaches, improved computational speed of the mathematical model, and enhanced overall accuracy and reliability.

1.1 Derivation of the laws of motion for a physical suspension model with hydraulic support

The study examines a rigorously defined vibration suppression scheme in mechanical systems, as illustrated in Figure 1. The configuration consists of a uniaxial hydraulic support with inertial transformer parameters denoted by subscript "2". It includes the hydraulic support's linear stiffness coefficient c_2, damping coefficient b_2 of the hydraulic medium, and inertial mass m_2; and the passive isolation elements: isolated mass m, cylindrical-spring stiffness c_1, and hydraulic-shock-absorber damping b_1 (linear in vertical velocity).

The investigation focuses on how the additional inertial mass within the hydraulic support affects the attenuation of both natural and forced vibrations in the parallel-connected hydraulic-mechanical damper system shown in Figure 1.

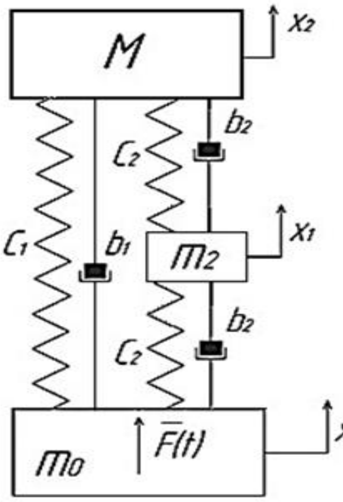

Figure 1: Dynamic model of a two-stage vibration isolation system comprising a main body of mass *M* and an inertial mass m_2 in the hydraulic support

The dynamic diagram in Fig. 1 represents the motion scheme of a three-mass mechanical system, where the masses m_2 and M have finite magnitudes. The base mass m_0 is considered infinite, to which a periodically varying vibrational load $F(t) = A\cdot\sin(\omega t)$ is applied, causing kinematic excitation with vibration displacement x_0, amplitude a, and frequency ω, as well as well as base velocity $\dot{x}_0$ and acceleration $\ddot{x}_0$.

The equations of motion of the physical model of the sprung mass are derived using Lagrange's equations of the second kind. Figure 1 shows the simplest dynamic schematic of the physical model. The dynamic diagram in Fig. 1 represents the motion of the vibration-isolated body of mass M. The sprung inertial mass m_2 of the hydraulic support is connected to the body by an elastic element c_2 and a damping element b_2, and an elastic element c_1 together with a damping element b_1 are placed between the body and the base. These characteristics can be readily determined experimentally, but their mathematical description limits the analysis to purely vertical oscillations.

Based on these definitions, the general equations of motion for the two-mass portion of the system are:

$$\begin{cases} \ddot{x}_1 = b_{10}\dot{x}_0 + c_{10}x_0 - b_{11}\dot{x}_1 - c_{11}x_1 + b_{12}\dot{x}_2 + c_{12}x_2, \\ \ddot{x}_2 = b_{20}\dot{x}_0 + c_{20}x_0 + b_{21}\dot{x}_1^2 + c_{21}x_1 - b_{22}\dot{x}_2^2 - c_{22}x_2. \end{cases} \tag{1}$$

Let the system of differential equations of motion be written after applying Lagrange's formalism and transformed. We omit the full form here for brevity. Introducing the notations

$$\begin{aligned} &b_{10} = b_2/m_2, \quad b_{11} = 2b_2/m_2, \quad b_{12} = b_2/m_2, \\ &b_{20} = b_1/M, \quad b_{21} = b_2/M, \quad b_{22} = (b_1+b_2)/M, \\ &c_{10} = c_2/m_2, \quad c_{11} = 2c_2/m_2, \quad c_{12} = c_2/m_2, \\ &c_{20} = c_1/M, \quad c_{21} = c_2/M, \quad c_{22} = (c_1 + c_2)/M, \\ &x_0 = -\,a\cdot\omega^{-2}\cdot\sin(\omega t), \\ &\dot{x}_0 = -\,a\cdot\omega^{-1}\cdot\cos(\omega t), \\ &\ddot{x}_0 = a\cdot\sin(\omega t), \\ &F(t) = m_0\ddot{x}_0(t), \end{aligned} \tag{2}$$

the system can be presented in canonical form [3–14]. The initial conditions at time $t = 0$ are:

$$x_1(0) = 0.01 \text{ м}, \quad \left.\frac{dx_1}{dt}\right|_{t=0} = 0.03 \text{ м/с}, \quad x_2(0) = -0.02 \text{ м}, \quad \left.\frac{dx_2}{dt}\right|_{t=0} = 0.005 \text{ м/с}. \tag{3}$$

In this canonical form, the system of equations (1) becomes:

$$\begin{cases} L_1(x_1) - d_{12}(x_2) = -a\left(\dfrac{c_{10}}{\omega^2}\sin(\omega t) + \dfrac{b_{10}}{\omega}\cos(\omega t)\right), \\ -d_{21}(x_1) + L_2(x_2) = -a\left(\dfrac{c_{20}}{\omega^2}\sin(\omega t) + \dfrac{b_{20}}{\omega}\cos(\omega t)\right), \end{cases} \tag{4}$$

where L_1, L_2, d_{12}, d_{21} are differential operators of the form:

$$L_1(\square) = \frac{d^2}{dt^2} + b_{11}\frac{d}{dt} + c_{11},$$
$$L_2(\square) = \frac{d^2}{dt^2} + b_{22}\frac{d}{dt} + c_{22},$$
$$d_{12}(\square) = b_{12}\frac{d}{dt} + c_{12},$$
$$d_{21}(\square) = b_{21}\frac{d}{dt} + c_{21}.$$

Using this notation, the equations in $x_1(t)$ and $x_2(t)$ can be separated, and the mathematical model transforms to a new form:

$$L_1(L_2(x_i)) - d_{12}(d_{21}(x_i)) = \frac{d^4x_i}{dt^4} + (b_{11} + b_{22})\frac{d^3x_i}{dt^3} + (c_{11} + c_{22} + b_{11}b_{22} - b_{12}b_{21})\frac{d^2x_i}{dt^2} + \\ +(c_{11}b_{22} + c_{22}b_{11} - c_{12}b_{21} - c_{21}b_{12})\frac{dx_i}{dt} + (c_{22}c_{11} - c_{12}c_{21})x_i, \quad i = 1, 2. \tag{5}$$

then the mathematical model transforms into a new form

$$\begin{cases} L_1(L_2(x_1)) - d_{12}(d_{21}(x_1)) = \eta_{11}\sin(\omega t) + \eta_{12}\cos(\omega t), \\ L_1(L_2(x_2)) - d_{12}(d_{21}(x_2)) = \eta_{21}\sin(\omega t) + \eta_{22}\cos(\omega t), \end{cases} \tag{6}$$

where the coefficients are determined by the following expressions:

$$\eta_{11} = a\frac{\left(\left(Mc_2 + b_2(2b_1 + b_2)\right)\omega^2 - c_2(2c_1 + c_2)\right)}{\omega^2 m_2 M},$$
$$\eta_{12} = a\frac{\left(b_2\left(M\omega^2 - 2(c_1 + c_2\right) - 2b_1c_2\right)}{\omega m_2 M},$$
$$\eta_{21} = a\frac{\left(\left(b_2\left(2b_1 + b_2\right) + c_1m_2\right)\omega^2 - c_2(2c_1 + c_2)\right)}{\omega^2 m_2 M},$$
$$\eta_{22} = a\frac{\left(b_1m_2\omega^2 - 2\left(b_1c_2 + b_2(c_1 + c_2)\right)\right)}{\omega m_2 M}.$$

in which the coefficients are determined by the expressions:

$$\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0. \tag{7}$$

The overall characteristic equation of the system is constructed from the left-hand sides of system (6) according to (5):

$$x_i(t) = \mathrm{e}^{-\lambda_1 t}\left(C_{i1}\sin(\lambda_2 t) + C_{i2}\cos(\lambda_2 t)\right) + \mathrm{e}^{-\lambda_3 t}\left(C_{i3}\sin(\lambda_4 t) + C_{i4}\cos(\lambda_4 t)\right) + C_{i5}\sin(\omega t) + C_{i6}\cos(\omega t), \quad i = 1,2 \tag{8}$$

with $\lambda_1>0$ and $\lambda_3>0$. The unknown constants C_{ij}, $i=1,2$, $j = \overline{1,6}$ are determined from the solution of the homogeneous system of differential equations (with initial conditions (3)) and from the particular solution of the non-homogeneous system (4). The first four terms in (8) ensure a rapid convergence of the free vibration trajectories $x_1(t)$ and $x_2(t)$ to their equilibrium positions $x_1(t) \to 0$ and $x_2(t) \to 0$ for $\lambda_1>0$ and $\lambda_3>0$. This leads to a rapid decay of these motions, with their amplitudes asymptotically decreasing to zero. Meanwhile, the forced oscillations (terms with C_{i5} and C_{i6}) should have minimal magnitudes.

1.2 Application of stability criteria for dynamic systems to ensure rapid damping of oscillations

Let us investigate the stability of the dynamic system using the well-known algebraic criteria of V.S. Voronov [2]. For this purpose, we extract the coefficients from the characteristic equation (7) in accordance with the notation adopted in those criteria

$$\begin{aligned}
&a_0 = c_{11}c_{22} - c_{12}c_{21},\\
&a_1 = c_{11}b_{22} + c_{22}b_{11} - c_{12}b_{21} - c_{21}b_{12},\\
&a_2 = c_{11} + c_{22} + b_{11}b_{22} - b_{12}b_{21},\\
&a_3 = b_{11} + b_{22},\\
&a_4 = 1.
\end{aligned} \tag{9}$$

The necessary condition for the system's stability is the fulfillment of the inequalities:

$$\frac{a_0}{a_2} < \frac{a_1}{a_3} < \frac{a_2}{a_4} < \ldots < \frac{a_{n-2}}{a_n}. \tag{10}$$

The sufficient stability conditions correspond to constraints of the form:

$$\Omega_k = \frac{a_k^2}{a_{k-1}a_{k+1}} > \sqrt{2.148}, \quad k = \overline{1, n-1}. \tag{11}$$

The stability margin conditions are defined by requirements:

$$W_k = \frac{a_k a_{k+1}}{a_{k-1}a_{k+2}} > 3, \quad k = \overline{1, n-2}. \tag{12}$$

Finally, the stability conditions derived from the system's performance indices must satisfy the inequalities:

$$\Omega_k > \sqrt{3}, \quad k = \overline{1, n-1}., \tag{13}$$

To ensure asymptotically stable and fast-decaying oscillations, the last two inequalities (12) and (13) must hold simultaneously. Substituting expressions (2) into conditions (9), (12), and (13) yields nonlinear multiparameter inequalities which, in general, cannot be solved analytically. To simplify the analysis, we introduce dimensionless combinations of parameters (ratios of physical coefficients having the same dimensions)

$$\frac{c_1}{c_2} = k, \quad \frac{b_1}{b_2} = l, \quad \frac{M}{m_2} = n. \tag{14}$$

Let $\alpha = c_2 m_2 / b_2^2$ then, substituting α into the coefficients (9), we obtain new expressions for those coefficients:

$$a_0 = \frac{c_2^2(2k+1)}{m_2^2 n}, \quad a_1 = \frac{2b_2c_2(k+l+1)}{m_2^2 n}, \quad a_2 = \frac{b_2^2(2l+3)+c_2m_2(k+2n+1)}{m_2^2 n}, \quad a_3 = \frac{b_2(l+2n+1)}{nm_2}, \quad a_4 = 1. \quad (15)$$

Substituting these values into the expressions for the system's performance indices and stability margins yields:

$$\begin{cases} \Omega_1 = \dfrac{4(k+l+1)^2}{(2k+1)(\alpha(k+2n+1)+2l+3)}, \\ \Omega_2 = \dfrac{(\alpha(k+2n+1)+2l+3)^2}{2\alpha(k+l+1)(l+2n+1)}, \\ \Omega_3 = \dfrac{(l+2n+1)^2}{n(\alpha(k+2n+1)+2l+3)}, \end{cases} \quad (16)$$

$$\begin{cases} W_1 = \dfrac{2(k+l+1)(\alpha(k+2n+1)+2l+3)}{\alpha(2k+1)(l+2n+1)}, \\ W_2 = \dfrac{(l+2n+1)(\alpha(k+2n+1)+2l+3)}{2\alpha n(k+l+1)}. \end{cases} \quad (17)$$

Next, let us determine the behavior of Ω_i and W_i as the parameters k, n, l grow without bound. To do this, we compute the corresponding limits:

$$\begin{aligned}
&1)\ \lim_{k\to\infty}\Omega_1 = \frac{2}{\alpha} > \sqrt{3}, \\
&2)\ \lim_{n\to\infty}\Omega_1 = 0, \\
&3)\ \lim_{l\to\infty}\Omega_1 = \infty, \\
&4)\ \lim_{k\to\infty}\Omega_2 = \infty, \\
&5)\ \lim_{n\to\infty}\Omega_2 = \infty, \\
&6)\ \lim_{l\to\infty}\Omega_2 = \frac{2}{\alpha} > \sqrt{3}, \\
&7)\ \lim_{k\to\infty}\Omega_3 = 0, \\
&8)\ \lim_{n\to\infty}\Omega_3 = \frac{2}{\alpha} > \sqrt{3}, \\
&9)\ \lim_{l\to\infty}\Omega_3 = \infty, \\
&10)\ \lim_{\alpha\to 0}\Omega_1 = \frac{4(l+k+1)^2}{(2l+3)(2k+1)} > \sqrt{3}, \\
&11)\ \lim_{\alpha\to\infty}\Omega_1 = 0, \\
&12)\ \lim_{\alpha\to 0}\Omega_2 = \infty, \\
&13)\ \lim_{\alpha\to\infty}\Omega_2 = \infty, \\
&14)\ \lim_{\alpha\to 0}\Omega_3 = \frac{(l+2n+1)^2}{n(2l+3)} > \sqrt{3}, \\
&15)\ \lim_{\alpha\to\infty}\Omega_3 = 0.
\end{aligned} \quad (18)$$

Based on these calculations, we conclude that the limits in points 3–5, 9, 12, and 13 are always satisfied, whereas conditions 2, 7, 11, and 15 indicate that parameters n, k, and α should never be taken too large. Conditions 1, 6, and 8 all lead to the same requirement (yielding an identical constraint)

$$\alpha < \frac{2}{\sqrt{3}} \cdot \quad (19)$$

Following the same approach, we will determine the limits for the stability margins W_1 and W_2

$$\begin{aligned}
&16)\ \lim_{k\to\infty} W_1 = \infty,\\
&17)\ \lim_{n\to\infty} W_1 = \frac{2(l+k+1)}{2k+1} > 3,\\
&18)\ \lim_{l\to\infty} W_1 = \infty,\\
&19)\ \lim_{k\to\infty} W_2 = \frac{l+2n+1}{2n} > 3,\\
&20)\ \lim_{n\to\infty} W_2 = \infty,\\
&21)\ \lim_{l\to\infty} W_2 = \infty,\\
&22)\ \lim_{\alpha\to 0} W_1 = \infty,\\
&23)\ \lim_{\alpha\to\infty} W_1 = \infty,\\
&24)\ \lim_{\alpha\to 0} W_2 = \infty,\\
&25)\ \lim_{\alpha\to\infty} W_2 = \infty.
\end{aligned} \quad (21)$$

Eight conditions from points 16, 18, and 20–25 are satisfied automatically. Solving conditions 10, 14, 17, and 19 together gives relationships between l, n, and k in the form:

$$l > \frac{1}{2}, \quad (22)$$

$$0 < n < \frac{l+1}{4}, \quad (23)$$

$$0 < k < \frac{l}{2} - \frac{1}{4} \cdot \quad (24)$$

On the basis of the obtained criteria (19) and (22)–(24) for choosing the physical coefficients of the dynamic system shown in Fig. 1 – in order to ensure the fastest possible damping of vibrations – one should adhere to the following recommended procedure for selecting *l*, *n*, *k* and α:

1) Using the known input parameters m_2, b_2 and α, calculate c_2 from formula (19).
2) Choose an arbitrary value for coefficient l that satisfies inequality (22).
3) Select *n* from the range given by the double inequality (23).
4) Select *k* from the range given by the double inequality (24).

5) At the next stage, verify that condition (25) is satisfied.

$$8n - 4k > 3, \quad (25)$$

6) Using the obtained values, compute: $b_1 = l \cdot b_2$, $M = n \cdot m_2$, $c_1 = k \cdot c_2$.
7) Finally, using formulas (15)–(17), verify that all necessary and sufficient stability conditions, as well as the stability margin and performance conditions (10)–(13), are fulfilled.

II. RESULT AND DISCUSSION

1.3 Determination of Physical Coefficients According to the Proposed Methodology

For the hydraulic support scheme shown in Fig. 1, assume the physical parameters m_2 = 0.25 kg and b_2 = 10 N·s/m are known a priori. To determine the remaining variables, we will apply the above methodology:

1) From the given m_2, b_2, and an assumed α, formula (19) yields c_2 = 396 N/m.

2) Let l = 1500 (this value satisfies inequality (22)), which gives b_1 = 15000 N·s/m.
3) From double inequality (23), choose n = 375, hence M = 93.75 kg.
4) From double inequality (24), choose k = 749, hence c_1 = 296604 N/m.
5) Next, verify condition (25).
All inequalities (10)–(13) are then checked:

$$\begin{array}{l} \frac{a_0}{a_2} = 524.002; \quad \frac{a_1}{a_3} = 3166.6; \quad \frac{a_2}{a_4} = 19140.27; \\ \Omega_1 = 3.011; \quad \Omega_2 = 2.007; \quad \Omega_3 = 3.012; \\ W_1 = 6.043; \quad W_2 = 6.044. \end{array} \quad (26)$$

As demonstrated, all necessary and sufficient stability conditions, as well as stability margin and performance criteria, are fully satisfied.

At the final stage, we must determine how the newly selected parameters of the vibration isolation system align with the goal of rapid damping of both natural (free) and forced oscillations. To achieve this, all the 9 new values of the physical quantities in the dynamic system (Fig. 1) are substituted into the problem setup (4) with initial data (3). The newly imposed coefficient constraints should result in asymptotic decay of the natural oscillation amplitude and minimization of the forced oscillation amplitude. Exact solutions of the original system of differential equations were obtained for an excitation frequency ω = 5 rad/s and vibration acceleration amplitude a = 0.57 m/s² (external load). From these solutions, the natural oscillation components $x_1^o(t)$ and $x_2^o(t)$ were isolated:

$$\begin{aligned} x_1^o(t) = &- 0.008206959494 \cdot e^{-23.12213432 \cdot t} - 0.001073282948 \cdot e^{-137.0114208 \cdot t} + \\ &+ 0.01459319212 \cdot e^{-39.9865558 \cdot t} \cdot \sin(39.58493245 \cdot t) + \\ &+ 0.01923553517 \cdot e^{-39.9865558 \cdot t} \cdot \cos(39.58493245 \cdot t), \end{aligned} \quad (27)$$

$$\begin{aligned} x_2^o(t) = &- 0.02307089788 \cdot e^{-23.12213432 \cdot t} + 0.003024230672 \cdot e^{-137.0114208 \cdot t} - \\ &+ 0.00002239367731 \cdot e^{-39.9865558 \cdot t} \cdot \sin(39.58493245 \cdot t) + \\ &+ 3.188322958 \cdot 10^{-6} \cdot e^{-39.9865558 \cdot t} \cdot \cos(39.58493245 \cdot t). \end{aligned} \quad (28)$$

Likewise, the forced components $x_1^*(t)$ and $x_2^*(t)$ are obtained as:

$$x_1^*(t) = - 0.02306443308 \cdot \sin(5.0 \cdot t) + 0.00004470727293 \cdot \cos(5.0 \cdot t), \quad (29)$$

$$x_2^*(t) = - 0.02297064413 \cdot \sin(5.0 \cdot t) + 0.00004347888442 \cdot \cos(5.0 \cdot t). \quad (30)$$

The general solutions (combining free and forced parts) satisfy the problem (4) with initial conditions (3) and must be combined as $x_1(t) = x_1^o(t) + x_1^*(t)$ и $x_2(t) = x_2^o(t) + x_2^*(t)$. All arbitrary coefficients have been expressed as functions of the parameters, but are omitted here due to their cumbersome form [3 – 14].

From the derived formulas, it is straightforward to plot the motion laws of the bodies in the system – namely, the vibration-isolated body and the inertial mass of the hydraulic support. The decaying free-vibration responses are shown in Figs. 2 and 3, and the combined (total) vibration responses are shown in Figs. 4 and 5.

To evaluate the effectiveness of the vibration damping model with the hydraulic mount (Figure 1) using the newly selected coefficients, we compare it with a single-mass model lacking both the hydraulic mount and coefficients m_2, b_2, and c_2. The system parameters remain unchanged: M = 93.5 kg, b_1 = 15000 N·s/m, c_1 = 296604 N/m, ω = 5 rad/s, a = 0.57 m/s².

We will determine which damping configuration proves superior - the single-stage (one-mass) or two-stage (hydraulic mount) system. The governing differential equation for the body of mass M is:

$$\frac{d^2 x_2(t)}{dt^2} + \frac{b_1}{M} \cdot \frac{dx_2(t)}{dt} + \frac{c_1}{M} \cdot x_2(t) = -\frac{ab_1}{M\omega} \cdot \cos(\omega t) - \frac{ac_1}{M\omega^2} \cdot \sin(\omega t) \cdot \quad (31)$$

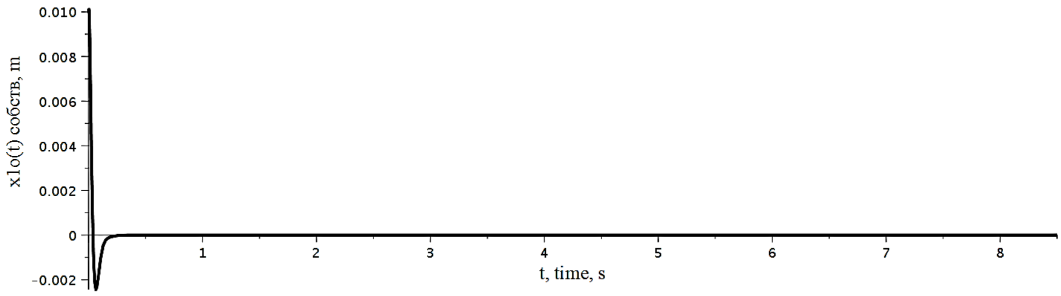

Figure 2: Natural vibrations of the hydraulic support's inertial mass

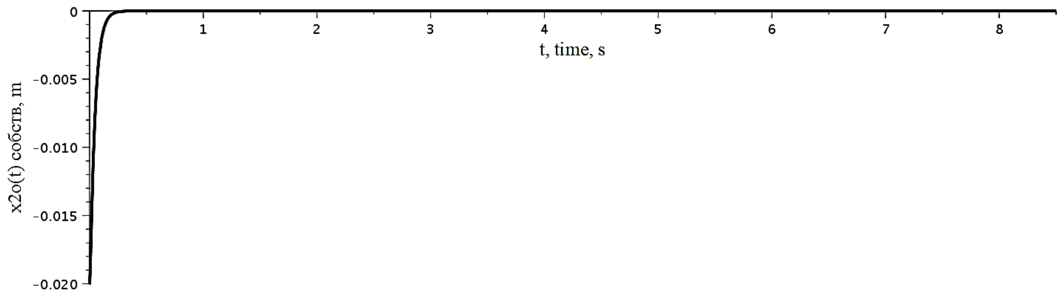

Figure3: Natural vibrations of the vibration-isolated body.

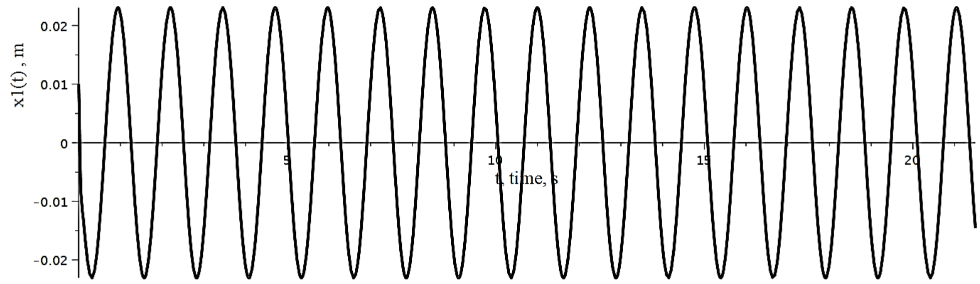

Figure 4: Combined vibrations (natural + forced) of the hydraulic support's inertial mass

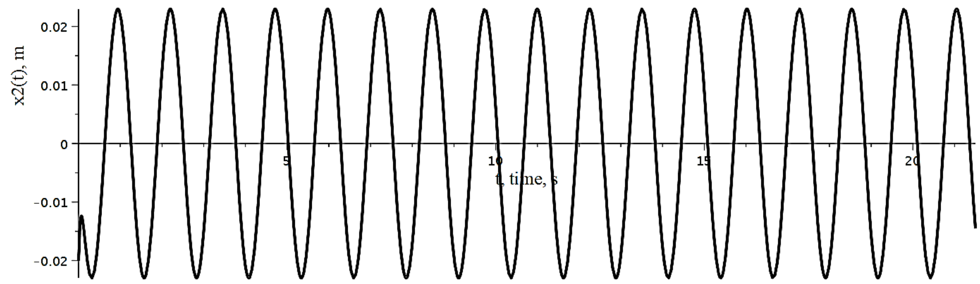

Figure 5: Combined vibrations (natural + forced) of the vibration-isolated body

Exact integration of equation (31) (with all known coefficients) yields the response $x_2(t) = x_2^o(t) + x_2^*(t)$, consisting of the natural part $x_2^o(t)$ and the forced part $x_2^*(t)$

$$x_2^o(t) = -\,0.02308020512 \cdot e^{-23.14330560 \cdot t} + 0.003036680462 \cdot e^{-136.4311625 \cdot t}, \qquad (32)$$

$$x_2^*(t) = -\,0.02297087896 \cdot \sin(5 \cdot t) + 0.00004352465656 \cdot \cos(5 \cdot t), \qquad (33)$$

The time-domain plot of $x_2(t)$ is shown in Figure 6.

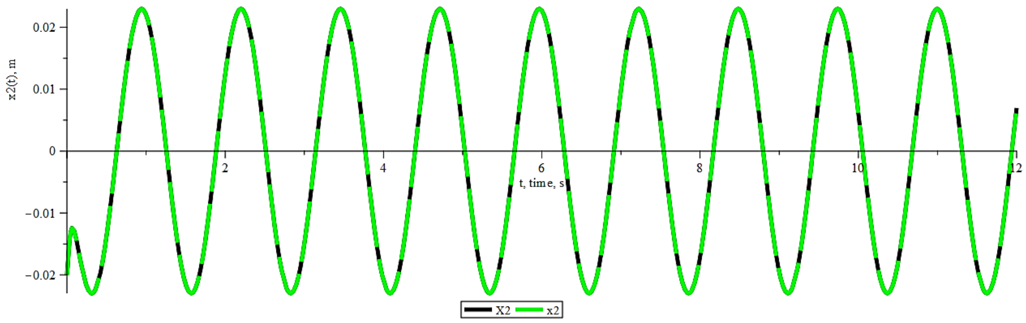

Figure 6: Total vibration response of the vibration-isolated body – without hydraulic support – $x_2(t)$, and with hydraulic support – $X_2(t)$

III. CONCLUSION

For the two-stage vibration isolation system shown in Figure 1, it is possible to select physical parameters that ensure asymptotic stability with a margin for natural dynamic modes. However, significant reduction in forced random vibration amplitudes cannot be achieved. This practical outcome was observed by the authors during testing of a prototype driver's seat suspension, and the experimental results proved unsatisfactory. Contrary to expectations, the introduction of a second stage did not redistribute or dissipate high levels of kinetic energy in the isolated object. Analytical proofs (Eqs. 28, 30, 32, 33) demonstrate that the Figure 1 isolation system cannot substantially reduce vibration amplitudes of mass M to permissible levels, regardless of admissible damping coefficients b_2. Figure 6 presents the kinematic responses: $X_2(t)$ (calculated via Eqs. 28, 30) for parallel-connected viscoelastic elements and the hydraulic mount, $x_2(t)$ (derived from Eqs. 32, 33). Despite differing formulations, their near-perfect overlap is evident. Thus, alternative series-coupled viscoelastic module configurations [14] more effective for vibration isolation.

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