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A Note on the Vorticity of MHD Flow Through A Porous Medium Bounded By An Oscillatingporous Plate

Sanjeev Kumar Saxena[1*], Ashish Kumar Saxena[2]
[1*] *Department of Mathematics, N.M.S.N. Dass (P.G.) College, Budaun, U.P., India, 243601*
[2] *Department of Physics, N.M.S.N. Dass (P.G.) College, Budaun, U.P., India, 243601*

Abstract:
Some features of the flow of a viscous incompressible fluid of small electrical conductivity in a porous medium near an oscillating infinite porous flat plate in the presence field of uniform strength, fixed relative to the fluid, have been studied in this paper. It is shown that this MHD flow is selfsuperposable and an irrotational flow on which it is superrposable is determined. Some observations have been made about the vorticity and stream function of the flow by using the property of superposability and selfsuperposability.

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I. Introduction

Flow through porous media are very much prevalent in nature and therefore, the study of flow through porous media has become of principal interest in many scientific and engineering applications. This type of flows have shown their great importance in petroleum engineering to study the movement of natural gas, oil and water through oil reservoirs, in chemical engineering for filtration and water purification processes. Further to study the underground water resources, seepage of water in river beds also one needs to investigate the fluids through porous media.

Stokes and Rayleigh [1, 2] investigated the flow about an infinite flat wall which executes linear harmonic oscillations parallel to itself. Stuart [3] considered the response of skin friction and temperature of an infinite, plate to fluctuations in the stream with suction at the plate. Ong and Nicholls [4] extended the method to obtain the flow in a magnetic field near an infinite flat wall which oscillates in its own plane. Ahmadi and Manvi [5] have derived a general equation of motion through a porous medium.Yamamoto and Iwamura [6] investigated the flow with convective acceleration through a porous medium. Singh [7] has discussed the influence of magnetic field on the flow of a viscous incompressible fluid of small electrical conductivity in a porous medium near an oscillating infinite porous flat plate.

Ram Ballabh [8, 9] introduced the idea of superposability and self-superposability regarding fluid motions. He laid down those two flows with velocities $\overline{v}_1$ *and* $\overline{v}_2$ are super posable if,

$$\mathrm{Curl}[\overline{v}_1 \times \mathrm{Curl}\overline{v}_2 + \overline{v}_2 \times \mathrm{Curl}\overline{v}_1] = 0 \qquad \ldots\ldots\ldots\ldots..(1)$$

and one flow with velocity $\overline{V}$ with self-superposable if,

$$\mathrm{Curl}(\overline{v} \times \mathrm{Curl}\overline{v}) = 0 \qquad \ldots\ldots\ldots..(2)$$

The purpose of this paper is to discuss the influence of an applied magnetic field on the vorticity of flow of a viscous incompressible fluid of small electrical conductivity in a porous medium near an oscillating infinite porous flat plate by drawing some peculiar vorticity profiles and using the property of super-posability and self-superposability. It is shown that this flow is self-superposable and an irrotational flow on which this MHD flow is superposable has also been determined.

1.2 FORMULATION OF THE PROBLEM

Let us consider the flow of an electrically conducting fluid of density ρ and viscosity μ through a porous medium of permeability k occupying a semi-infinite region of the space bounded by a porous plate. Let us assume that the porosity of the medium is equal to one. This assumption is valid only for a highly porous medium such as an air filter. A uniform magnetic field H_0 is acting along y-axis. The magnetic Reynolds number R_0 is usually small. Under such conditions the induced magnetic field due to the flow may be neglected with respect to tre applied magnetic field. As the plate is infinite in length and uniform suction is imposed over it, the physical variables depend only on y and time t. The pressure p in the fluid is assumed constant.

Let u and $\mathcal{V}$ be the component of velocity in x and y directions respectively taken along and perpendicular to the porous plats. If v represents the velocity of suction or injection at the plate, the equation of continuity.

$$\frac{\partial v}{\partial y} = 0 \qquad \text{……..(3)}$$

with the condition y = 0, $\mathcal{V}$ = V leads to the $\mathcal{V}$ = V every where.

The boundary layer equation describing the flow of an incompressible viscous electrically conducting fluid through a porous medium (assumed highly porous) is,

$$\rho\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\mu u}{k} - \sigma\mu_e^2 H_o^2 u \qquad \dots\dots\dots\dots(4)$$

Where **σ** is the electrical conductivity and Me the μ_e magnetic permeability. As the plate executes linear harmonic oscillutions in its own plane, the boundary conditions are.

y=0. u =U_0 (cont)

y → ∞. $u \to 0$ ….

…...... (5)

Introducing the variable

$$\bar{t} = nt, \qquad \eta = y\sqrt{\frac{n}{v}}, \lambda = \frac{v}{\sqrt{nv}} \qquad \dots\dots\dots\dots..(6)$$

$$m = \frac{\sigma\mu_e^2 H_o^2}{\rho}, M_1 = \frac{m}{n} = R_m^2$$

$$K = \frac{n}{v}k \qquad \dots\dots\dots\dots\dots\dots(7)$$

The equation (4)) becomes

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial \eta} + \left(M_1 + \frac{1}{K}\right)u = \frac{\partial^2 u}{\partial \eta^2} \qquad \dots\dots\dots\dots\dots\dots.(8)$$

with the condition

$\eta = 0, u = U_0 \cos\bar{t}$

$\eta \to \infty, u \to 0$ $\dots\dots\dots\dots\dots\dots.(9)$

Singh [7] has given a closed form solution of equation (7) as,

$$u = U_0 \exp(\eta/2)[\lambda - (\eta/2)\{\sqrt{\left(\lambda^2 + 4M_1 + \frac{4}{K}\right)^2 + 16}$$

$$+(\lambda^2 + 4M_1 + 4/K)\}^{\frac{1}{2}}] \times \cos[\bar{t} - \frac{\eta}{2\sqrt{2}}\{\sqrt{\left(\lambda^2 + 4M_1 + \frac{4}{K}\right)^2 + 16}$$

$$-(\lambda^2 + 4M_1 + 4/K)\}^{\frac{1}{2}}] \qquad \dots\dots\dots\dots\dots(10)$$

1.3 FLOW SUPER POSABLE ON THE MHD FLOW UNDER DISCUSSION

Let us suppose that a flow

$$\bar{v}_1 = (v_x, v_y, v_z) \qquad \dots\dots\dots\dots\dots\dots.(11)$$

is superposable on the MHD flow (10). Here we have assumed that v_x,v_y and v_z are independent of x and z.i,e. ,these are either function of y or constants.

It can be shown that under the condition (1) the flow (11) will be super posable on the flow (10) if,

$$v_y = \frac{C}{U_0} \exp\left\{-\frac{1}{2}\left(\lambda - \frac{1}{A}\right)\sqrt{\frac{n}{v}}y\right\}$$

$$\left[\frac{1}{2} - \left(\lambda - \frac{1}{A}\right)\sqrt{\frac{n}{v}}\cos\left(\bar{t} - A\sqrt{\frac{n}{v}}y\right) + A\sqrt{\frac{n}{v}}\sin\left(\bar{t} - A\sqrt{\frac{n}{v}}y\right)\right]^{-1} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

$$= \frac{C}{U_0}\exp\{-\frac{1}{2}(\lambda - 1/\mathrm{A})\eta\}[\frac{1}{2}\left(\lambda - \frac{1}{\mathrm{A}}\right)\sqrt{\frac{n}{v}}\cos(\bar{t} - A\eta))$$

$$+A\sqrt{\frac{n}{v}}\sin(\bar{t} - A\eta)]^{-1}$$

Where

$$A = \frac{1}{2\sqrt{2}}\left[\sqrt{\left(\lambda^2 + 4M_1 + \frac{4}{K}\right)^2 + 16} - \left(\lambda^2 + 4M_1 + \frac{4}{K}\right)\right]^{\frac{1}{2}} \qquad \ldots\ldots\ldots\ldots\ldots..(13)$$

where C is a constant.

If we consider the motion in z-x plane only and v_z= constant then flow $\bar{v}_1$ becomes

$$\bar{v} = (0, v_y, v_z) \qquad \ldots\ldots\ldots\ldots\ldots.(14)$$

where v_y is given by equation (12). For the flow (11) we get the vorticity of flow ζ=0 which means that the motion dented by (11) is irrotional. since it satisfies the condition (1) with the flow under discussion,

hence we can say that an irrotational flow given by (11) is superposable on the flow (10).

It was shown by Ballabh [9] that an irrotational flow is superposable on a rotational flow if and only if the vorticity of the latter is constant along the stream lines of the former.

The equation of the stream lines of te motion (10) can be easily be deduced as,

$$x = \text{ constant, } z = \exp\{\frac{1}{2}(\lambda - 1/\mathrm{A})\}[\frac{1}{2}\{(\lambda - 1/\mathrm{A})\sqrt{\frac{n}{v}} - (1/A)(\lambda - 1/\mathrm{A})\}$$

$$\sin(\bar{t} - A\eta) + A\sqrt{\frac{n}{v}}\cos(\bar{t} - A\eta)] \qquad \ldots\ldots(15)$$

Hence the vorticity of theMHD flow through a porous medium bounded by an oscillating porous plate with a transverse magnetic field is constant along the curve (15).

1.4 SELF SUPERPOSABILITY OF THEFLOW

It can readily be shown that flow (10) satisfies the condition (2) of self superposability. It was found be ballabh [9] that, if the axis of symmetry in the axially symmentric flow be x-axis, and the axis perpendicular to it be R axis, *t*he condition for the self-*s*uperposability of the flow will be,

$$\zeta = R\mathrm{f}(\Psi) \qquad \ldots\ldots\ldots\ldots\ldots..(16)$$

where ζ is the vorticity of the flow, $\mathrm{f}(\Psi)$ is any function of the stream function Ψ. condition(16)in our case reduces to.

$$\zeta = yf(\psi) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

Now the vorticity of the flow(9)is,

$$\zeta = U_0 \quad \exp\{\frac{1}{2}(\lambda - 1/\mathrm{A})\eta\}\cdot[\frac{1}{2}\left(\lambda - \frac{1}{\mathrm{A}}\right)\sqrt{\frac{n}{v}}\cos(\bar{t} - A\eta))$$

$$+A\sqrt{\frac{n}{v}}\sin(\bar{t} - A\eta)] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

Hence from (4.16) we have

$$\mathrm{f}(\psi) = \frac{U_0\sqrt{\frac{n}{v}}}{y} \quad \exp\{\frac{1}{2}(\lambda - 1/\mathrm{A})\eta\}[\frac{1}{2}\left(\lambda - \frac{1}{\mathrm{A}}\right)\cos(\bar{t} - A\eta))$$

$$+\mathrm{A}\sin(\bar{t} - A\eta)] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

$$= \frac{U_0\sqrt{\frac{n}{v}}}{\eta} \quad \exp\{\frac{1}{2}(\lambda - 1/\mathrm{A})\eta\}[\frac{1}{2}\left(\lambda - \frac{1}{\mathrm{A}}\right)\cos(\bar{t} - A\eta))$$

$$+\mathrm{A}\sin(\bar{t} - A\eta)]$$

It is now evident that for a flow through a porous medium bounded by en oscillating porous plate with a transverse magnetic field, the right hand side of equation(19) is function of η i.e.,y alone. It means that the stream function ψ of the flow under sondideration is function of y.

$i.e. \quad \psi = \psi(y)$ ………………………(20)

1.5 VORTICITY OF FLOW

The vorticity of the flow (9) is

$$\zeta = U_0 \quad \exp\{\frac{1}{2}(\lambda - 1/\mathrm{A})\eta\}[\frac{1}{2}\left(\lambda - \frac{1}{\mathrm{A}}\right)\sqrt{\frac{n}{v}}\cos(\bar{t} - A\eta)) + A\sqrt{\frac{n}{v}}\sin(\bar{t} - A\eta)]$$

or [$\zeta = \exp\{\frac{1}{2}(\lambda - 1/\mathrm{A})\eta\}[\frac{1}{2}(\lambda - 1/\mathrm{A})\cos(\bar{t} - A\eta) + A\sin(\bar{t} - A\eta)]$] …………………………(21)

where $\zeta^* = \frac{\zeta}{U_0}\sqrt{\frac{v}{n}}$

From equation (21), it is clear that the vorticity of the flow depends on the value of A, λ, and $\bar{t}$ i.e., on applied magnetic field, the magnetic permeablity, electrical conductivity, and the density of the fluid and time etc.

For different values of K, M_1, λ we have calculated ζ^* and vorticity profiles have been plotted for

$\bar{t}$= 0 and $\bar{t}$= π/2

Case I

Table - 1.1

$\bar{t}$= 0, λ=0, K=1

	η	0.0	0.2	0.4	0.6	0.8	1.0
M_1=0	ζ^*	-1.0989	-0.8226	-0.7098	-0.5694	-0.4573	-0.3673
M_1=1	ζ^*	-1.4556	-1.0883	-0.8136	-0.6082	-0.4547	-0.3400

Case II

Table - 1.2

$\bar{t}$= 0, λ=0, K=1

	η	0.0	0.2	0.4	0.6	0.8	1.0
M_1=0	ζ^*	-0.8994	-0.7525	-0.6291	-0.5262	-0.4401	-0.3680
M_1=1	ζ^*	-1.2853	-1.9940	-0.7692	-0.5951	-0.4604	-0.3562

Case III

Table - 1.3

$\bar{t}$= 0, λ=0, K=1

	η	0.0	0.2	0.4	0.6	0.8	1.0
M_1=0	ζ^*	-2.4556	-1.5029	-0.9198	-0.5629	-0.3445	-0.2108
M_1=1	ζ^*	-2.7556	-1.5882	-0.9154	-0.5275	-0.3041	-0.1753

Case IV

Table - 1.4

$\bar{t}$= 0, λ=0, K=1

	η	0.0	0.2	0.4	0.6	0.8	1.0
M_1=0	ζ^*	-2.2850	-1.4471	-0.9165	-0.5804	-0.3676	-0.2328
M_1=1	ζ^*	-2.6113	-1.5491	-0.9190	-0.5452	-0.3234	-0.1918

Case V

Table - 1.5

$\bar{t}$= 0, λ=0, K=1

	η	0.0	0.2	0.4	0.6	0.8	1.0
M_1=0	ζ *	0.4550	0.3638	0.2909	0.2325	0.1859	0.1487
M_1=1	ζ^*	0.3436	0.2554	0.1889	0.1412	0.1050	0.0780

1.6 DISCUSSION

The effects of porous medium and the suction have been shown in fig4.1 by the vorticity profiles at $\bar{t}$= 0 and in figure 4.1 by vorticity profiles at $\bar{t}$= π /2, from these it is evident that:

(i) For the same value of $\bar{t}$, an increase in K leads to decrease in vorticity.

(ii) For the same value of $\bar{t}$, and K, the vorticity increases with increase in magnetic field.

(iii) As we pass from $\bar{t}$= 0 to $\bar{t}$= π /2, the vorticity changes its direction i.e., the rotation of the fluid particles changes their mode. It indicates that somewhere in between $\bar{t}$= 0 and $\bar{t}$= π /2, irrotational flow is expected.

(iv) At $\bar{t}$= π /2, the vorticity decreases with increase in magnetic field while it increases with increase in K

Thus, we conclude that the porous medium increases the vorticity field while magnetic field decreases the vorticity at $\bar{t}$= 0. At $\bar{t}$= π /2 just reverse effect is observed.

Case VI

Table-1.6

$\bar{t}$= π/2 λ=0 K=2

	η	0.0	0.2	0.4	0.6	0.8	1.0
M_1=0	ζ *	0.5559	0.4629	0.3854	0.3210	0.2672	0.2261
M_1=1	ζ *	0.3890	0.2994	0.2305	0.1775	0.1366	0.1051

Case VII

Table-1.7

$\bar{t}$= π/2 λ=-2 K=1

	η	0.0	0.2	0.4	0.6	0.8	1.0
M_1=0	ζ *	0.3435	0.2084	0.1264	0.0766	0.0465	0.0282
M_1=1	ζ *	0.2848	0.1625	0.0927	0.0529	0.0302	0.0177

Case VIII

Table-1.8

$\bar{t}$= π/2 λ=-2 K=2

	η	0.0	0.2	0.4	0.6	0.8	1.0
M_1=0	ζ *	0.3890	0.2443	0.1534	0.0963	0.0605	0.0379
M_1=1	ζ *	0.3103	0.1823	0.1072	0.0629	0.0370	0.0217

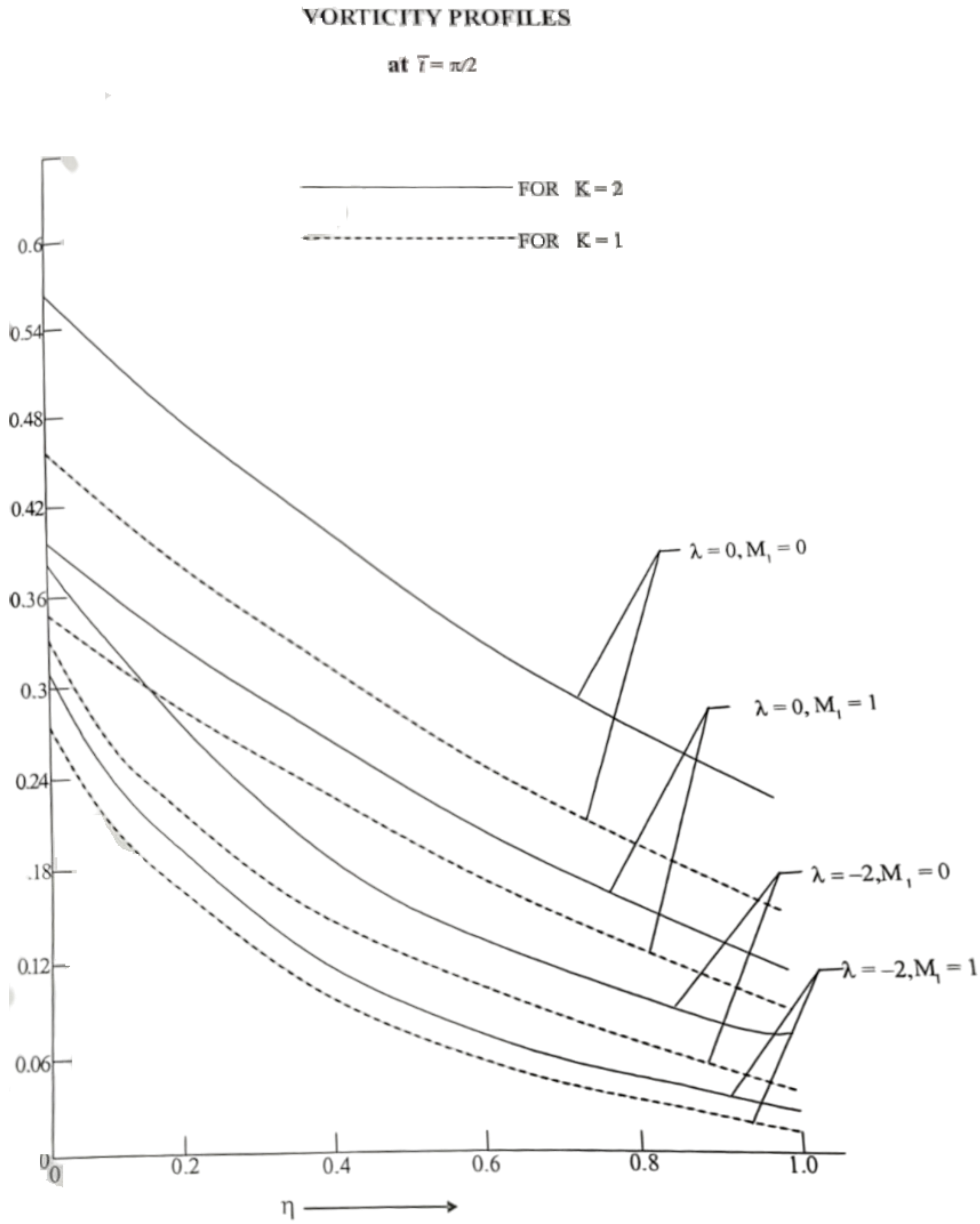

Fig. 1.1

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