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k-Total Modulo Labeling Graphs

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ABSTRACT. Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a map where $k \in N$ and $k > 1$. For each edge uv, assign the label $(f(u) + f(v) + f(uv)) \mod k$ where $k > 1$. f is called a k-total modulo labeling of G if $|t_m(i) - t_m(j)| \leq 1$, $i, j \in \{0, 1, 2, \ldots, k-1\}$ where $t_m(x)$ denotes the total number of vertices and the edges labeled with x. A graph with admits a k-total modulo labeling is called a k-total modulo graphs. We investigate k-total modulo labeling of some graphs and study the 3-total modulo labeling behaviour of star,bistar,complete bipartiate graph,comb,wheel,helm,armed crown etc.

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1. Introduction

All graphs in this paper are finite, simple and undirected only. Cahit [1] introduced notion of cordial labeling of graphs. The cocept of k-difference cordial graph was introduced in [4] . Recently Ponraj etal [4] has been introduced the concept of k-total difference cordial graphs. Motivated by this, we introduce k-total modulo labeling of graphs. Also we prove that every graph is a subgraph of a connected k-total modulo labeling graphs and investigate 3-total modulo labeling of sevarel graphs like path,star, bistar, complete bipartite graph etc .

2. k-TOTAL MODULO LABELING OF GRAPHS

Definition 2.1. Let G be a graph. Let $f : V(G) \to 0, 1, 2, \ldots, k-1$ be a map where $k \in N$ and $k > 1$. For each edge uv, assign the label $(f(u) + f(v) + f(uv)) \mod k$ where $k > 1$. f is called a k-total modulo labeling of G if $|t_{ml}(i) - t_{ml}(j)| \leq 1$, $i, j \in 0, 1, 2, \ldots, k-1$ where $t_{ml}(x)$ denotes the total number of vertices and the edges labeled with x. A graph with admits a k-total modulo labeling is called a k-total modulo graphs.

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3. Preliminaries

Definition 3.1. *Armed crown AC_n is the graph obtained from the cycle $C_n : u_1u_2 \dots u_nu_1$ with $V(AC_n) = V(C_n) \cup \{v_i, w_i : 1 \leq i \leq n\}$ and $E(AC_n) = E(C_n) \cup \{u_iv_i, v_iw_i : 1 \leq i \leq n\}$.*

Definition 3.2. A *bipartite graph* is a graph whose vertex set $V(G)$ can be partitioned into two subsets V_1 and V_2 such that every edge of G joins a vertex of V_1 with a vertex of V_2. If G contains every edge joining V_1 and V_2, then G is a *complete bipartite graph.* If $|V_1| = m$ and $|V_2| = n$, then the complete bipartite graph is denoted by $K_{m,n}$.

Definition 3.3. $K_{1,n}$ is called a *star*.

Definition 3.4. The graph $W_n = C_n + K_1$ is called a *wheel*. In a wheel, a vertex of degree 3 is called a *rim vertex*. A vertex which is adjacent to all the rim vertices is called the *central vertex*. The edges with one end incident with the rim and the other incident with the central vertex are called *spokes*.

Definition 3.5. The *helm* H_n is obtained from a wheel W_n by attaching a pendent edge at each vertex of the cycle C_n. A *closed helm* is the graph obtained from a helm by joining each pendent vertex to form a cycle.

Definition 3.6. The graph $C_n \odot K_1$ is called a *crown*.

Definition 3.7. The *Lilly graph* $I_n : n \geq 2$ is constructed by 2 stars $2K_{1,n}$,$n \geq 2$, joining 2 path graphs $2P_n$,$n \geq 2$ with sharing of a common vertex. Let $V(I_n) = \{u_i, v_i : 1 \leq i \leq n\} \bigcup \{x_i : 1 \leq i \leq n\}$ $\bigcup \{y_i : 1 \leq i \leq n-1\}$ and $E(I_n) = \{x_nu_i, x_ny_i : 1 \leq i \leq n\} \bigcup$ $\{x_ix_{i+1} : 1 \leq i \leq n-1\} \bigcup \{x_ny_1\} \bigcup \{y_iy_{i+1} : 1 \leq i \leq n-2\}$.

4. Main Results

Remark. 2- total modulo labeling graph is 2-total product cordial graph.

Theorem 4.1. The crown $C_n \odot K_1$ is 3-total modulo labeling for all values of n.

Proof. Let C_n be the cycle $u_1, u_2, \dots, u_nu_1$.
Let $V(C_n \odot K_1) = V(C_n) \bigcup v_i : 1 \leq i \leq n$ and
$E(C_n \odot K_1) = E(C_n) \bigcup u_iv_i : 1 \leq i \leq n$.

Case 1. $n \equiv 0 \pmod 3$.
Assign the label 1 to the cycle vertices $u_1, u_2, \dots, u_n$. Next we move to

the pendent vertices v_i. Assign the label 2 to the vertices $v_1, v_2, \ldots, v_{\frac{2n}{3}}$. Next assign 1 to the remaining vertices $v_{\frac{2n+3}{3}}, v_{\frac{2n+6}{3}}, \ldots, v_n$.

Case 2. $n \equiv 1 \pmod 3$.
Assign the label 1 to the cycle vertices $u_1, u_2, \ldots, u_n$. Next we move to the pendent vertices v_i. Assign the label 2 to the vertices $v_1, v_2, \ldots, v_{\frac{4n-1}{3}}$. Next assign 1 to the remaining vertices $v_{\frac{4n-4}{3}}, v_{\frac{4n-1}{3}}, \ldots, v_n$.

Case 3. $n \equiv 2 \pmod 3$.
Assign the label 1 to the cycle vertices $u_1, u_2, \ldots, u_n$. Next we move to the pendent vertices v_i. Assign the label 2 to the vertices $v_1, v_2, \ldots, v_{\frac{2n-1}{3}}$. Next assign 1 to the remaining vertices $v_{\frac{2n+2}{3}}, v_{\frac{2n+5}{3}}, \ldots, v_n$.

Values of n	$t_{ml}(0)$	$t_{ml}(1)$	$t_{ml}(2)$
$n \equiv 0 \pmod 3$	$\frac{4n}{3}$	$\frac{4n}{3}$	$\frac{4n}{3}$
$n \equiv 1 \pmod 3$	$\frac{4n-1}{3}$	$\frac{4n-1}{3}$	$\frac{4n+2}{3}$
$n \equiv 2 \pmod 3$	$\frac{4n+1}{3}$	$\frac{4n+1}{3}$	$\frac{4n-2}{3}$

TABLE 1

□

Theorem 4.2. All Wheels are 3-total modulo labeling.

Proof. Let $W_n = C_n + K_1$ where C_n is the cycle $u_1u_2 \ldots u_nu_1$ and $V(K_1) = \{u\}$. Assign the label 2 to the central vertex u and assign the label 1 to the all the rim vertices $u_i (1 \leq i \leq n)$.Clearly $t_{ml}(0) = t_{ml}(1) = \frac{3n}{3}$ and $t_{ml}(2) = \frac{3n+3}{3}$. Hence W_n is 3-total modulo labeling. □

Theorem 4.3. Helms H_n is 3-total modulo labeling.

Proof. Helm H_n is obtained from the wheel $W_n = C_n + K_1$ where C_n is the cycle $u_1u_2 \ldots u_n$ and $V(K_1) = \{u\}$ by attaching pendent edges to the rim vertices. Let $v_1, v_2, \ldots, v_n$ be the pendent vertices adjacent to $u_1, u_2, \ldots, u_n$ respectively.
Fix the label 2 to the central vertex and 1 to the rim vertices $u_1, u_2, \ldots, u_n$

Case 1. $n \equiv 0 \pmod 3$.
Assign the label 2 to the vertices $v_1, v_2, \ldots, v_{\frac{n}{3}}$. Next assign the label 1 to the vertices $v_{\frac{n+3}{3}}, v_{\frac{n+6}{3}}, \ldots, v_n$.

Case 2. $n \equiv 1 \pmod 3$.
Assign the label 2 to the vertices $v_1, v_2, \ldots, v_{\frac{n-1}{3}}$. Next assign the label 1 to the remaining vertices $v_{\frac{n+2}{3}}, v_{\frac{n+5}{3}}, \ldots, v_n$.

Case 3. $n \equiv 2 \pmod 3$.
Assign the label 2 to the vertices $v_1, v_2, \ldots, v_{\frac{n-1}{3}}$ and assign the label 1 to the vertices $v_{\frac{n+1}{3}}, v_{\frac{n+4}{3}}, \ldots, v_n$.

The table 2 given below shows that this labeling f is a 3-total modulo labeling of H_n.

Values of n	$t_{ml}(0)$	$t_{ml}(1)$	$t_{ml}(2)$
$n \equiv 0 \pmod 3$	$\frac{5n}{3}$	$\frac{5n}{3}$	$\frac{5n+3}{3}$
$n \equiv 1 \pmod 3$	$\frac{5n+1}{3}$	$\frac{5n+1}{3}$	$\frac{5n+1}{3}$
$n \equiv 2 \pmod 3$	$\frac{5n+2}{3}$	$\frac{5n+2}{3}$	$\frac{5n-1}{3}$

TABLE 2

Example 4.1. A 3-total modulo labeling of H_n is shown in Figure 1

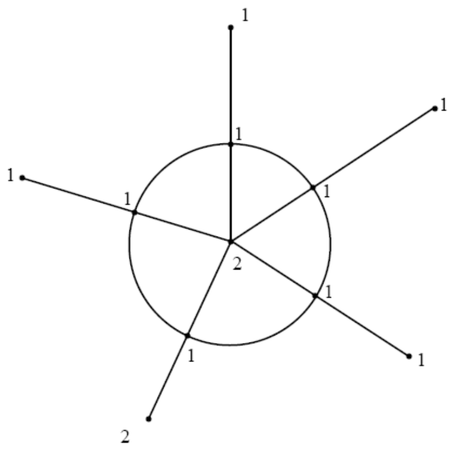

FIGURE 1

Theorem 4.4. AC_n is 3-Total modulo labeling for all n

Proof. Clearly AC_n has $3n$ vertices and $3n$ edges. Fix the label 1 to all the cycle vertices $u_1u_2\dots u_n$.
Case 1. n is odd.
Next assign the label 2 to the all the vertices with degree 2. That is assign the label 2 to the vertices $v_1, v_2, \dots, v_{\frac{n-1}{2}}$ and $w_1, w_2, \dots, w_{\frac{n-1}{2}}$.

Now we assign the label 1 to all the pendent vertices $v_{\frac{n+1}{2}}, v_{\frac{n+3}{2}}, \dots, v_n$ and $w_{\frac{n+1}{2}}, w_{\frac{n+3}{2}}, \dots, w_{n-1}$. The last vertex w_n receive the label 2 when n is odd.
Case 2. n is even.
Assign the label 2 to the vertices $v_1, v_2, \dots, v_{\frac{n}{2}}$ and $w_1, w_2, \dots, w_{\frac{n}{2}}$. Now we assign the label 1 to the vertices $v_{\frac{n+2}{2}}, v_{\frac{n+4}{2}}, \dots, v_n$ and $w_{\frac{n+2}{2}}, w_{\frac{n+4}{2}}, \dots, w_n$.

The table 3 given below establish that this vertex labeling pattern is a 3 total modulo labeling.

Values of n	$t_{ml}(0)$	$t_{ml}(1)$	$t_{ml}(2)$
n is odd	$\frac{6n}{3}$	$\frac{6n}{3}$	$\frac{6n}{3}$
n is even	$\frac{6n}{3}$	$\frac{6n}{3}$	$\frac{6n}{3}$

TABLE 3

□

Theorem 4.5. The complete bipartite graph $K_{2,n}$ is k-total modulo labeling.

Proof. Let $V_1 = u, v$ and $V_2 = u_1, u_2, \dots, u_n$ where V_1, V_2 is the bipartition of $K_{2,n}$. We now give the vertex labeling. Assign the label 2 to the vertex u. Next assign the label 1 to the vertices v and $u_1, u_2, \dots, u_n$. It is easy to verify that $t_{ml}(0) = \frac{3n}{3}$,$t_{ml}(1) = t_{ml}(2) = \frac{3n+3}{3}$. □

Theorem 4.6. Any star $K_{1,n}$ is 3-Total modulo labeling except $n \equiv 1 \pmod 3$

Proof. Let $V(K_{1,n}) = \{u, u_i : 1 \le i \le n\}$ and $E(K_{1,n}) = \{uu_i : 1 \le i \le n\}$. Clearly $K_{1,n}$ has $|V(K_{1,n}) + E(K_{1,n}| = 2n + 1$ vertices and edges.

Case 1. $n \equiv 0 \pmod 3$.
Assign the label 1 to the vertex u. Then assign the label 2 to the vertices $u_1, u_2, \ldots, u_{\frac{n}{3}}$. We now assign the label 1 to the remaining vertices $u_{\frac{n+3}{3}}, u_{\frac{n+6}{3}}, \ldots, u_n$.
Case 2. $n \equiv 2 \pmod 3$.
Assign the label 1 to the vertex u. Then assign the label 2 to the vertices $u_1, u_2, \ldots, u_{\frac{n+1}{3}}$. Next assign the label 1 to the remaining vertices $u_{\frac{n+4}{3}}, u_{\frac{n+7}{3}}, \ldots, u_n$.

The table 4 given below establish that this vertex labeling pattern is a 3 total modulo labeling.

Values of n	$t_{ml}(0)$	$t_{ml}(1)$	$t_{ml}(2)$
$n \equiv 0 \pmod 3$	$\frac{2n}{3}$	$\frac{2n+3}{3}$	$\frac{2n}{3}$
$n \equiv 2 \pmod 3$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$	$\frac{2n+2}{3}$

TABLE 4

Theorem 4.7. *The Flower graph Fl_n is 3-total modulo labeling for all n.*

Proof. Let $W_n = C_n + K_1$ where C_n is the cycle $u_1u_2 \ldots u_nu_1$ and $V(K_1) = u$. The Helm H_n is a graph with $V(H_n) = V(W_n)$ $\cup \{V_i : 1 \leq i \leq n\}$ and $E(H_n) = E(W_n) \cup \{u_iv_i : 1 \leq i \leq n\}$. The Flower Fl_n is obtained from H_n by joining v_i to u $(1 \leq i \leq n)$. We now assign the labels to the vertices as follows. Assign label 2 to the vertices u and 1 to the vertices $u_1, u_2, \ldots, u_n$. Next assign the label 1 to the vertices $v_1, v_2, \ldots, v_n$. Clearly $t_{ml}(0) = t_{ml}(1) = \frac{6n}{3}$,$t_{ml}(2) = \frac{6n+3}{3}$. □

Example 4.2. *A 3-total modulo labeling of Fl_4 is shown in Figure 2*

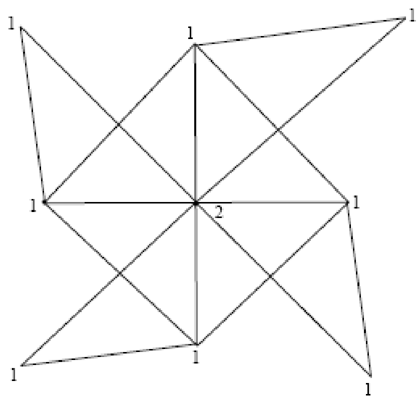

Figure 2

Theorem 4.8. The *Lilly graph I_n* is 3-total modulo labeling except $n \in 3, 6, \ldots, 3t$

Proof. **Case 1.** $n \equiv 1 \pmod 3$.
Take the vertex set and edge set as in definition 3.7. Clearly $|V(I_n)| + |E(I_n)| = 8n - 3$. Assign the label 1 to the path vertices $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_{n-1}$. Next we assign the label 2 to the vertices $u_1, u_2, \ldots, u_n$ and assign the label 1 to the vertices $v_1, v_2, \ldots, v_n$.
Case 2. $n \equiv 2 \pmod 3$.
Assign the label 1 to the path vertices $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_{n-1}$.

Next we assign the label 2 to the vertices $u_1, u_2, \ldots, u_n$ and assign the label 1 to the vertices $v_1, v_2, \ldots, v_n$.

Clearly $t_{df}(0) = t_{df}(1) = t_{df}(2) = 2n - 1$, $t_{df}(3) = 2n$. □

Example 4.3. A 3-total modulo labeling of I_5 is shown in Figure 3

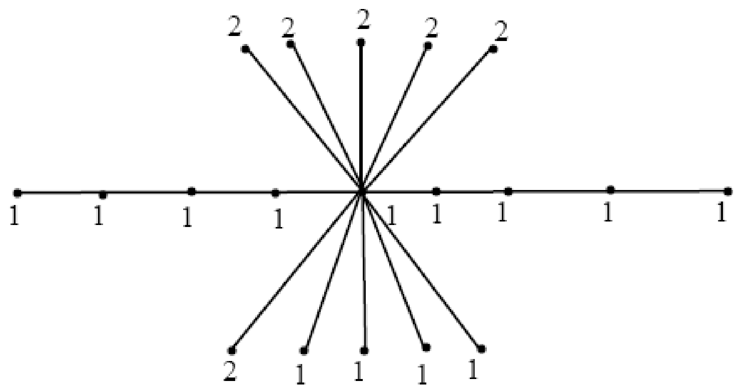

FIGURE 3

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