International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 13 Issue 3 ǁ March 2025 ǁ PP. 23-29

Dimensionless Modelling of Piezoelectric Energy Harvesting with d_{33}-mode Multilayer Piezoceramics using Mass-Spring Oscillator

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Abstract
This study presents a dimensionless modeling approach to piezoelectric energy harvesting using d_{33}-mode multilayer piezoceramics based on a mass-spring oscillator framework, with a particular focus on the impact of varying the damping ratio. The damping ratio is identified as a key parameter that significantly influences the harvester's performance by affecting resonance frequency, bandwidth, peak energy output, transient response, and overall system durability. Frequency-domain analysis reveals that lower damping ratios result in sharp resonance peaks, leading to high energy output at specific frequencies but with a narrow bandwidth and increased mechanical stress. In contrast, higher damping ratios broaden the bandwidth, enabling more stable energy harvesting across a wider frequency range but at the expense of peak power output. Time-domain analysis further demonstrates that decreasing the damping ratio leads to increased oscillatory behavior, resulting in higher peak voltages, energy, and power due to prolonged mechanical strain and extended charge accumulation. Additionally, while harvested voltage remains in phase with the system's structural vibration regardless of damping, power and energy exhibit a phase delay as damping increases. This delay arises from the time-dependent nature of energy accumulation, which is influenced by the rate of power transfer and the system's oscillatory response. These findings provide valuable insights into optimizing damping conditions for enhanced energy harvesting efficiency and structural reliability in piezoelectric systems.

Keywords: *Piezoceramics, Damping, Mass-Spring, Voltage, Energy, Power*

Date of Submission: 22-02-2025 Date of acceptance: 03-03-2025

I. INTRODUCTION

In recent years, energy harvesting technologies have become increasingly important as we seek sustainable and self-sufficient power sources for small electronic devices, wireless sensors, and Internet of Things (IoT) applications. One of the standout methods in this area is piezoelectric energy harvesting (PEH), according to Osheidu *et. al.* [1], which can transform everyday mechanical vibrations into usable electrical energy. Piezoelectric materials, especially piezoceramics, have a unique ability to generate an electric charge when they are subjected to mechanical stress, making them ideal for capturing energy from vibrations [2, 3].

One promising approach in PEH is using multilayer piezoceramics in the d_{33}-mode operation, taking advantage of their ability to utilize the longitudinal piezoelectric effect [4, 5]. This method is gaining attention because it offers high energy conversion efficiency and a compact design.

To better understand how these piezoelectric energy harvesters work, researchers often use a mass-spring oscillator model [5-7]. This model simplifies how we analyze the harvester's response to vibrations, making it easier to see how mechanical and electrical components interact. However, the efficiency of these systems can vary significantly based on several factors, with the damping ratio being particularly critical. Damping occurs due to both mechanical and electrical losses and can influence how much energy the harvester produces by affecting the amplitude of vibrations and the system's resonance frequency according to Iqbal *et. al.* [8].

Dimensionless modeling has emerged as a valuable technique in studying PEH systems. By normalizing equations, researchers can generalize results and lessen the complexity that comes with dimensional parameters. This method makes it easier to identify key factors that affect performance, allowing comparisons between different systems and helping optimize designs for better results.

One of the major challenges in improving piezoelectric energy harvesters is finding the right balance between vibration amplitude, resonance frequency, and bandwidth to boost power output while keeping the system durable. The damping ratio is crucial in this optimization process [9, 10]. However, there aren't many generalized models that examine how the damping ratio affects the performance of d_{33}-mode multilayer piezoceramic energy harvesters. This knowledge gap makes it harder to create design guidelines that help achieve optimal energy harvesting performance across various settings and applications.

This research aims to fill that gap by developing a dimensionless model that looks at how the damping ratio impacts the energy harvesting efficiency of d_{33}-mode multilayer piezoceramic systems. Through this study, we hope to provide insights that will aid in the design and optimization of high-performance PEH systems.

II. MATHMATICAL FORMULATION

Consider a single-degree- of- freedom (SDOF) mass-spring-damper-piezo encased in a rigid housing microgenerator system in which only a linear small displacement is allowed and is excited around its resonance frequency. Figure 1 shows a schematic representation of the piezoelectric vibration and the electromechanical model, M represents the equivalent rigid mass of the block, while x and y are the displacements of the base and the rigid mass under the external acceleration $\ddot{y}$(t), K is the equivalent stiffness, and C is the mechanical loss coefficient (viscous damper).

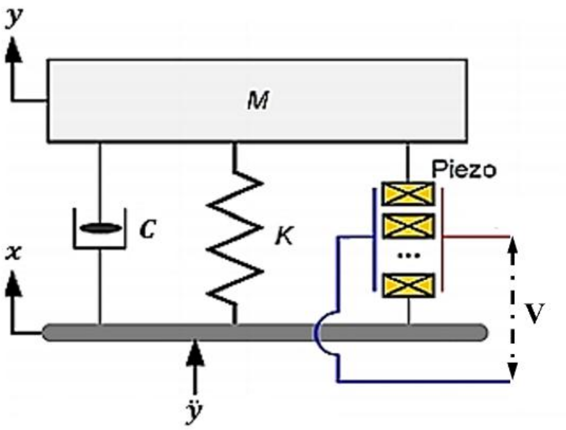

Figure 1: Schematic representation of SDOF piezoelectric and electromechanical coupling model [11]

Assuming that the difference between the base displacement x and the rigid mass displacement y is defined as the relative displacement. Then, the dynamic characteristics of piezoceramics under sinusoidal excitation and very weak electromechanical coupling can be modelled using Newton's second law according to Guyomar *et al.* [11]; Gatti *et al.* [12], Li *et al.* [5], Zhang *et al.*[13] and Osheidu *et al.*[1],

$$M\ddot{Z}\,(t) + 2\xi\omega_n\dot{Z}(t) + \omega_n^2\,Z(t) = F_0\,\omega^2\text{Sin}(\omega t) \tag{1}$$

With initial conditions

$$Z(0) = 0, \dot{Z}(0) = 0 \tag{2}$$

Where, the dot over the variable denotes differentiation with respect to time. Furthermore, assume a harmonic vibration excitation of the form $y(t) = F_0\,Sin(\omega t)$ where ω is the excitation frequency and F_o is the force. Also, $\omega_n\big(= \sqrt{K/M}\big)$ is the natural frequency and $\xi(= C/2M\omega_n)$ is the damping ratio. Equations (1) and (2) are transformed into dimensionless forms as,

$$\ddot{z}(\tau) + 2\xi\dot{z}(\tau) + z(\tau) = \Omega^2\sin(\Omega\tau) \tag{3}$$

$$z(0) = 0, \dot{z}(0) = 0 \tag{4}$$

With the following dimensionless parameters, $z = \frac{Z}{F_0}$, $\tau = t\omega_n$, $\Omega = \frac{\omega}{\omega_n}$ (is the normalized frequency). The dimensionless Equation (3) subject to the initial conditions (4), representing the forced damping vibration of the piezoelectric energy harvester, is a second non-homogeneous differential equation whose solution is well known. The dimensionless vibration displacement can be expressed according to Osheidu *et al.* [1],

$$z(\tau) = \; e^{-\xi\tau}(c_1\cos(\beta\tau) + c_2\sin(\beta\tau)) + \Gamma[sin(\Omega\tau - \varphi)] \tag{5}$$

Where $\Gamma = \Omega^2[(1-\Omega^2)^2 + (2\xi\Omega)^2]^{-1/2}$, $\varphi = tan^{-1}\left(\frac{2\xi\Omega}{1-\Omega^2}\right)$ and $\beta = \sqrt{1-\xi^2}$. The solution expressed in equation (5) consists of two parts, where the first part represents the transient response, the second part represents the steady – state response of the system. Assuming that the transient term dies quickly as time, τ, increases as the normalized relative displacement reduces to a steady state harmonic, $z(\tau)$, which continues indefinitely. In other words,

$$z(\tau) = \Gamma[sin(\Omega\tau - \varphi)] \tag{6}$$

Consequently, the magnitude of vibration acceleration takes the form given by,

$$a(\tau) = \left|\ddot{Z}(\tau)\right| = \Omega^2 \Gamma sin(\Omega\tau - \varphi) \quad (7)$$

To proceed with the analysis, we assume that the piezoelectric material is subjected to a vibrational energy and the applied stress are in the same direction (direction 3), resulting in the d_{33}-mode. Then, for a very low electromechanical coupling, the electrical displacement ($D_3(t)$) and electrical charge ($Q_3(t)$) in the direction 3 are given respectively according to Li *et al.* [5] and Alaya *et al.* [14].

$$D_3(t) = d_{33} T_3(t) \quad (8)$$

$$Q_3(t) = nD_3(t)s \quad (9)$$

Where n is the number of layers (patches) in the harvester, $T_3(t)$ is the mechanical stress in direction 3 (MPa), d_{33} is the piezoelectric coefficient (C/N), S is the area of a single piezoelectric layer (patch) and $Q_3(t)$ is the electric charge (C).

The voltage generated by the piezoelectric material across the piezoelectric device, according to Li *et al.* [5], Alaya *et al.* [14] and Pandiev *et al.* [15], can be defined as the ratio of the stored charge and the capacitance across the piezoelectric device;

$$V_p(t) = \frac{Q_3}{C_P} \quad (10)$$

Where C_p is the capacitance of the piezoelectric material defined as $C_P = \frac{\varepsilon_o \varepsilon_r A}{h}$, ε_o is the permittivity of the space (F/N), ε_r is the relative permittivity, A is the area of the piezoelectric material (m^2) and h is the thickness of the piezoelectric material (m).

By utilizing Equations (8) and (9) in Equation (10) the total voltage generated by the piezoelectric energy harvester becomes

$$V_p(t) = \frac{nd_{33}}{C_p} F_3(t) \quad (11)$$

Relating to Li *et al.* [5] and Zhang *et. al.* [13], the mechanical stress in direction 3 is defined as,

$$T_3(t) = \frac{F_3(t)}{s} \quad (12)$$

Following Li *et al.* [5], the force, $F_3(t)$, exerted by the spring on the piezoelectric material can be expressed as,

$$F_3(t) = KZ(t) \quad (13)$$

Where K is the spring constant and $Z(t)$ is the vibrational displacement. The utilization of Equation (13) in Equation (11) gives the harvested voltage across the piezoelectric energy harvesting device as,

$$V_p(t) = \frac{nKd_{33}}{C_p} Z(t) \quad (14)$$

By defining the following dimensionless parameters,

$$z = \frac{Z}{F_o}, \qquad \tau = t\omega_n, \qquad V_p(\tau) = \frac{C_p}{F_o K d_{33}} V(t) \quad (15)$$

Relative to equation (14), the generated dimensionless voltage can then be expressed as,

$$V_p(\tau) = n\Gamma Sin(\Omega\tau - \varphi) \quad (16)$$

The electrical energy that can be obtained at maximum voltage is defined according to Xu *et al.* [16]; Pandiev *et al.* [15] and Alaya *et al.* [14] and is given as,

$$E_p(t) = \frac{1}{2} C_p V_p(t)^2 \quad (17)$$

The dimensionless electrical energy stored in the piezoelectric energy devices is defined as,

$$E_p(\tau) = \frac{n^2\Gamma^2}{2} Sin^2(\Omega\,\tau - \varphi) = \frac{n^2\Gamma^2}{4}(1 - Cos[2\Omega\,\tau - 2\varphi]) \quad (18)$$

Where $E(\tau) = \frac{C_p}{(F_0 k d_{33})^2} E_p(t)$. Then, based on Equation (17) the maximum dimensionless power output from the piezoelectric energy harvester is,

$$P(\tau) = \frac{dE(\tau)}{d\tau} = \frac{n^2\Gamma^2\Omega}{2} Sin[2\Omega\,\tau - 2\varphi] \quad (19)$$

III. RESULTS

The harvested voltage, energy and power as defined by equations (16), (18) and (19) was then modelled relative to normalized frequency (Ω) and normalized time (τ) as affected by the damping ratio (ξ) with varying model parameters as shown in Figures 2 to 8.

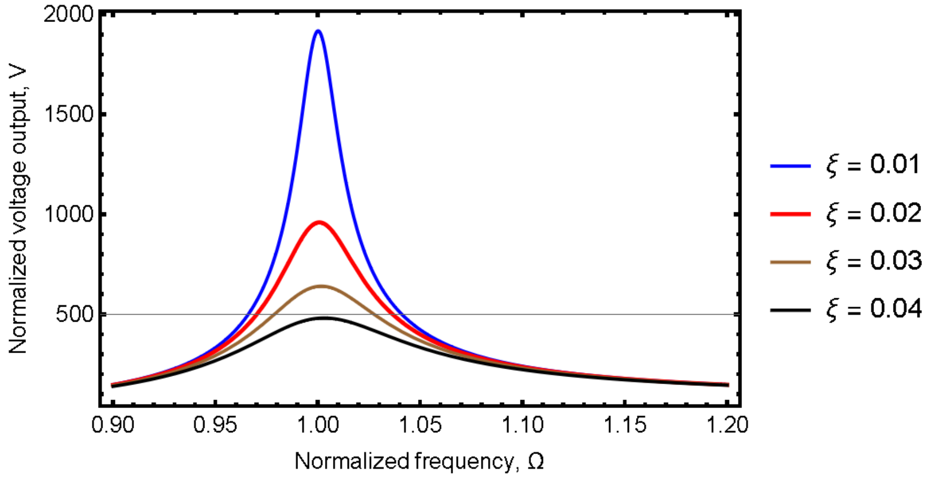

Figure 2: Normalized voltage against normalized frequency for different damping ratio with $\tau = 0.5$ and $n = 80$

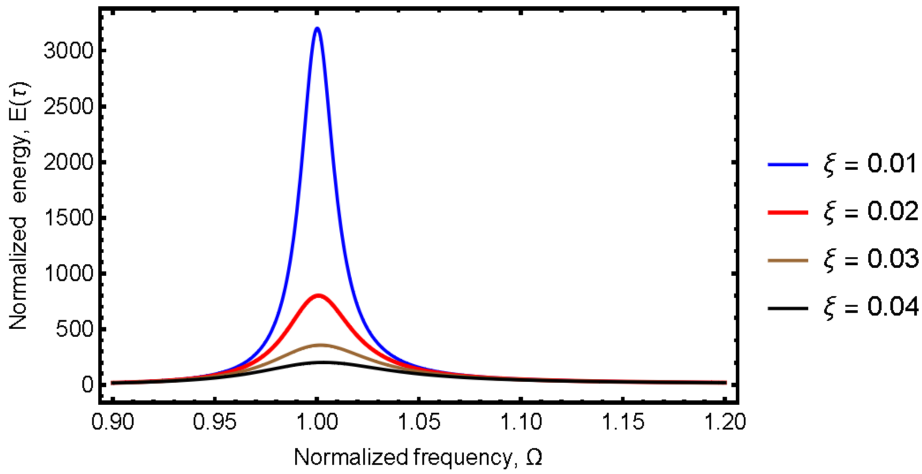

Figure 3: Normalized energy against normalized frequency for different damping ratio with $\tau = 0.02$ and $n = 80$

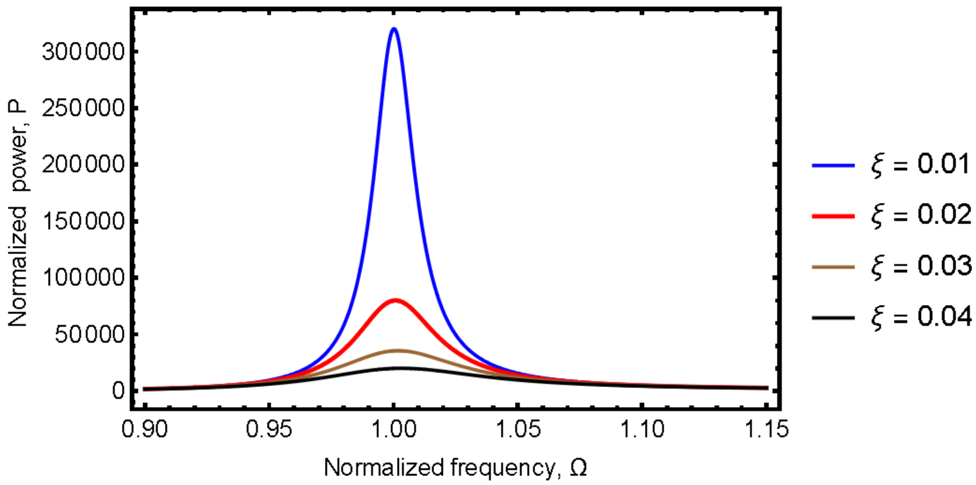

Figure 4: Normalized power against normalized frequency for different damping ratio with $\tau = 0.02$ and $n = 80$

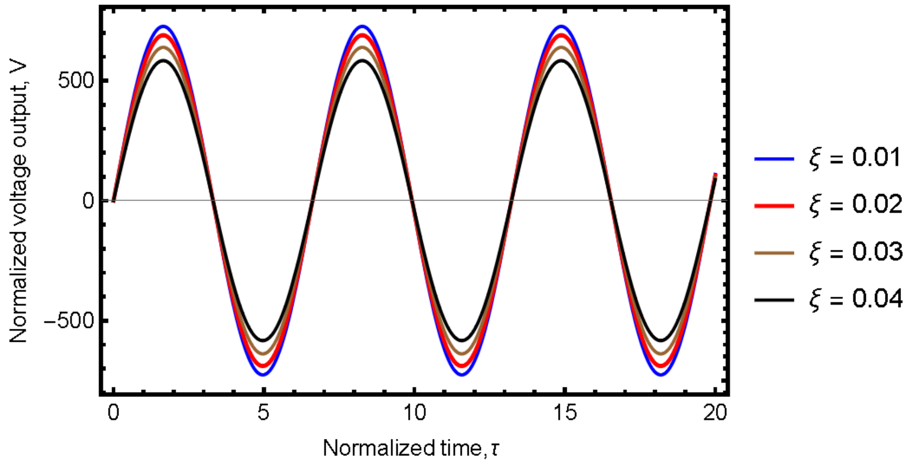

Figure 5: Normalized voltage against normalized time for different damping ratio with $\Omega = 0.95$ and $n = 80$

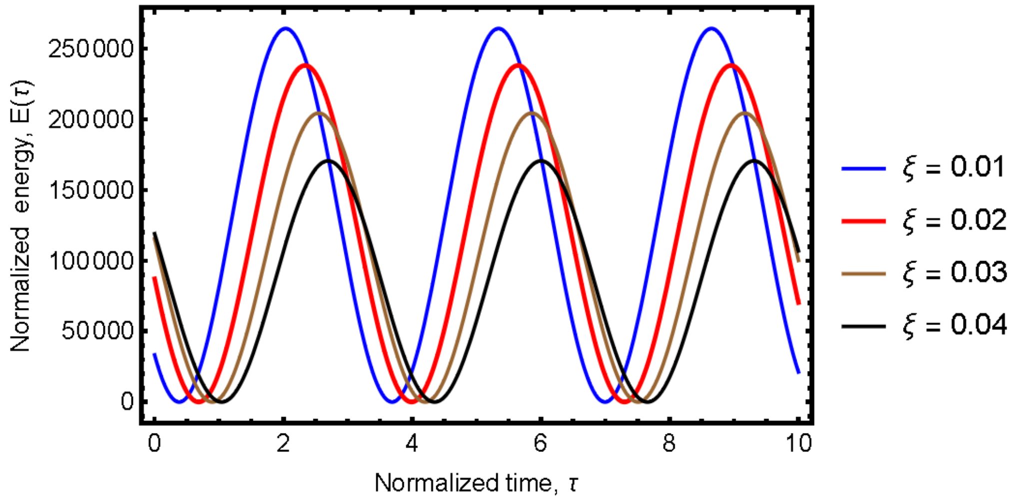

Figure 6: Normalized energy against normalized time for different damping ratio with $\Omega = 0.95$ and $n = 80$

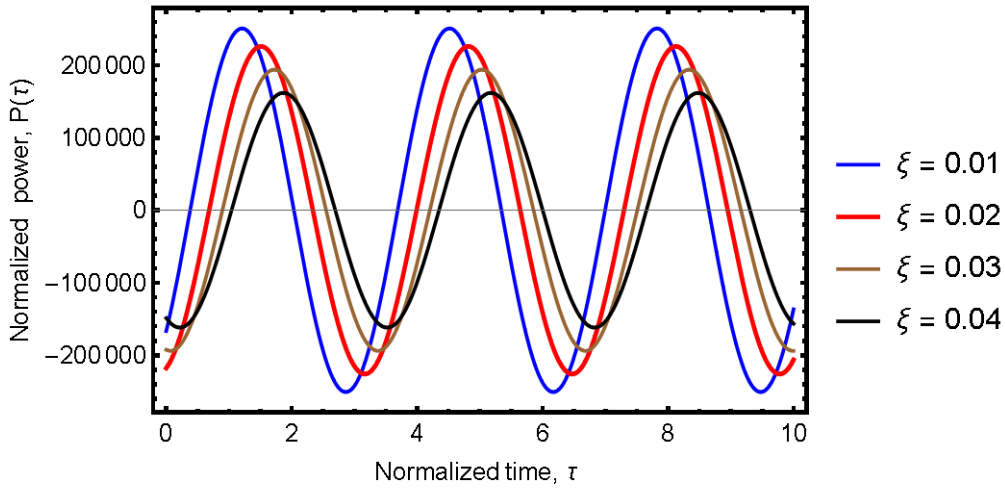

Figure 7: Normalized power against normalized time for different damping ratio with $\Omega = 0.95$ and $n = 80$

IV. DISCUSSION

The effect of damping ratio on the system's energy harvesting performance was investigating by making plots of normalized voltage, energy and power against normalized frequency and time. The damping ratio is a key factor that affects how well a piezoelectric energy harvester works. According to Iqbalet al. [8], it influences several important aspects, including the resonance frequency, bandwidth, energy output, transient response, and overall durability.

Relative to normalized frequency, the modelled system variables (voltage, energy and power) all had similar responses to increased damping ratio as seen in figures 2, 3 and 4. When the damping ratio is low, the harvester has a sharp resonance peak, which means it can generate a lot of energy at a specific frequency according to Ahmad *et. al.* [17]. However, this also results in a narrow bandwidth, placing extra mechanical stress on the system and making it less stable when the frequency changes according to Maamer *et. al.* [18]. On the flip side, a higher damping ratio creates a wider bandwidth, allowing the harvester to collect energy more consistently over a range of frequencies. This reduces mechanical stress, but it usually results in lower peak energy output at resonance.

Furthermore, investigating the effect of damping ratio on modelled system variable relative to normalize time (figures 5, 6 and 7), there were noticeable increases in the peaks of the harvested system variables as the damping ratio increased. When the damping ratio drops below 1, as the case in our investigation, piezoelectric harvesters tends to oscillate more vigorously [19]. This indicates that the piezoelectric material gets stretched and compressed more, which leads to greater mechanical strain. Since the voltage produced is directly linked to this strain, lower damping results in higher peak voltages as observed in our result. This more compression and stretching also means that the harvesting system can keep oscillating for longer, allowing it to gather more charge, resulting in greater energy and power peaks, which could be the reason why the harvested energy and power in our model showed increasing peaks with reducing damping ratio.

Further investigation showed that increasing the damping ratio produced a delay in the phases of the harvested energy and power but not in the harvested voltage. In a piezoelectric harvester, the voltage generated is closely linked to the immediate mechanical strain in the material, meaning it reacts in real time with the vibrations of the system according to Perez-Alfaro *et. al.* [20]. So, no matter how much damping there is, the voltage stays in sync with the structural movements. On the other hand, power is a bit more complex. It depends on both the voltage and the current, which is affected by how quickly the voltage changes and the electrical resistance of the system [21]. When we increase the damping ratio, the system's oscillations become less pronounced. This suppression means that energy isn't transferred as efficiently, leading to a delay in how quickly energy accumulates over time. This delay becomes even more pronounced when we look at the harvested energy itself, which is essentially the total power collected over time. Since energy is cumulative, any lag in power generation results in a noticeable phase shift. In simpler terms, while the voltage reacts immediately to the material's deformation, the energy and power take longer to catch up. This delay is caused by the changes in the system's behavior due to increased damping, which affects how quickly energy can be harvested according to Lesieutre *et. al.* [22].

V. CONCLUSION

This study highlights the critical role of the damping ratio in dimensionless modeling of piezoelectric energy harvesting using d33-mode multilayer piezoceramics. The findings show that a lower damping ratio results in sharp resonance peaks, leading to higher peak energy and power output but with a narrow bandwidth and increased mechanical stress. Conversely, a higher damping ratio broadens the bandwidth, ensuring more stable energy harvesting over a wider frequency range, albeit with reduced peak output. Time-domain analysis further reveals that lower damping leads to greater oscillations, increasing peak values of harvested voltage, energy, and power. However, as damping increases, power and energy exhibit a phase delay, while voltage remains in sync with the system's structural motion. This delay arises because energy accumulation is influenced by the rate of power transfer, which is affected by damping. Overall, the results emphasize the need for an optimal damping balance to maximize energy efficiency while ensuring system stability. Future work should explore adaptive damping strategies to enhance both performance and durability in practical energy harvesting applications.

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