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An EOQ Inventory Model Using Generalised WEIBULL Distribution under Ramp Type Demand with Shortages

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Abstract:
In this paper an inventory model is developed in which inventory is depleted not only by demand but also by deterioration at a Generalized Weibull distributed rate, assuming the demand rate a ramp type function of time. The model is solved analytically by enumerating two possible shortage models to obtain the optimal solution. Finally numerical examples are provided to assess the solution procedure.

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I. INTRODUCTION

In earlier days classical inventory model like Harris [1] assumes that the depletion of inventory is due to a constant demand rate. But subsequently, it was noticed that depletion of inventory may take place due to deterioration also, and then the problem of decision makers is how to control and maintain inventories of deteriorating items. Many researchers like Ghare and Schrader [2], Goyal et al. [3], Covert and Philip [4], Aggarwal and Jaggi [5], Cohen [6], Mishra [7] are very important in this connection. As the time progressed, several other researchers developed inventory models with deteriorating items with time dependent demand rates. In this connection, related works may refer to Ritchie [8], Hariga [9], S.P. Singh and V.K. Sehgal [10] Ghose and Chaudhari [11], Donaldson [12], Silver [13], Datta and pal [14], Deb and Chaudhari [15], Pal and mandal [16], Goel and Aggarwal [17]. Mishra [7] developed a two parameter Weibull distribution deterioration for an inventory model. This was followed by many researchers like S.P. Singh and G.C. Panda [18] Dev and Patel [19], Shah, and Jaiswal [20], Giri et al. [21] etc.

In the present paper, we drive the three EOQ inventory models for items that deteriorate at a Weibull rate, assuming the demand rate with a ramp type function of time. Shortages are allowed. The demand rate for such items increases with time up to certain time and then ultimately stabilizes and becomes constant. Finally numerical examples are proposed to demonstrate our developed model and the solution procedure.

II. Assumptions and Notations

The fundamental assumptions and notations used in this paper are given as follows:

(a) Replenishment is instantaneous.
(b) Lead time is zero.
(c) T is the fixed length of each production cycle.
(d) R is the replenishment cost per cycle.
(e) C_1 is the inventory holding cost per unit per unit time.
(f) C_2 is the shortage cost per unit per unit time.
(g) C_3 is the unit deterioration cost.
(h) The deterioration rate function follows a two parameter Weibull distribution

$$Z(t) = \alpha\beta t^{\beta-1}, 0 < \alpha << 1,\ \beta > 0,\ t > 0$$

When $\beta = 1$, Z(t) becomes a constant which is the case of an exponential decay. When $\beta < 1$, the rate of deterioration is decreasing with time t and $\beta > 1$, it is increasing with t.

(i) Shortages are allowed and completely backlogged.
(j) S is the maximum inventory level of each ordering cycle.
(k) The demand rate D(t) is assumed to be a ramp type function of time:

$D(t) = D_0[t-(t-\mu)\, H(t-\mu)],\ D_0 > 0$

Where $H(t-\mu)$ is the well-known Heaviside's function defined as follows;

$H(t-\mu) = 1, t \geq \mu$
$= 0, t < \mu$

III. Mathematical Models and Solutions

The objective of the inventory problem here is to find the optimal order quantity to keep the total relevant cost minimum. Based on whether the inventory starts with shortages or not, there are two possible models under the assumptions described in section 2.

1. Model I : Inventory starts without shortages

In the subsection, we will analyze the inventory starts without shortages. Replenishment is made at time t = 0 when the inventory level is at its maximum, S. Due to reasons of market demand and deterioration of the items, the inventory level gradually diminishes during the period (0, t_1) and finally falls to zero at t = t_1. Shortages are allowed during the period (t_1, T) which are completely backlogged. The total number of backlogged items is replaced by the next replenishment.

The inventory level I(t) of the system at any time t over [0, T] can be described by the following equations:

$$\frac{dI(t)}{dt} + Z(t)\,I(t) = -D(t),\ 0 \leq t \leq t_1 \qquad (1)$$

and

$$\frac{dI(t)}{dt} = -D(t),\ t_1 \leq t \leq T \qquad (2)$$

The boundary condition are $I(t_1) = 0$ and $I(0) = S$ (3)

By assumptions of (h) and (k) in section (2) and assuming $\mu < t_1$, the two governing equations (1) and (2) becomes:

$$\frac{dI(t)}{dt} + \alpha\,\beta\,t^{\beta-1}\,I(t) = -D_0(t),\ 0 \leq t \leq \mu \qquad (4)$$

$$\frac{dI(t)}{dt} + \alpha\,\beta\,t^{\beta-1}\,I(t) = -D_0\mu,\ \mu \leq t \leq t_1$$

(5)

and

$$\frac{dI(t)}{dt} = -D_0\mu,\ t_1 \leq t \leq T \qquad (6)$$

Now by using the boundary conditions (3), the solution of above three equations are respectively given as:

$$I(t) = S\,e^{-\alpha t^{\beta}} - D_0\,e^{-\alpha t^{\beta}} \int_0^t x\,e^{\alpha x^{\beta}}\,dx,\ 0 \leq t \leq \mu$$

(7)

$$I(t) = S\,e^{-\alpha t^{\beta}} - D_0\,e^{-\alpha t^{\beta}} \int_0^{\mu} x\,e^{\alpha x^{\beta}}\,dx + \mu \int_{\mu}^{t} e^{\alpha x^{\beta}}\,dx,\ \mu \leq t \leq t_1 \qquad (8)$$

and

$$I(t) = -D_0\mu\left(t - t_1\right),\ t_1 \leq t \leq T$$

(9)

When 0 < α << 1, we neglect the second and higher terms of α, equation (7) and (8) becomes:

$$I(t)=\frac{-D_0t^2}{2}\left(1-\frac{\alpha\beta t^{\beta}}{\beta+2}\right)+S\left(1-\alpha t^{\beta}\right),\ 0\le t\le\mu \tag{10}$$

and

$$I(t)=S\left(1-\alpha t^{\beta}\right)-D_0\mu t\left(1-\frac{\alpha\beta t^{\beta}}{\beta+1}\right)+\frac{D_0\mu^2}{2}\left[1-\alpha t^{\beta}+\frac{2\alpha\mu^{\beta}}{(\beta+1)(\beta+2)}\right],\ \mu\le t\le t_1 \tag{11}$$

Since $I(t_1) = 0$, we get from equation (8) with neglecting second and higher order terms of α as:

$$S=D_0\mu t_1\left(1+\frac{\alpha\, t_1^{\beta}}{\beta+1}\right)-\frac{D_0\mu^2}{2}\left[1+\frac{2\alpha\mu^{\beta}}{(\beta+1)(\beta+2)}\right] \tag{12}$$

Hence the total number of deteriorated units during $[0, t_1]$ is
D_t = Initial inventory – Total demand during $[0, t_1]$

$$=S-\int_0^{t_1}D(t)dt$$
$$=S-\left[\int_0^{\mu}D_0t\,dt+\int_{\mu}^{t_1}D_0\mu\,dt\right]$$

Evaluating the above two integrals and using equation (12), we get as

$$D=\frac{D_0\mu\alpha}{\beta+1}\left(t_1^{\beta+1}-\frac{\mu^{\beta+1}}{\beta+2}\right) \tag{13}$$

The total number of inventory holding during the period $[0, t_1]$ is

$$I_1=\int_0^{t_1}I(t)dt$$
$$=\int_0^{\mu}I(t)dt+\int_{\mu}^{t_1}I(t)dt$$
$$=\int_0^{\mu}\left[S\,e^{-\alpha t^{\beta}}-D_0e^{-\alpha t^{\beta}}\int_0^{t}x\,e^{\alpha x^{\beta}}dx\right]dt+\int_{\mu}^{t_1}\left[D_0\mu e^{-\alpha t^{\beta}}\int_t^{t_1}e^{\alpha x^{\beta}}dx\right]dt$$

[From (7) & (8)]

Evaluating the above integrals and neglecting the second and higher order of α, we get as:

$$I_1=D_0\mu\left[-\frac{\mu^2}{6}+\frac{t_1^{2}}{2}-\frac{\alpha\beta\mu^{\beta+2}}{(\beta+1)(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

(14)

The total shortage quantity during the interval $[t_1, T]$ is

$$I_2 = -\int_{t_1}^{T} I(t)\,dt$$
$$= \int_{t_1}^{T} D_0 \mu (t - t_1)\,dt \qquad [\text{from}\,(9)]$$
$$= \frac{1}{2} D_0 \mu (T - t_1)^2 \tag{15}$$

Then the average total cost per unit time is given by

$$C_1(t_1) = \frac{R}{T} + \frac{C_3 D_t}{T} + \frac{C_1 I_1}{T} + \frac{C_2 I_2}{T}$$

Now substituting the expressions for D_t, I_1, I_2 given by the equations (13), (14) and (15) respectively and then eliminating S by the equation (12), we get as:

$$C_1(t_1) = \frac{R}{T} + \frac{D_0 \mu C_1}{T}\left[\frac{t_1^{\,2}}{2} - \frac{\mu^2}{6} - \frac{\alpha\beta\mu^{\beta+2}}{(\beta+1)(\beta+2)(\beta+3)} + \frac{\alpha\beta t_1^{\,\beta+2}}{(\beta+1)(\beta+2)}\right] + \frac{C_2 D_0 \mu}{2T}(T - t_1)^2 + \frac{C_3 D_0 \mu\alpha}{T(\beta+1)}\left(t_1^{\,\beta+1} - \frac{\mu^{\beta+1}}{\beta+2}\right) \tag{16}$$

To minimize the average total cost per unit time, the optimal value of t_1, say $t_1^{\,*}$ can be obtained by solving the following equation

$$\frac{dC_1(t_1)}{dt_1} = 0$$

Which also satisfies the condition

$$\left.\frac{d^2 C(t_1)}{dt_1^{\,2}}\right|_{t=t_1^{\,*}} > 0$$

After solving, the condition $\frac{dC(t_1)}{dt_1} = 0$ gives the equation:

$$C_3 \alpha t_1^{\,\beta} + C_1\left(t_1 + \frac{\alpha\beta t_1^{\,\beta+1}}{\beta+1}\right) + C_2(t_1 - T) = 0\,, \qquad [\text{say } \phi(t_1)] \tag{17}$$

Since $\phi(0) < 0$ and $\phi(T) > 0$, then $\phi(0).\phi(T) < 0$. So there exits one solution $t_1 = t_1^* \in (0, T)$ of equation (3.1.17), which can be easily solved by Newton-Raphson method.
Also,
Substituting $t_1 = t_1^*$ in equation (12), we find the optimum value of S, given as:

$$S^* = D_0 \mu t_1^{\,*}\left(1 + \frac{\alpha\, t_1^{\,*\,\beta}}{\beta+1}\right) - \frac{D_0 \mu^2}{2}\left[1 + \frac{2\alpha\mu^{\beta}}{(\beta+1)(\beta+2)}\right] \tag{18}$$

Again the total amount of backorder at the end of the cycle is $D_0 \mu (T - t_1)$. Therefore $Q = S + D_0 \mu (T - t_1)$. so the optimal value of Q is given by

$$Q^* = S^* + D_0\mu\left(T - t_1^*\right)$$
$$= D_0\mu\left[\frac{\alpha t_1^*}{\beta+1} - \frac{\mu}{2} - \frac{\alpha \mu^{\beta+1}}{(\beta+1)(\beta+2)} + T\right] \tag{19}$$

And the minimum value of the average total cost $C_1(t_1)$ is $C_1\left(t_1^*\right)$ as from equation (16).

2. Model II : Inventory Starts with Shortages

In this subsection, we have considered the deterministic inventory model for deteriorating items, where the inventory is allowed to start with shortages. Here the two situations may arise, depending on procurement time t_1. (i) $\mu < t_1$ and (ii) $\mu > t_1$. The inventory system starts with zero inventory level at $t = 0$ and shortages are allowed to accumulate up to t_1. Replenishment is done at time t_1. The quantity received at t_1 is used partly to make up for the shortages accumulated in the previous cycle from time 0 to t_1. The rest of the procurement accounts for the demand and deterioration in $\left[t_{1,}T\right]$. The inventory level gradually falls to zero at time T.
The inventory level of the system at time t over the period [0, T] can be described by the following equations.

$$\frac{dI(t)}{dt} = -D(t),\ 0 \le t \le t_1 \tag{20}$$

and

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -D(t),\ t_1 \le t \le T \tag{21}$$

A. Situation I: $(\mu < t_1)$

In this situation, the above equations become

$$\frac{dI(t)}{dt} = -D_0 t,\ 0 \le t \le \mu \tag{22}$$

$$\frac{dI(t)}{dt} = -D_0\mu,\ \mu \le t \le t_1 \tag{23}$$

And

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -D_0\mu,\ t_1 \le t \le T \tag{24}$$

Now using the boundary conditions $I(0) = 0$ and $I(T) = 0$, neglecting the second and higher order terms of α, the solutions of the differential equations (22)-(24) are respectively given as:

$$I(t) = -\frac{D_0}{2} t^2,\ 0 \le t \le \mu \tag{25}$$

$$I(t) = D_0\mu\left(\frac{\mu}{2} - t\right),\ \mu \le t \le t_1 \tag{26}$$

And

$$I(t) = D_0\mu\left[T - t + \frac{\alpha T^{\beta+1}}{\beta+1} - \alpha T t^{\beta} + \frac{\alpha\beta t^{\beta+1}}{\beta+1}\right],\ t_1 \le t \le T \qquad (27)$$

Since $I(t_1) = S$,

$$S = D_0\mu\left[T - t_1 + \frac{\alpha T^{\beta+1}}{\beta+1} - \alpha T t_1^{\beta} + \frac{\alpha\beta t_1^{\beta+1}}{\beta+1}\right] \qquad (28)$$

As discussed in subsection III.1,

$$D_t = S - D_0\mu(T - t_1)$$

$$D_t = D_0\mu\left[\frac{\alpha T^{\beta+1}}{\beta+1} - \alpha T t^{\beta} + \frac{\alpha\beta t^{\beta+1}}{\beta+1}\right] \qquad (29)$$

$$I_1 = D_0\mu\left[\frac{1}{2}\left(T^2 + t_1^{2}\right) - T t_1 + \frac{\alpha T t_1}{\beta+1}\left(t_1^{\beta} - T^{\beta}\right) + \frac{\alpha\beta}{(\beta+1)(\beta+2)}\left(T^{\beta+2} - t_1^{\beta+2}\right)\right] \qquad (30)$$

$$I_2 = \frac{D_0\mu}{6}\left[\mu^2 + 3t_1(t_1 - \mu)\right] \qquad (31)$$

Now the average total cost of the system per unit time is

$$TC_2(t_1) = \frac{(R + C_3 D_t + C_1 I_1 + C_2 I_2)}{T}$$

$$= \frac{R}{T} + \frac{C_3 D_0\mu}{T}\left[\frac{\alpha T^{\beta+1}}{\beta+1} - \alpha T t_1^{\beta} + \frac{\alpha\beta t_1^{\beta+1}}{\beta+1}\right] + \frac{C_1 D_0\mu}{T}\left[\begin{array}{l}\frac{1}{2}\left(T^2 - t_1^{2}\right) - T t_1 + \frac{\alpha T t_1}{\beta+1}\left(t_1^{\beta} - T^{\beta}\right) \\ + \frac{\alpha\beta}{(\beta+1)(\beta+2)}\left(T^{\beta+2} - t_1^{\beta+2}\right)\end{array}\right]$$

$$+ \frac{C_2 D_0\mu}{6T}\left[\mu^2 + 3t_1(t_1 - \mu)\right] \qquad (32)$$

The optimal value of t_1 for the minimum average total cost per unit of time is the solution

$$\frac{dTC_2(t_1)}{dt_1} = 0 \qquad (33)$$

Provided that the value of t_1 satisfies the condition

$$\left.\frac{d^2TC_2(t_1)}{dt_1^{2}}\right|_{t=t_1^*} > 0.$$

From equation (33), we have as:

$$C_3\left[\alpha\beta t_1^{\beta-1}(t_1 - T) + C_1\left(\alpha T t_1^{\beta} - \frac{\alpha T^{\beta+1}}{\beta+1}\right) - \frac{\alpha\beta t_1^{\beta}}{\beta+1} - t_1 - T + C_2\left(t_1 - \frac{\mu}{2}\right)\right] = 0 \qquad (34)$$

For different values of the various parameters, Eq.(34) can be solved numerically using the Newton-Raphson method.
Furthermore, the total backorder amount for the entire cycle is

$$\frac{D_0\mu^2}{2} + D_0\mu\left(t_1^* - \mu\right)$$

Therefore, the optimal order quantity, Q^* is

$$\begin{aligned} Q^* &= S^* + \frac{D_0\mu^2}{2} + D_0\mu\left(t_1^* - \mu\right) \\ &= S^* + D_0\mu\left(t_1^* - \frac{\mu}{2}\right) \\ &= D_0\mu\left[T - \frac{\mu}{2} + \frac{\alpha T^{\beta+1}}{\beta+1} - \alpha T\, t_1^{*\beta} + \frac{\alpha\beta t_1^{*\beta+1}}{\beta+1}\right] \end{aligned} \tag{35}$$

B. Situation II ($\mu > t_1$)

In this situation, the two governing Eqs. (20) and (21), become

$$\frac{dI(t)}{dt} = -D_0 t,\ 0 \le t \le t_1 \tag{36}$$

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -D_0 t,\ t_1 \le t \le \mu \tag{37}$$

And

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -D_0\mu,\ \mu \le t \le T \tag{38}$$

The solution of the differential equations (36)-(38) with the boundary conditions I(0) = 0 and I(T) = 0 are

$$I(t) = -\frac{D_0 t^2}{2},\ 0 \le t \le t_1 \tag{39}$$

$$I(t) = D_0\left[\mu\left(T + \frac{\alpha T^{\beta+1}}{\beta+1}\right) - \frac{1}{2}\left(\mu^2 + T^2\right) - \frac{\alpha\mu^{\beta+2}}{(\beta+1)(\beta+2)} + \alpha\mu t^{\beta}\left(\frac{\mu}{2} - T\right) + \frac{\alpha\beta t^{\beta+2}}{2(\beta+2)}\right], t_1 \le t \le \mu \tag{40}$$

And

$$I(t) = D_0\mu\left[t - t + \frac{\alpha T^{\beta+1}}{\beta+1} - \alpha T t^{\beta} + \frac{\alpha\beta t^{\beta+2}}{\beta+1}\right], \mu \le t \le T \tag{41}$$

Since $I(t_1) = S$, from equation (40), we have as:

$$S = D_0\left[\mu\left(T + \frac{\alpha T^{\beta+1}}{\beta+1}\right) - \frac{1}{2}\left(\mu^2 + T^2\right) - \frac{\alpha\mu^{\beta+2}}{(\beta+1)(\beta+2)} + \alpha\mu t_1^{\beta}\left(\frac{\mu}{2} - T\right) + \frac{\alpha\beta t_1^{\beta+2}}{2(\beta+2)}\right] \quad (42)$$

Proceeding as in earlier case,

$$D_t = S + \frac{D_0}{2}\left(\mu^2 + t_1^{2} - 2\mu T\right)$$
$$= D_0\left[\frac{\mu\alpha T^{\beta+1}}{\beta+1} - \frac{1}{2}\left(T^2 - t_1^{2}\right) - \frac{\alpha\mu^{\beta+2}}{(\beta+1)(\beta+2)} + \alpha\mu t_1^{\beta}\left(\frac{\mu}{2} - T\right) + \frac{\alpha\beta t_1^{\beta+2}}{2(\beta+2)}\right] \quad (43)$$

$$I_1 = \int_{t_1}^{\mu} I(t)dt + \int_{\mu}^{T} I(t)dt \quad (44)$$

And

$$I_2 = \frac{D_0 t_1^{3}}{6} \quad (45)$$

Therefore, average total cost of the system per unit time is

$$TC_3(t_1) = \frac{\left(R + C_3 D_t + C_1 I_1 + C_2 I_2\right)}{T} \quad (46)$$

To minimize TC_3 the optimal value of t_1 can be obtained by solving the equation

$$\frac{dTC_3(t_1)}{dt_1} = 0$$

Which also satisfies the condition

$$\left.\frac{d^2 TC_3(t_1)}{dt_1^{2}}\right|_{t=t_1^*} > 0$$

The total backorder amount for the cycle is $\frac{D_0 t_1^{*2}}{2}$

Therefore, the optimal order quantity Q^* is

$$Q^* = S^* + \frac{D_0 t_1^{*2}}{2}$$

IV. Numerical Examples

Example 1: To illustrate the theory developed above, the following numerical example has been considered. Let the input parameters are as follows:
C_1 = \$ 3 per unit per year, C_2 = \$ 15 per unit per year, C_3 = \$ 5 per unit per year,
D_0 = 100 units, μ = 0.8 year, α = 0.001, β = 2, T = 1 year

Here Model I denote that the inventory model starts without shortages and Model II denote that the inventory model starts with shortages, and then applying the procedure described in previous section, the optimal solution for Model I and Model II are those given in Table 1.

Optimal Solution	Model I	Model II	
		Situation I: $\mu < t_1^*$	Situation II $\mu. > t_1^*$
t_1^*	0.8331	0.2178	….
S^*	9.2792	9.3997	….
Q^*	11.2823	11.2837	….
TC^*	14.9378	13.3968	….

Table 1. optimal solution of the proposed Inventory System – Example 1

Example 2:

The parameters are like those in Example 1, except that C_1 is changed to \$0.8 and C_3 is changed to \$0.5. Table 2 shows the corresponding results.

Optimal Solution	Model I	Model II	
		Situation I: $\mu < t_1^*$	Situation II $\mu. > t_1^*$
t_1^*	0.9492	….	0.1071
S^*	10.6934	….	10.7208
Q^*	11.3054	….	11.2931
TC^*	20.5789	….	10.9046

Table 2. optimal solution of the proposed Inventory System – Example 2

From Table 1 and 2, it is found that the minimum average total cost per unit of time and the optimal order quantity both are smaller in the case when the inventory starts with shortages.

VI. Conclusions

We have analyzed two order- level inventory models for deteriorating items with a ramp type demand function of time and deterioration rate is assumed to follow a two-parameter Weibull distribution. Analytical solutions of the model are discussed and are illustrated with the help of two numerical examples.

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