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The alternative quantization of local fields and open cosmological questions

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Abstract: *An alternative quantization method of local fields was proposed in the papers [1, 2] (cf.: [3]). In this paper as a continuation of the paper [4] we discuss three cosmological aspects in connection with this quantization method called quantum local field theory (QLFT).*
Aspect 1.: The lowest energy state of the generator of the time translation (the local Hamiltonian), i.e. its "vacuum state" in this approach, results in a non zero, but finite expectation value, thus it gives in the Universe a finite but non zero vacuum energy density.
Aspect 2.: The submicroscopic structure of space-time should be studied by analysing the Hawking-radiations of black holes. If Nature uses a space-time quantum as a new Natural constant then the analysis of Hawking radiations should display this new constant.
Aspect 3.: One should consider the question: what are the consequences if an Observer in reference frame L(v=0) wants to transfer in reference frame L(v=V)? Of course one has to accelerate the Observer to the speed of V. Therefore the acceleration process needs an elementary resolution and description using an accelerating field of force. Such a force is for example the electromagnetic field but theoretically it may be the newly discovered fifth force, too. Then one has to answer the question, at least theoretically, until the direct measurement, if the speed of the action of the fifth force is equal to the speed of the light or it differs from c? We discuss this question and find that theoretically it could differ from c, even it could be greater than c. Moreover the discussion admits the theoretical possibility that the fifth force might need a space of dimension greater the 3.

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I. Introduction

An alternative quantization method of local field theory (LFT) was proposed four decades ago in the papers [1, 2] (cf.: [3]). This approach is a direct extension of the canonical quantization method of Quantum Mechanics (QM) of finitely many degrees of freedom to Quantum Field Theory (QFT) of infinitely many degrees of freedom. It preserves the structure of LFT, namely the global observables of the system are generated by local ones integrated them over the underlying space-time structure. Since a classical local field theory is built up from an infinite collection of identical classical mechanical systems of finitely many degrees of freedom connected in space, we expect that the quantization preserves this structure. In fact, generally one can say that this quantization method substitutes the individual members of the infinite collection by their quantum mechanical refinements connected in space.

In this way an alternative approach of quantized fields was constructed which applies a new (quantum) conception of space-time, in accordance with Schwinger's observation ([5], see below point B)) and it lacks the main difficulties of the conventional theory. These difficulties are as follow:

A) If the basic hypotheses of the conventional theory hold true then the interaction picture is not applicable to describe nontrivial interactions. This is stemmed by two difficulties: a) in the framework of the conventional theory, by means of the interaction picture, one can derive only the trivial S matrix (Haag´s theorem), and b) the interaction Hamiltonian consisting of higher power than quadratic of the field does not possess a definite mathematical meaning in the Fock space of the free field (ultraviolet catastrophe).

B) The quantum fields defined at the points of the Minkowski space do not exist as operators in a Hilbert space.

In the alternative approach the local bounded observables of the system are represented by bounded self-adjoint operators (A-module homomorphism) in the local state space H_A where H_A is an A-valued Hilbert space (Hilbert A-module) and A is the algebra of bounded operators in the Hilbert-space $L^2(\boldsymbol{R}^3)$ (the Hilbert-space of the square integrable functions over the space $\boldsymbol{R}^3$). The local states are represented by the rays of norm

1 (the unity operator of *A*) in H_A. The expectation value of a local bounded observable *F* in the local state *Ψ* in H_A is given by the formula

$$Exp\ F = <\Psi|F|\Psi>_A \in A \qquad (1)$$

using the *A*-valued Hermitian inner product of H_A.

The global description of QLFT of local state space H_A corresponds to the rules:

a1) The global state space of QLFT of local state space H_A in the measuring procedures characterized by the statistical operator $\rho \in A$ is the complex separable Hilbert space $H^\rho = \mathrm{Tr}\ \rho H_A$; its global states are described by the rays of H^ρ.

a2) The global observables are represented by self-adjoint operators in H^ρ.

a3) The expectation value of the global observable *f* generated by the local one, *F*, in the global state ϕ generated by the local one, *Φ*, is

$$f_\rho = <\phi|f|\phi>_\rho = \mathrm{Tr}\ \rho <\Phi|F|\Phi>_A\ , \qquad \Phi \in \boldsymbol{\Phi} \qquad (2)$$

H^ρ carries all the information about the infinite collection of the connected quantum mechanical systems constituting QLFT, which are obtained in the common quantum statistical state ρ of the local measuring apparatuses in (quantum) space-time.
We note that it was demonstrated in the paper [6] that this approach of QLFT does contain the physical implications of the conventional theory legitimating in this way this alternative quantization method of local fields.

In this paper we discuss three cosmological aspects in connection with this quantization method leading to QLFT. In accordance with ref. [1] we consider the basic observables as operator valued functions over the underlying quantum space-time. Namely

$$\varphi: \boldsymbol{f} \rightarrow \varphi(\boldsymbol{f}) \qquad (3)$$

where φ is a Hermitian operator, representing the observable considered, in the local state space H_A, and $\boldsymbol{f}$ is a ray of the event space of the underlying (quantum) space-time represented by a countably separable Hilbert space, more precisely, the Hilbert space of the square integrable functions of the space (of the space-like coordinates of events, i.e. the set of the spectral values of the position observables $\boldsymbol{x}$ of the test particle probing the (sub)microscopic structure of space-time).
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1. The lowest energy state of the time translation operator, the vacuum state

First we shortly present the alternative method of field quantization in references [1, 2]. By the sake of simplicity we consider here the illustrative case of *N* real classically relativistic scalar fields of Lagrangian density,

$$L(t,\boldsymbol{x}) = [1/2\textstyle\sum_{\alpha=1}^{N}(\partial_\mu\phi_\alpha\partial^\mu\phi_\alpha - m_\alpha^2\phi_\alpha^2) - V(\phi_1,\ \ldots,\ \phi_N)](t,\boldsymbol{x}),\ \ (t,\boldsymbol{x})\in \boldsymbol{M}^4, \quad (1.1)$$

to demonstrate the methodology of the alternative quantization.
This system consists of an infinite collection of identical classical anharmonic oscillators of *N* degrees of freedom connected in space. Then the corresponding QLFT should consist of an infinite collection of identical quantum anharmonic oscillators of *N* degrees of freedom connected in space.
Really, as it was shown in [2] the alternative quantization method substitutes the individual members of the system by their quantum mechanical counterparts. The local state pace H_A is of the form $L^2(\boldsymbol{R}^N)\otimes A$ [the tensor product of the complex separable Hilbert space $L^2(\boldsymbol{R}^N)$ and the C^*-algebra *A* of bounded operators of $L^2(\boldsymbol{R}^3)$]. The quantized system is described coherently in this approach because the algebra of bounded operators $B(H_A) = B(H)\otimes A$ of the local state space H_A is a ***factor*** [6].
Von Neumann's basic theorem of QM, namely that the canonical commutation relations (CCR) have a unique solution up to unitary equivalence, has an extended form in this framework: a *B*-irreducible set of unitary

operators in the *A*-valued Hilbert space H_A satisfying the CCR's is uniquely determined up to *A*-unitary equivalence (see [2, pp.198, 199].[1] In this way this extension of von Neumann's theorem offers the possibility that one formulates QLFT in terms of the *A*-valued Hilbert spaces in the same unique way, up to *A*-unitary equivalence, as QM is formulated in terms of complex Hilbert spaces up to unitary equivalence.

The dynamics of the system is described by the unitary map

$$t \rightarrow exp\ (-iHt)$$

of H_A onto itself, where H is the local Hamiltonian of the system obtained by replacing the Hamiltonian density of the classical system with its local operator counterpart one gets by the quantization algorithm, i.e.

$$H = H(\phi, \pi, \partial\phi) =$$

$$= 1/2\sum_{\alpha=1}^{N}[\pi_\alpha^2 + (\partial\phi_\alpha)^2 + m_\alpha^2\phi_\alpha^2] + V(\phi_1, \ldots, \phi_N),$$

where the fields ϕ_α and their canonical momentum densities π_α as operators in H_A satisfy the CCR's [2]. The classical equations of motion become well-defined operator equations in H_A and the local states are governed by the local Schrödinger equation [2] :

$$i\partial\Phi(t)/\partial t = 1/2\sum_{\alpha=1}^{N}[\pi_\alpha^2 + (\partial\phi_\alpha)^2 + m_\alpha^2\phi_\alpha^2]\Phi(t) + V(\phi_1, \ldots, \phi_N)\Phi(t), \ \ \Phi\in H_A \quad (1.2)$$

Then one can apply the extension of the perturbation theory of QM to solve this equation by using the interaction picture [2]. The local Hamiltonian of the free fields is

$$H_0 = 1/2\sum_{\alpha=1}^{N}[\pi_\alpha^2 + (\partial\phi_\alpha)^2 + m_\alpha^2\phi_\alpha^2] =$$

$$= \sum_{\alpha=1}^{N}(N_\alpha + 1/2)p_0^\alpha, \quad (1.3)$$

where $p_0^\alpha = (\partial^2 + m_\alpha^2)^{1/2}$ and $N_\alpha = a_\alpha^+ a_\alpha$, a_α^+ is the creation while a_α is the annihilation operator in the local Fock space F_A of the free fields [2].
Then we see that there is a ***no zero*** *but finite* local energy density

$$E_{vacuum} = 1/2\sum_{\alpha=1}^{N} \langle\Phi_0|\, p_0^\alpha|\Phi_0\rangle_A \quad (1.4)$$

in the lowest energy local state Φ_0, in the vacuum state. Thus this approach of QLFT results a vacuum energy density in the Universe which then should contribute to the gravitation and one should see its action in cosmological observations.

II. The sub-microscopic structure of space-time

Let we summarise shortly the main point in the alternative field quantization referring to the underlying space-time structure.
The causal structure of the Minkowski space $\boldsymbol{M}^4$ was studied by W. Cegla and A.Z. Jadczyk in [8]. They showed that the causal lattice of $\boldsymbol{M}^4$ is a nondistributive, irreducible, atomic, complete orthomodular lattice. Thus it is almost a quantum propositional lattice of Piron [9], i.e. it satisfies all of the postulates of a quantum propositional lattice except one, the covering law. However the covering law plays a central role in the Piron's approach to quantum physics. Thus the Minkowski space belongs to the part of the Nature described by the classical physics which does not know and apply the Heisenberg's uncertainty principle.
Then the question is: what kind of "space-time" is the underlying structure over which the quantized fields of the foregoing section are exist? We studied this question in details in the paper [10]. To resolve the paradoxical and semantically inconsistent nature of conventional field quantization that they should result quantum theories over a classical space-time construction, we proposed a quantization method of space-time in the paper [10] by introducing a "space-time quantum hypothesis". The point of this is that we are not able to distinguish two events by measurements inside an $\check{h}$ size region of the Minkowski space $\boldsymbol{M}^4$, or equivalently the time and place

1 A system of bounded operators in the *A*-valued Hilbert space H_A is called a *B*-irreducible system if the set of bounded operators in H_A commuting with all the members of the system is equal to the Abelian von Neumann algebra *B*.

of an event cannot both be measured simultaneously, in principle, with arbitrary precision. Then this hypothesis leads us to propose the Heisenberg-type uncertainty relation

$$\Delta t\Delta r \geq \tfrac{1}{2}\,\check{h},\quad r^2 = x_1^2 + x_2^2 + x_3^2 \qquad (2.1)$$

where $\check{h}$ is a new Plank-constant characteristic for space-time and (2.1) will lead to a "canonically" quantized version of the Minkowski space $\boldsymbol{M}^4$, to a specific quantum space-time. Namely, one can derive the uncertainty principle (2.1) from the "canonical" commutation relation

$$[\mathrm{t}, \mathrm{r}] = i\check{h} \qquad (2.2)$$

We can represent the solutions satisfying this commutation relation by self-adjoint operators in the Hilbert space $L^2(\boldsymbol{R}^3)$. Thus the event space of this quantum space-time is this Hilbert space. We mean events here the events, the (measurable) states of the test particle probing the submicroscopic structure of space-time: the arena of the natural processes in the Universe. Those events for which the relation (2.1) has minimal values, i.e.

$$\Delta t\Delta r = \tfrac{1}{2}\,\check{h} \qquad (2.3)$$

provide the classical limit of this quantum space-time. The radial component of these events represented by elements (rays) of the Hilbert space $L^2(\boldsymbol{R}^3)$ has the form

$$\Phi(r) = (\gamma/\pi\check{h})^{1/4}\exp\,[-(\gamma/2\check{h})(r - \mathrm{r})^2 + (i\mathrm{t})/\check{h})r] \qquad (2.4)$$

where $\gamma \in (0, +\infty)$, $\mathrm{r} \in (-\infty, +\infty)$, $\mathrm{t} \in (-\infty, +\infty)$ and for these events

$$\Delta t = (\check{h}\gamma/2)^{1/2}, \qquad \Delta r = (\check{h}/2\gamma)^{1/2} \qquad (2.5)$$

The radial component (2.4) of the event in this quantum space-time describes a wave packet around the point r, with the width $(4\check{h}/\gamma)^{1/2}$. In the formal classical limit $\check{h} \to 0$, $\Phi(r)$ concentrates at the point r, and thus at an event of the classical space-time recovering the events of the classical (macroscopic) space-time when one can disregard the space-time quantum $\check{h}$ [10].

If Nature has such a constant, a „space-time quantum", then its measurement procedure is the question, i. e. how one can measure the value of this constant?

We suggest here again (see [4]) that the analysis of the Hawking radiations of black holes must reveal the existence of this type of a natural constant if Nature really applies it. According to the interpretation in reference [10] the dispersions Δt of the time observable t corresponds to the lifetime of the quantum test particle, while the dispersion Δr of the observable r, describing the position of this point like test particle in its rest frame, corresponds to its "extension" in space. If we consider the case when the equation (2.3) holds true we get for the lifetime of the quantum test particle

$$\Delta t = \check{h}/2\Delta r \qquad (2.6)$$

Then this relation tells us that the smallest the dispersion of r (the smallest the spread of the test particle in its rest frame) the longest the dispersion of its proper time, its lifetime. Then it implies that the lifetime of an elementary particle as a test particle must become longer approaching the event horizon of a black hole where the spread of its position shrinks to the centre of its rest frame. Therefore, in principle the Hawking radiations of black holes should display the relation (2.6).

III. The speed of the action of the fifth force

As we know the greatest news of the year 2023 in physics was the announcement of a measurement indicating the possibility of the fifth force [11, 12].
Now we turn to discuss the question of this fifth force. Namely how could we insert this force into our theoretical physics quantitative scheme.
Then first one should ask the question: what is the speed of the action of this new force? By special relativity it should be not greater than the speed of the light. However one must check this statement by experiment.

Nevertheless, until the direct measurement, we can not do anything else than to discuss this question theoretically.
First of all one should consider the question: what are the consequences if an Observer resting in reference frame L($v=0$) wants to transfer in reference frame L($v=V$) of velocity V? Of course one has to accelerate the Observer to the speed of V. Therefore the acceleration process needs an elementary resolution and description using an accelerating field of force. We know by Quantum Electrodynamics that the acceleration of a charge particle is a process of absorbing the quanta of the electromagnetic field, namely the photons taking their momenta and energy. This process rises both the speed and the mass of the charged particle. In this case the accelerating force is provided by the electromagnetic field. However we already know from the year 2023 that theoretically such an accelerating force might be also provided by the newly discovered fifth force, too. In that case the acceleration process should mean nothing else than the accelerated (dark) particle is absorbing the dark photons of the fifth force taking their momenta and energy. Then one has to answer the question, at least theoretically, until the direct measurement, if the speed of the action of the fifth force is equal to the speed of the light or it differs from c? We consider now this question and find that theoretically it could differ from c, even it could be greater than c.
(We distinct in what follow the electro-magnetic field with the prefix *white* from the hypothetical "electro-magnetic field" of the dark matter named *dark electro-magnetic field.*)

3.1 For this reason let us study a simple thought experiment (see [4], too). Using the simple natural phenomena known by everybody experiencing by walking next to a stream and throwing a piece of wood into the water, the wood slowly takes the speed of the water and then it travels with the water. In this way let us imagine a stream of uniform quanta each of the same velocity v and mass m. Let we insert in this stream a particle of mass M being in rest relative to the quanta of the stream. Let the interaction between this particle and the quanta of the stream be the inelastic collision. Then the conservation law of momentum yields, after a simple algebra, the velocity $v(N)$ of the inserted particle after the Nth collision

$$v(N) = [Nm/(Nm + M)]v \qquad (3.1)$$

We see from this relation that if N goes to infinity the velocity of the particle does go to the velocity v of the quanta of the stream, while the mass of the particle does go also to the infinity. If the accelerating stream is provided by the electromagnetic force then v is the velocity c of the light. However if the accelerating stream is provided by the fifth force then v is the velocity of the quanta of this force, i.e. the velocity of the dark photons and the relation (3.1) is also hold true when v is not equal to c. Thus this simple thought experiment might suggest that the speed of the action of the fifth force might be different, even greater than the speed of the light. The observed volume of the Universe does not contradict to this theoretical possibility. Then the QLFT of the fifth force needs a reconsideration of this method, first of all we should determine the Lagrangian density and the Hamiltonian density of the local field theory of the fifth force.

3.2 Let $A_k^{(d)}$ denote the vector potential of the fifth force named here tentatively as (the hypothetical) "*dark electro-magnetic force*", where $k = 0, 1, 2,..., n$, i.e. $A^{(d)}$ is a $1 + n$ dimensional vector. We know by experience that n equals to 3 in the case of the "white" photon, the quanta of the electromagnetic field. $A_0^{(w)}$ is the time-like component while $A_1^{(w)}$, $A_2^{(w)}$, $A_3^{(w)}$ are the space-like components of the vector potential of the (white) electro-magnetic field. We also know by experience that one can polarize the light in 3 different ways, i.e. its polarization has three degrees of freedom. We could say that the propagation of the light needs or *does span* a 3-dimensional manifold to fit in. Then what we humans, who are principally based on this "white" electro-magnetic interaction, experience as a 3 dimensional space around us and we call it *the space.* Since we do not know yet by experience the "polarization" degrees of freedom of the dark "light" corresponding to the dark photons we only suppose that it should be at least 3 or greater to be able to incorporate the white light into the space spanned by the dark light.
Then an event in this $1 + n$ dimensional manifold is denoted by $(x_0, x_1, x_2, ..., x_n)$ where $x_0 = c^{(d)}t$ and $c^{(d)}$ denotes the speed of the dark light. One can write for the dark force-field tensor using the standard notations (see e.g. [13])

$$F_{ik}^{(d)} = \partial A_k^{(d)}/\partial x^i - \partial A_i^{(d)}/\partial x^k \qquad (3.2)$$

and the dark energy-impulse tensor is

$$T_{ik}^{(d)} = (1/4\pi)[F_{il}^{(d)}F_k^{(d)l} + (1/4)F_{lm}^{(d)}F^{(d)lm}g_{ik}^{(d)}] \qquad (3.3)$$

where $g_{ik}^{(d)}$ denotes the metric tensor of the $1 + n$ dimensional manifold, the space-time fitting to the dark force (or another way of saying: the space-time of the Universe which is able to incorporate the fifth force). Then the Einstein equations of the dark matter should take the form

$$R_{ik} - \tfrac{1}{2} g_{ik}^{(d)}R = (8\pi k/c^{(d)4})T_{ik}^{(d)} \qquad (3.4)$$

where $R = R_i^i$ is the Riemann curvature of the $1 + n$ dimensional space-time manifold[2]. For the case $n \geq 3$ by contracting the indexes in (3.4) and using the relation $g_i^{i(d)} = \delta_i^i = 1 + n$ we get

$$R_i^i - \tfrac{1}{2} \delta_i^i R = R_i^i - [(n + 1)/2]R = [(1 - n)/2]R = (8\pi k/c^{(d)4})T_i^{i(d)}$$

where $T_i^{i(d)} = T^{(d)}$ is the trace of the energy – impulse tensor $T_{ik}^{(d)}$ of the dark matter. From this relation we obtain for the speed of the dark light/photon

$$c^{(d)4} = [16\pi k/(1-n)](T^{(d)}/R)$$

or

$$c^{(d)} = 2\{[\pi k/(1-n)](T^{(d)}/R)\}^{1/4} \qquad (3.5)$$

Then $c^{(d)}$ is positive for $n > 1$ if the trace of the energy-impulse tensor of the dark matter is negative, i.e. $T^{(d)} < 0$. We could approximate the energy – impulse tensor of the dark matter as

$$T_{ik}^{(d)} = (p + \varepsilon)u_i u_k - p g_{ik}^{(d)}$$

or

$$T_i^{(d)k} = (p + \varepsilon)u_i u^k - p\delta_i^k \qquad (3.6)$$

where ε is the energy density while p is the pressure (on the wall of a unit cube in the dark matter) and u denotes the $(1 + n)$-velocity. Then the trace of the energy-impulse tensor approximately is

$$T_i^{(d)i} = \varepsilon - np \qquad (3.7)$$

Therefore $T_i^{(d)i} < 0$ if $np > \varepsilon$ or $p > \varepsilon/n$. In the case of the white electro-magnetic field, when $n = 3$, we know that in the relativistic limit $p = \varepsilon/3$ [13]. Then one should conclude that the dark electro-magnetic field could propagate in a space which should have more than 3 dimensions.
To be able to use at least theoretically the parallel between the white electro-magnetic and dark electro-magnetic fields one needs the notion of the vectorial product. However this operation has a mathematical meaning, next to 3 dimension, in 7 dimension [15], therefore the dimension of the underlying space-time, able to fit the dark electro-magnetic field in, should be $1 + n = 8$.

One can check with a formal calculation that the energy density of the dark electro-magnetic field far from its sources consists of a sum of Hamiltonian functions each term in this sum corresponding to a member of an infinite collection of independent harmonic oscillators, likewise to the case of the white electro-magnetic fields [13]. Of course this checking needs to choose a large cube in the seven dimensional space $\boldsymbol{R}^7$.

Remarks:
1) If the dark matter would have such a feature then we should substitute the event space $L^2(\boldsymbol{R}^3)$ in section 2 by the event space $L^2(\boldsymbol{R}^7)$ and the radius r in the relation (2.1) should mean $r^2 = x_1^2 + x_2^2 + \ldots + x_7^2$, otherwise all other suggestions of the section remain valid.
2) Also, in the alternative quantization procedure of section 1 the local state space H_A remains of the form $L^2(\boldsymbol{R}^N)\otimes A$ [the tensor product of the complex separable Hilbert space $L^2(\boldsymbol{R}^N)$ and the C^*-algebra A of bounded

2 In this line of thoughts, by applying the least action principle, one must use the generalization of the Stoke theorem for the $1 + n$ dimensional manifold which theorem offers the possibility of reducing an n dimensional surface integral to an $1 + n$ dimensional volume integral (see [14], Theorem 16.11).

operators of $L^2(\boldsymbol{R}^7)$]. The quantized system is of course described coherently because the algebra of bounded operators $B(H_A) = B(H) \otimes A$ of the local state space H_A is a ***factor*** [6], too. The specific model of the relation in (1.1) should extend to the Minkowski space $\boldsymbol{M}^8$ generalized to $1 + 7$ dimensions.

3) As to the quantization of the dark electro-magnetic field we note here that one could choose the non gauge invariant Lagrange function form of Fermi [16] to start[3] with, and imposing a boundary condition to guarantee that the field equations hold true in mean value in the *A*-valued Hilbert space H_A.

IV. Discussion

In the first section the alternative quantization of fields was presented using scalar fields to demonstrate the mathematical apparatus of this proposition. It was shown that the lowest energy state, the vacuum state has in principle a fine, non zero contribution to the energy density of the Universe which then should be taken into account in the energy-momentum tensor of the Einstein equations of the space-time.

In the second section a space-time quantum hypothesis was discussed and suggested that how one should measure its value by analysing the Hawking radiations of black holes.

In the third section we discussed the speed of the action of the fifth force. First we showed that the elementary resolution of the acceleration process allows the possibility that this action propagates differently, even with greater speed than the light. Then we derived from the Einstein equations a relation (3.5) for the velocity of the "dark light" by supposing that the fifth force behaves similarly to the (white) electro-magnetic field, i.e. there is a vector potential which is responsible for the dark electro-magnetic field. The main observation of this discussion is that it may happen that the propagation of the dark electro-magnetic field needs a space of dimension greater than 3 and it should have dimensions 7. Then this theoretical possibility might imply the consequence that we humans see, through the white electro-magnetic interaction, only a fraction of the Universe.

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3 To avoid the problem of vanishing canonical momentum densities.