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Magnetohydrodynamic Flow Characteristics in a Porous Rotating Disk under Mixed Convection, Buoyancy, and Thermal Radiation Effects

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Abstract
This study delves into the magnetohydrodynamic (MHD) flow characteristics over a porous rotating disk placed in a mixed convective fluid environment. The primary focus is on understanding the intricate interaction between buoyancy effects, thermal radiation, and MHD forces in the flow dynamics. The system of governing equations, which comprises coupled nonlinear partial differential equations, is carefully formulated to capture the key physical phenomena governing the flow, including the influence of the applied magnetic field, the porosity of the disk, and the thermal properties of the surrounding fluid. The study explores how the magnetic field affects the flow structure by inducing a resistive force that influences the velocity distribution, while the porous nature of the disk modifies the flow characteristics by allowing for fluid penetration, further affecting the heat and mass transfer. Additionally, buoyancy-driven convection plays a crucial role in enhancing or hindering the fluid motion depending on the temperature and concentration gradients. The effect of thermal radiation is also considered, particularly its impact on the heat transfer rates and temperature distribution within the fluid. Numerical solutions are obtained to investigate the behavior of key flow quantities, including velocity, temperature, and concentration profiles. These solutions provide valuable insights into how variations in physical parameters, such as the strength of the magnetic field, porosity, and radiation effects, influence the overall dynamics of the system. This analysis contributes to a deeper understanding of MHD flows in porous media, with potential applications in industrial processes and engineering systems where heat and mass transfer are critical

Keywords: *MHD flow, porous disk, rotating disk, mixed convection, buoyancy effects, thermal radiation, fluid dynamics, heat transfer.*

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I. INTRODUCTION

The study of MHD flows has garnered significant attention in fluid mechanics due to its applications in engineering and geophysical processes. The interaction of magnetic fields with electrically conducting fluids introduces complexities that have practical importance in areas such as cooling systems, astrophysical flows, and energy generation. Rotating disk systems are particularly relevant in industrial applications, including turbine design, chemical processing, and lubrication systems.

Porous media add an additional layer of complexity, allowing mass transfer and fluid injection or suction. When coupled with buoyancy-driven convection and radiative heat transfer, the system becomes a rich domain for theoretical and experimental exploration. This study aims to address these

Symbol	Meaning		Symbol	Meaning
u	Radial velocity component (m/s)		C	Species concentration
v	Azimuthal velocity component (m/s)		$C\infty$	Ambient species concentration
w	Axial velocity component (m/s)		ρcp	Thermal capacity (J/kg·K)
r	Radial coordinate (m)		k	Thermal conductivity (W/m·K)
z	Axial coordinate (m)		qr	Radiative heat flux (W/m2)
p	Pressure (Pa)		$\sigma*$	Stefan-Boltzmann constant (W/m2·K4)
ρ	Fluid density (kg/m3)		$K*$	Mean absorption coefficient (1/m)
ν	Kinematic viscosity (m2/s)		D	Diffusion coefficient (m2/s)
σ	Electrical conductivity (S/m)		**Dimensionless Group**	

$B0$	Magnetic field strength (T)	Grc	Mass Grashof number
g	Gravitational acceleration (m/s2)	Grt	Thermal Grashof number
β	Thermal expansion coefficient (1/K)	M	Magnetic parameter
βc	Species concentration expansion coefficient (1/K)	Pr	Prandtl number
T	Temperature (K)	R_d	Radiation parameter
$T\infty$	Ambient temperature (K)	Sc	Schmidt number

Table 1: List of Variables and Parameters in the Governing Equations

phenomena by analyzing the MHD flow properties over a porous rotating disk, considering mixed convection, buoyancy, and radiation effects.

The study of magnetohydrodynamic (MHD) flows over rotating disks has garnered significant attention due to its relevance in various engineering and industrial applications. Raptis and Perdikis (2006) explored the viscous and heat transfer characteristics of MHD flows over a moving plate, focusing on how magnetic fields modify viscosity and heat diffusion. These foundational studies have provided critical insights into the interplay of magnetic fields, thermal radiation, and fluid dynamics in MHD systems.

Mixed convection, which combines forced and natural convection, is another essential phenomenon influencing fluid flow and heat transfer. Sparrow and Gregg (1959) provided a foundational analysis of mixed convection in boundary layers, describing the interaction between buoyancy forces and imposed fluid motion. Building on this, Chen (2004) examined the effects of magnetic fields on mixed convection heat transfer along vertical plates. Chen's work demonstrated how Lorentz forces alter thermal gradients and flow stability, which are critical in understanding MHD flows coupled with buoyancy and radiation effects. These studies have contributed to the broader understanding of mixed convective flows near porous rotating disks and their applications in engineering systems.

Radiative heat transfer also plays a significant role in fluid dynamics, particularly in systems involving high temperatures or optically thick media. Rosseland (1936) introduced a theoretical framework for radiative heat flux, which remains a cornerstone in modern studies of radiative energy transport. In their study, Jha and Aina (2016) explored the influence of an induced magnetic field on magnetohydrodynamic (MHD) natural convection flow within a vertical microchannel formed by two electrically non-conducting infinite vertical parallel plates. Their findings highlighted the critical role of the induced magnetic field in modulating the flow dynamics and thermal behavior in such configurations, emphasizing its potential applications in microchannel heat transfer systems. Similarly, Fang (2014) investigated MHD viscous flow over a nonlinearly moving surface, deriving closed-form solutions to characterize the flow behavior. Fang's work provided valuable insights into the complex interplay between magnetic forces and nonlinear motion, contributing to the understanding of MHD flow phenomena in industrial and engineering applications. Together, these studies advance the theoretical and practical knowledge of MHD flows under varied boundary and magnetic conditions.

Recent studies have further advanced the understanding of magnetohydrodynamic (MHD) flows over rotating disks, particularly in the context of porous media, mixed convection, buoyancy, and radiation effects. Asibor and Osudia (2023) investigated the MHD flow properties due to a porous rotating disk in a mixed convective fluid flow, incorporating buoyancy and radiation influences. Their findings highlighted the significant impact of these parameters on flow stability and heat transfer characteristics. Similarly, Mustafa et al. (2023) examined unsteady MHD Casson fluid flow with Dufour and Soret effects due to a rotating cone, providing insights into the complex interactions between thermal diffusion and concentration gradients in rotating systems.

The dynamics of heat and mass transfer in nanofluid flows have also been a focal point in recent research. Khan et al. (2023) provided an in-depth analysis of nanofluid flow with MHD linear and nonlinear mixed convection, nonlinear thermal radiation, and activation energy, elucidating the intricate balance between magnetic forces and thermal effects in such systems. In another study, Reddy et al. (2023) explored chemical reaction attributes in mixed convection radiative hybrid nanofluid flow between permeable porous plates, emphasizing the role of chemical reactions and radiation in modifying flow and heat transfer behaviors.

Further investigations have delved into MHD mixed convection flows in various configurations. Reddy et al. (2023) examined MHD mixed convection flow in a permeable vertical channel, considering buoyancy and Dufour effects, and highlighted the interplay between magnetic fields and thermal diffusion in porous media. Additionally, the study by Reddy et al. (2023) on MHD Casson flow across a stretched surface in a porous material provided valuable insights into the influence of magnetic fields on non-Newtonian fluid behavior in porous structures. These contemporary studies, along with earlier foundational research, contribute to a comprehensive understanding of MHD flow properties in systems involving porous rotating disks, mixed convection, buoyancy, and radiation effects.

These contributions, together with the advancements in MHD and mixed convection studies, provide a comprehensive background for analyzing MHD flow properties due to porous rotating disks in mixed convective fluid flows with buoyancy and radiation effects.

Despite existing research on MHD flow in various settings, this study addresses gaps by exploring the combined effects of buoyancy, thermal radiation, and porosity on MHD flow over a rotating disk in a mixed convective fluid environment. While previous studies have looked at these factors separately, few have considered their interaction within a porous medium, particularly with radiation effects. The study aims to fill this gap by formulating a model that incorporates magnetic fields, porosity, and thermal properties, offering a comprehensive understanding of the flow dynamics. This research is crucial for improving models of heat and mass transfer in industrial applications like heat exchangers and energy systems, where such complex interactions are common.

II. MATHEMATICAL FORMULATION

2.1 Governing Equations

The governing equations for the Magnetohydrodynamic (MHD) flow characteristics in a porous rotating disk under mixed convection, buoyancy, and thermal radiation effects are derived from the Navier-Stokes equations, coupled with the energy and species concentration equations. In this study, the flow is assumed to be steady, axisymmetric, and incompressible, which simplifies the analysis by reducing the complexity of the governing equations. The Boussinesq approximation is employed to account for the buoyancy effects in the presence of temperature gradients, assuming small variations in fluid density with temperature. The magnetic field is considered to be constant and uniform, with the Lorentz force arising from the interaction between the velocity field and the magnetic field playing a critical role in altering the flow dynamics.

The equations governing the velocity, temperature, and species concentration profiles are coupled, nonlinear partial differential equations. The momentum equation, describing the fluid velocity, is affected by both viscous forces and the Lorentz force. The energy equation incorporates thermal conduction, convection, and radiation effects, with the radiative heat flux modeled using the Rosseland approximation. The species concentration equation is influenced by the flow of the fluid and mass transfer effects. Assuming a steady, axisymmetric flow and employing the Boussinesq approximation, the Radial Momentum, Azimuthal Momentum Axial Momentum, Continuity Equation, Energy and Species Concentration equations respectively are:

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\sigma B_0^2}{\rho}u, \tag{1}$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}=\nu\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{\sigma B_0^2}{\rho}v, \tag{2}$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right)+g\beta(T-T_\infty)+g\beta_c(C-C_\infty), \tag{3}$$

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial w}{\partial z}=0, \tag{4}$$

$$\rho c_p\left(u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)-\frac{\partial q_r}{\partial z}, \tag{5}$$

$$u\frac{\partial C}{\partial r}+w\frac{\partial C}{\partial z}=D\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right) \tag{6}$$

where q_r is the radiative heat flux given by the Rosseland approximation:

$$q_r=-\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial z}, \tag{7}$$

2.2 Boundary and Initial Conditions

The boundary conditions for the system are:

$$\begin{aligned}&\text{At the disk surface } (z=0): u=U, v=0, w=W, T=T_s, C=C_s,\\&\text{Far from the disk } (z\to\infty): u\to 0, v\to 0, w\to 0, T\to T_\infty, C\to C_\infty.\end{aligned} \tag{8}$$

Equation (7) is approximated as

$$\frac{\partial T^4}{\partial z}\approx 4T^3\frac{\partial T}{\partial z}.$$

Thus

$$q_r = -\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial z} = -\frac{4\sigma^*}{3K^*}4T^3\frac{\partial T}{\partial z},$$

Therefore

$$\frac{\partial q_r}{\partial z} = \frac{\partial}{\partial z}\left(\frac{4\sigma^*}{3K^*}4T^3\frac{\partial T}{\partial z}\right) = \frac{4\sigma^*}{3K^*}\left[12T^2\left(\frac{\partial T}{\partial z}\right)^2 + 4T^3\frac{\partial^2 T}{\partial z^2}\right]$$

Energy Equation (5) becomes

$$\rho c_p\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{4\sigma^*}{3K^*}\left[12T^2\left(\frac{\partial T}{\partial z}\right)^2 + 4T^3\frac{\partial^2 T}{\partial z^2}\right], \qquad (9)$$

2.3 Non-Dimensionalisation

To nondimensionalize the given governing equations and boundary conditions, we use the nondimensional variables:

$$u' = \frac{u}{U}, v' = \frac{v}{U}, w' = \frac{w}{U}, r' = \frac{r}{L}, z' = \frac{z}{L}, t' = \frac{t}{t_0},$$
$$p' = \frac{p}{\rho U^2}, \theta = \frac{T - T_\infty}{T_s - T_\infty}, \phi = \frac{C - C_\infty}{(C_s - C_\infty)}. \qquad (10)$$

and substitute these into the equations and conditions. We have

$$\frac{1}{r}\frac{\partial(ru')}{\partial r'} + \frac{W}{U}\frac{\partial w}{\partial z} = 0, \qquad (11)$$

$$u\frac{\partial u}{\partial r} + \frac{W}{U}w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial r} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial r^2}\right) - Mu, \qquad (12)$$

$$u\frac{\partial v}{\partial r} + \frac{W}{U}w\frac{\partial v}{\partial z} = \frac{1}{Re}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} + \frac{\partial^2}{\partial z^2}\right)v - Mv, \qquad (13)$$

$$u\frac{\partial w}{\partial r} + \frac{W}{U}w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) + \frac{1}{Re}Grt\theta + \frac{1}{Re}Grc\phi, \qquad (14)$$

$$u\frac{\partial \theta}{\partial r} + \frac{W}{U}w\frac{\partial \theta}{\partial z} = \frac{1}{Re}\frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2}\right) + \frac{1}{Re}\frac{4R_d}{Pr}\left(1 + \frac{(T_s - T_\infty)}{T_\infty}\theta\right)^3\frac{\partial^2 \theta}{\partial z^2}$$
$$+\frac{1}{Re}\frac{12R_d}{Pr}\frac{(T_s - T_\infty)}{T_\infty}\left(1 + \frac{(T_s - T_\infty)}{T_\infty}\theta\right)^2\left(\frac{\partial \theta}{\partial z}\right)^2 \qquad (15)$$

$$u\frac{\partial \phi}{\partial r} + \frac{W}{U}w\frac{\partial \phi}{\partial z} = \frac{1}{Sc}\frac{1}{Re}\left(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial r^2}\right) \qquad (16)$$

The boundary conditions becomes:

$$u = 1, v = 0, w = 1, \theta = 1, \phi = 1 : z = 0,$$
$$u \to 0, v \to 0, w \to 0, \theta \to 0, \phi \to 0 : z \to \infty \qquad (17)$$

Where the dimensionless governing parameters are obtained as:

$$\frac{UL}{\nu} = Re, Grt = \frac{g\beta\,(T_s - T_\infty)L^2}{\nu W}, \frac{g\beta_c(C_s - C_\infty)L^2}{\nu W} = Grc,$$
$$\frac{1}{Sc} = \frac{D}{\nu\rho}, \frac{k}{\nu\rho c_p} = \frac{1}{Pr}, \frac{\sigma B_0^2 L}{\rho U} = M, \frac{4\sigma^* T_\infty^3}{3kK^*} = R_d, \frac{L}{t_0 U} = St \qquad (18)$$

III. NUMERICAL SOLUTION

The solution of the coupled system of equations governing Magnetohydrodynamic (MHD) Flow Characteristics in a Porous Rotating Disk under Mixed Convection, Buoyancy, and Thermal Radiation Effects is a complex process due to the interdependencies of the governing equations, including the momentum equations, energy equation, and species concentration equation. These equations must be solved simultaneously to capture the interactions between the fluid velocity, temperature, and species concentration fields.

Assumptions for Simplification

To solve these equations, certain assumptions are often made:

(i). **Steady-State:** The flow is assumed to be steady, meaning the partial derivatives with respect to time are zero.

(ii). **Axisymmetry:** The system is assumed to be axisymmetric, implying that the flow does not change with the azimuthal angle θ, reducing the complexity of the problem.

(iii). **Boussinesq Approximation:** This is used for buoyancy-driven flows, where density differences are small and only affect the body forces (such as buoyancy), rather than the entire fluid behavior.

(iv). **Small Magnetic Field:** The applied magnetic field is assumed to be small enough for linear approximations.

Since the system is highly coupled and nonlinear, numerical method is required for solving the full system of equations. The main steps used in the numerical solution are:

a) Discretization of the Governing Equations

(i). Finite Difference Method (FDM) or Finite Element Method (FEM) is used to discretize the partial differential equations. In this process, the domain is divided into small intervals, and the derivatives in the equations are approximated by finite differences or piecewise polynomial approximations, respectively.

(ii). Spatial derivatives (like $\partial/\partial r$, $\partial/\partial z$) are replaced by finite difference approximations, and the equations are solved on a grid.

b) Solving the System of Equations

The resulting system of algebraic equations is solved iteratively using methods Gauss-Seidel, Successive Over-Relaxation (SOR). Boundary conditions (no-slip conditions for velocity at the disk, fixed temperature, and concentration values) are applied appropriately during the solution process. The solution procedure is implemented using Jupiter Notebook embedded in Anaconda language, the result of which is displayed using Figures (1) – (16)

IV. RESULTS AND DISCUSSION

After solving the system numerically, the results can be analyzed to understand the effects of various parameters such as the Magnetic parameter (M), Thermal and mass Grashof numbers (Grt, Grc), and Radiation parameter ($\boldsymbol{R_d}$) on the velocity, temperature, and concentration profiles. The results are generated in 3D plot o interpret the flow behavior and heat transfer characteristics.

The following sections will discuss how each of these parameters affects the velocity profiles, temperature distribution, and concentration profiles.

4.1 Velocity Profiles

Velocity Distributions for the flow is depicted by Figures 2, 5 – 10. From these figures Mass Grashof Number (Gr*c*) which is a measure of the buoyancy-driven flow due to concentration differences in a fluid is shown to leads to enhanced mass transfer and buoyancy forces in the fluid as Grc increase. This is due to the stronger influence of buoyancy forces, which promote a more significant flow in the radial and axial directions, potentially reducing the resistance caused by the porous medium. Similar to Grc, the Thermal Grashof number measures the strength of buoyancy forces arising from temperature differences within the fluid. Increasing Grt intensifies the buoyant forces, which in turn enhances the velocity profiles. In the context of MHD flows, higher Gr*t* values may cause greater upward movement (axial velocity) due to the heating of the fluid, especially in systems where thermal gradients play a significant role.

The Magnetic parameter describes the influence of the applied magnetic field on the conducting fluid As shown in Figures 8 – 10. A higher M increases the Lorentz force, which acts as a resistive force to the motion of the fluid. As a result, higher M values lead to a reduction in both radial and axial velocities, as the magnetic field opposes the flow. This reduction in velocity is particularly significant in regions of high magnetic field strength. The Radiation parameter measures the effect of radiative heat transfer in the system. Higher Rd values enhance the radiative heat transfer within the flow, which can alter the velocity profiles by introducing additional energy into the fluid. As a result, the presence of radiation tends to reduce the thermal boundary layer's thickness, which may cause a slight increase in axial velocity due to less resistance to motion in the heated region.

While the Prandtl number relates the fluid's momentum diffusivity (kinematic viscosity) to its thermal diffusivity. A high Pr indicates that the fluid has a lower thermal diffusivity compared to momentum diffusivity, which can lead to slower thermal energy transport in comparison to velocity. In this flow scenario, increasing Pr reduces the velocity profiles, particularly in the radial direction, as the thicker thermal boundary layer limits the fluid's ability to adjust to temperature changes and, the Schmidt number represents the ratio of momentum diffusivity to mass diffusivity. A higher Sc indicates that the fluid's momentum diffuses much more readily than its species concentration. This results in reduced velocity in the regions near the rotating disk since the concentration gradient impedes the motion of the fluid, especially near the boundary.

4.2 Temperature Distribution

In Figures 1, 11 – 13 are displayed. As the Mass Grashof number increases, the buoyancy driven flow causes increased temperature gradients in the fluid. The higher Grc enhances the upward motion of the fluid, causing regions of higher temperature to be transported more effectively, which results in a greater temperature difference between the wall and the fluid. Whereas, increasing the Thermal Grashof number intensifies the buoyant force due to the temperature difference. This leads to a thicker thermal boundary layer and, in turn, higher temperature at the fluid near the porous rotating disk. Higher Grt results in enhanced natural convection, causing a more pronounced temperature increase near the surface of the disk.

The Magnetic parameter influences the temperature distribution by introducing resistance to the fluid's motion, which limits the ability of the fluid to transport heat. The temperature distribution becomes more pronounced near the disk surface, as higher M values increase thermal resistance, leading to a higher local temperature in the boundary layer near the disk. The Radiation parameter affects temperature by enhancing the radiative heat transfer within the system. Higher Rd values lead to a reduction in the thermal boundary layer thickness, allowing for a more uniform temperature distribution. This results in a smoother and faster thermal adjustment, especially in regions away from the disk.

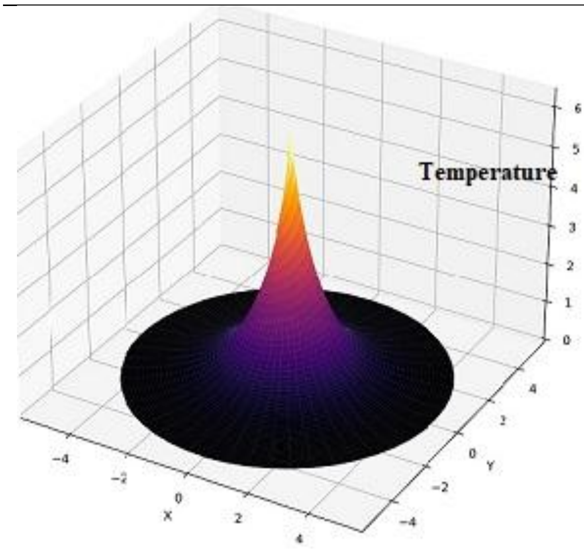

Figure 1: Temperature profile

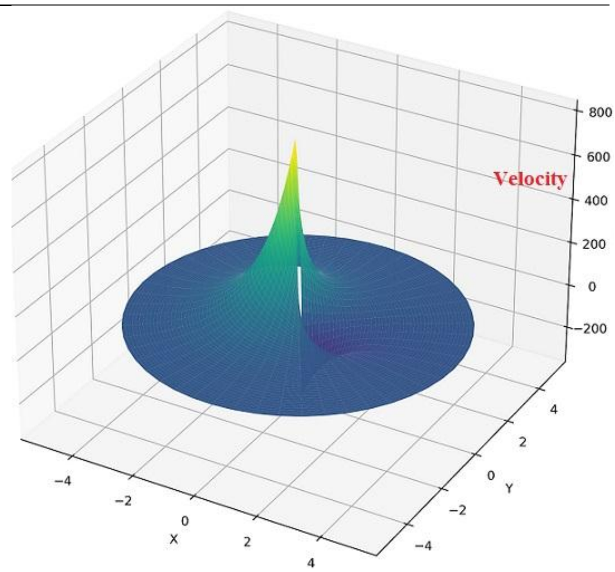

Figure 2: Velocity Profile

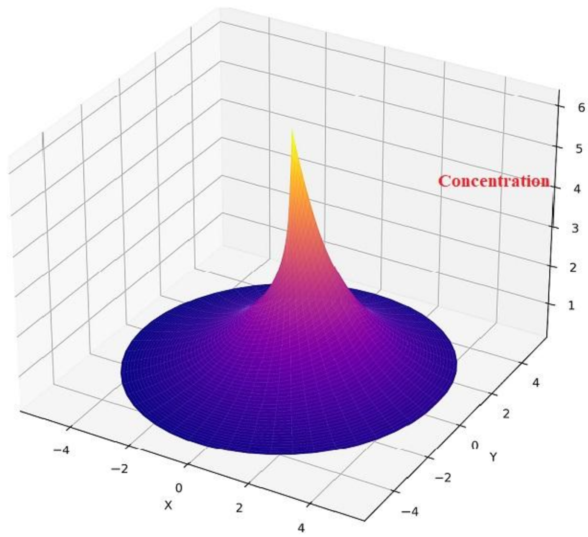

Figure 3: Concentration Profile

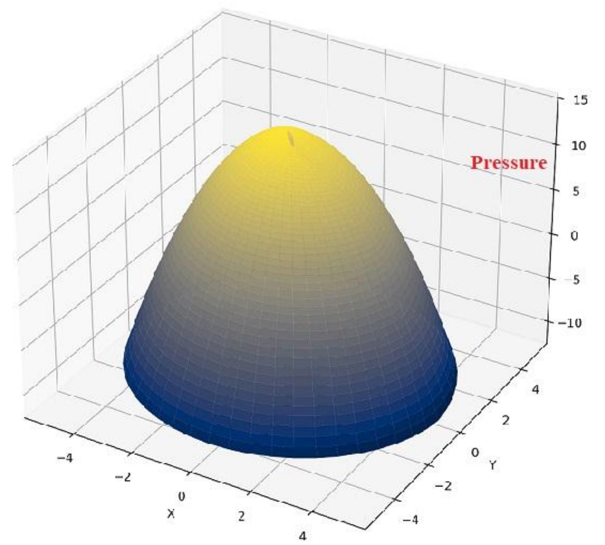

Figure 4: Pressure distribution

Also, higher Prandtl numbers tend to result in a thicker thermal boundary layer, as thermal diffusion is slower compared to momentum diffusion. In such a case, the temperature distribution will be more pronounced near the disk, and heat will be retained in the fluid for a longer time, creating a more significant temperature gradient. The Schmidt number affects the interaction between species concentration and thermal diffusion. A higher Sc leads to a more pronounced separation of concentration and thermal effects, potentially reducing the temperature gradients in areas of significant mass transfer and enhancing heat transfer resistance in the boundary layer.

4.3 Concentration Profiles

For Concentration distributions, we displayed the distributions using Figures 3, 14 – 16. From the figures, an increase in the Mass Grashof number strengthens the buoyancy-driven convection due to concentration gradients. This leads to enhanced mass transfer, which causes the species concentration to increase in the vicinity of the porous disk. The concentration profile becomes steeper as buoyant forces promote the upward transport of species. While Thermal Grashof number affects concentration profiles indirectly through its impact on temperature. Since temperature and concentration often have related effects on fluid motion, a higher Grt can lead to increased species concentration near the rotating disk, especially in regions where both buoyancy and thermal effects are significant. Magnetic parameter reduces the flow's ability to mix the species, thereby limiting the diffusion of concentration across the fluid. A higher M value leads to a reduction in species transport, resulting in a more stagnant concentration profile in the flow, especially near the boundary. The Schmidt number is directly related to mass transfer. A higher Sc reduces the mass diffusivity, which makes the species concentration gradient steeper near the disk. This leads to a less uniform distribution of concentration, with a more pronounced concentration boundary layer near the surface.

4.4 Pressure Distribution

Pressure distribution in the system is influenced by the interplay of the velocity field, magnetic forces, buoyancy effects, and radiative heat transfer.

The Mass Grashof Number (Grc) enhances upward fluid motion through increased buoyancy, reducing the pressure drop near the disk as the pressure gradient becomes less steep. Similarly, the Thermal Grashof Number (Grt) reduces the axial pressure gradient due to stronger thermal buoyancy forces, leading to a weaker pressure distribution near the disk as resistance to flow decreases. In contrast, the Magnetic Parameter (M) increases resistance to flow, intensifying the pressure gradient as the Lorentz force opposes fluid motion, significantly raising the pressure drop near the disk. The Prandtl Number (Pr) reduces heat diffusion, increasing temperature gradients and potentially raising pressure near the disk due to greater resistance to thermal changes. The Radiation Parameter (Rd) diminishes thermal gradients, reducing the pressure gradient as convection becomes less dominant, resulting in a less pronounced pressure distribution. Lastly, the Schmidt Number (Sc), though primarily affecting mass transfer, indirectly influences pressure distribution by slowing mass transfer at higher values, slightly modifying the flow resistance and pressure gradient.

V SUMMARY AND CONCLUSION

5.1. Summary

The numerical solution of Magnetohydrodynamic (MHD) flow in a porous rotating disk under mixed convection, buoyancy, and thermal radiation effects involves solving a complex system of coupled equations governing momentum, energy, and species concentration. Key assumptions such as steady-state flow, axisymmetry, Boussinesq approximation, and small magnetic fields simplify the problem. Using numerical methods like the Finite Difference or Finite Element Method, the equations are discretized and solved iteratively, incorporating boundary conditions to obtain results visualized through 3D plots. Analysis of velocity profiles reveals that the Mass Grashof Number (Grc) and Thermal Grashof Number (Grt) enhance buoyancy-driven flow, increasing velocity while the Magnetic Parameter (M) reduces it by opposing fluid motion through Lorentz force. The Radiation Parameter (Rd) facilitates radiative heat transfer, altering velocity and thermal boundary layers. The Prandtl Number (Pr) and Schmidt Number (Sc) influence thermal and mass diffusivity, respectively, impacting the velocity and concentration profiles. Temperature distribution is affected by buoyancy forces, with higher Grc and Grt intensifying temperature gradients near the disk. The Magnetic Parameter increases thermal resistance, while higher Rd reduces thermal boundary layer thickness, enhancing heat transfer uniformity. Concentration profiles show that buoyancy-driven convection and higher Grt increase species concentration near the disk, while M and Sc reduce diffusivity, steepening concentration gradients. Pressure distribution is modulated by buoyancy, magnetic forces, and radiative effects, with higher Grc and Grt reducing pressure gradients, while M increases resistance and pressure drop. These findings elucidate the intricate interplay between parameters governing MHD flow dynamics.

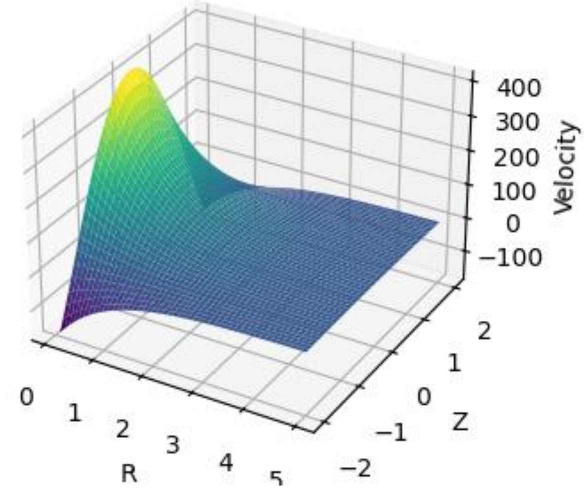

Figure 5: Velocity distribution for $Re = 500$

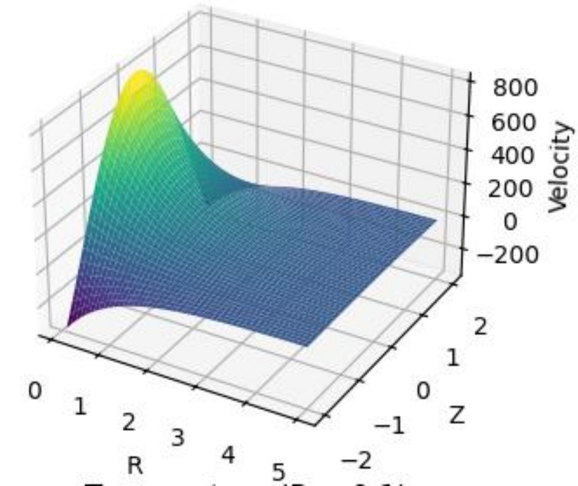

Figure 6: Velocity distribution for $Re = 1000$

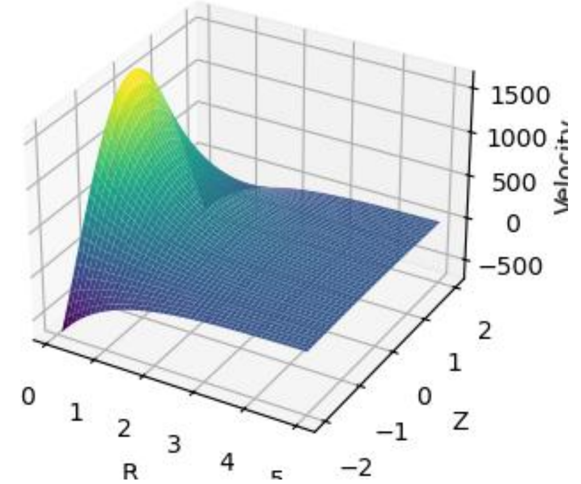

Figure 7: Velocity distribution for $Re = 2000$

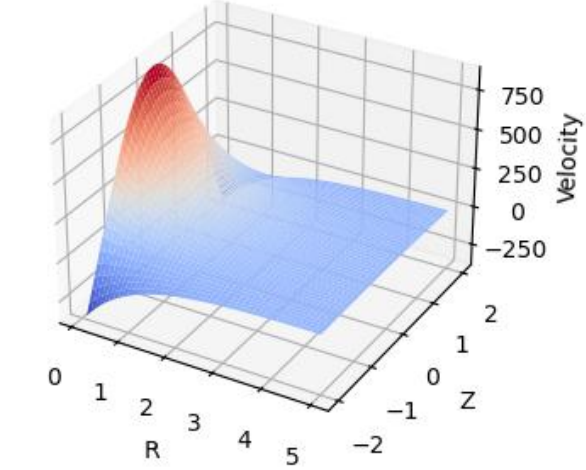

Figure 8: Velocity distribution for $M = 0.1$

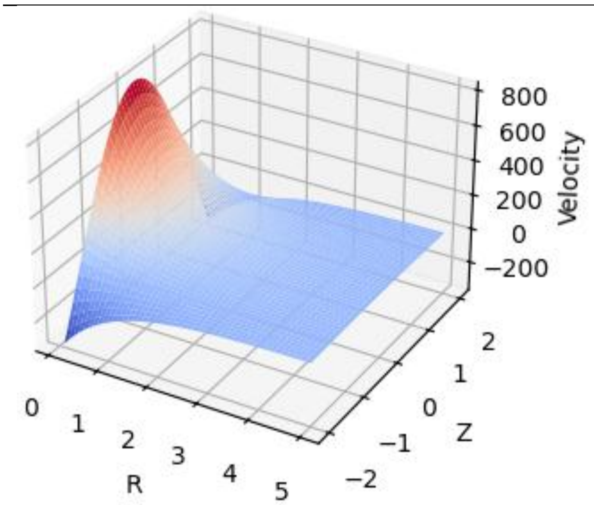

Figure 9: Velocity distribution for $M = 2.0$

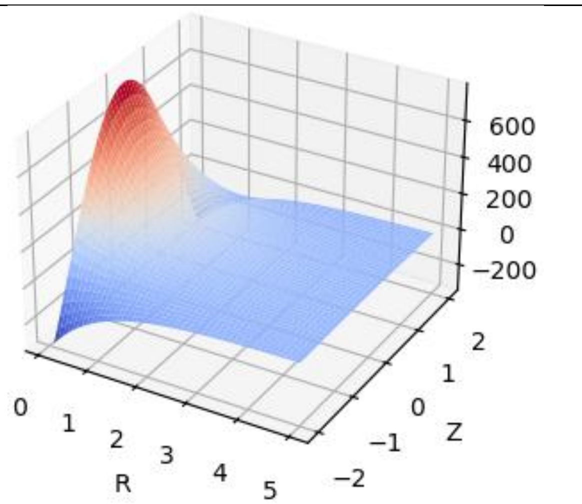

Figure 10: Velocity distribution for $M = 3.0$

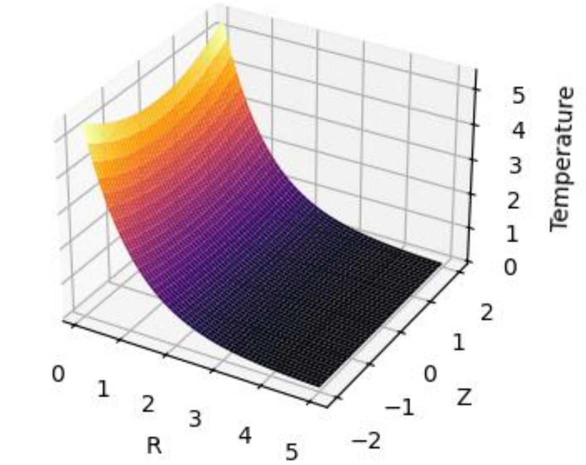

Figure 11: Temperature distribution for $Rd = 0.05$

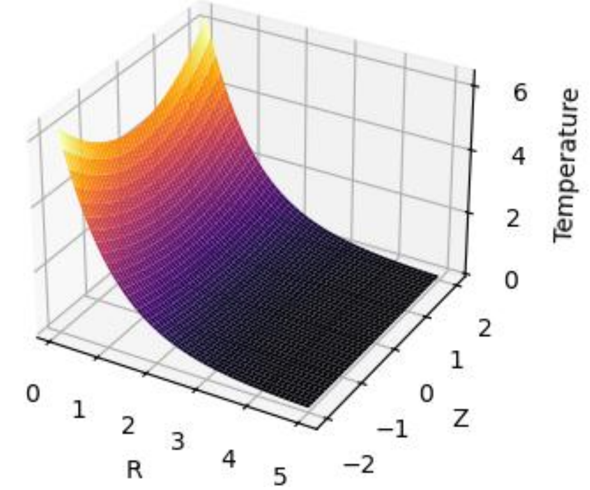

Figure 12: Temperature distribution for $Rd = 0.1$

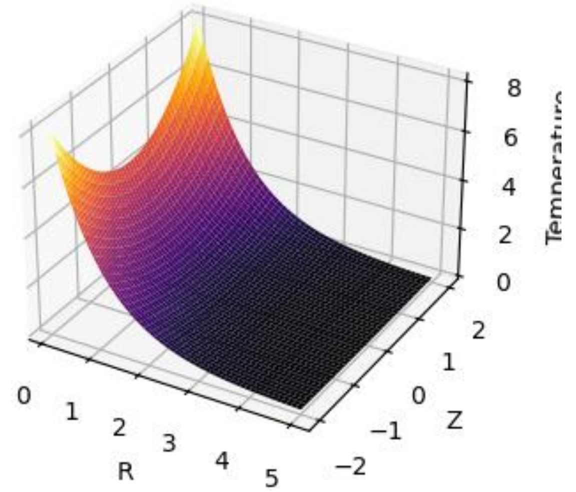

Figure 13: Temperature distribution for $Rd = 0.2$

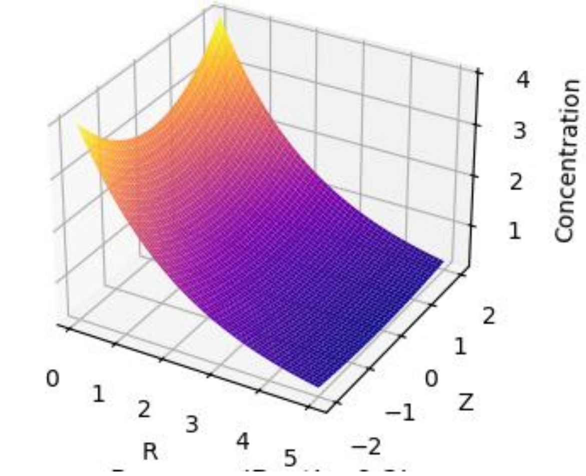

Figure 14: Concentration distribution for $M = 0.1$

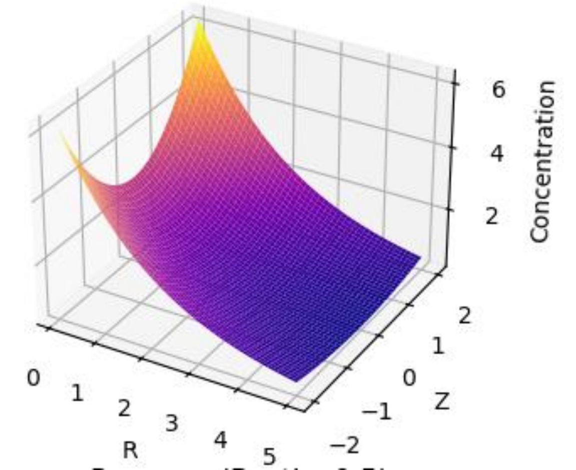

Figure 15: Concentration distribution for $M = 0.3$

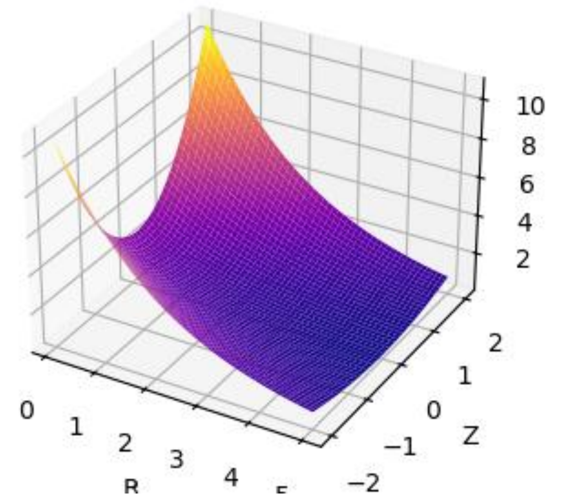

Figure 16: Concentration distribution for $M = 0.7$

5.2 CONCLUSION

The study of Magnetohydrodynamic (MHD) flow in a porous rotating disk under mixed convection, buoyancy, and thermal radiation effects highlights the complex interactions among velocity, temperature, and species concentration fields. Numerical simulations reveal that buoyancy forces, influenced by the Mass and Thermal Grashof Numbers (Grc, Grt), significantly enhance fluid motion and heat transfer near the disk. Conversely, the Magnetic Parameter (M) acts as a resistive force, reducing velocity and species transport by introducing Lorentz force effects. The Radiation Parameter (Rd) facilitates radiative heat transfer, reducing thermal boundary layer thickness and improving energy distribution. Parameters such as the Prandtl Number (Pr) and Schmidt Number (Sc) further modulate the flow by influencing thermal and mass diffusivity, respectively, leading to variations in pressure, temperature, and concentration gradients. These findings provide valuable insights into the optimization of MHD flows in engineering systems, such as cooling technologies, energy transport, and industrial processes, where magnetic fields and buoyancy effects play a critical role. The study highlights the importance of considering these parameters for applications involving heat and mass transfer in MHD systems. Future work may extend to unsteady flows and non-Newtonian fluids for broader applicability. From the discussion of this work, the following observations made:

1. Mass Grashof Number (Grc) enhances buoyancy-driven flow, increasing velocity and mass transfer near the disk.
2. Thermal Grashof Number (Grt) intensifies buoyant forces, enhancing velocity and temperature gradients.
3. Magnetic Parameter (M) increases resistance to flow, reducing velocity and concentration diffusion.
4. Radiation Parameter (Rd) improves radiative heat transfer, reducing thermal boundary layer thickness.
5. Prandtl Number (Pr) reduces thermal diffusivity, increasing temperature gradients and pressure resistance.
6. Schmidt Number (Sc) decreases mass diffusivity, steepening concentration gradients near the disk.

The interactions between buoyancy, magnetic fields, thermal radiation, and mass transfer lead to distinct effects on flow behavior, which we have carefully analyzed for a better and comprehensive understanding of fluid dynamics in such systems.

Conflict of interests: The authors declare that there is no conflict of interest regarding the publication of this article. The research was conducted independently, without any financial or personal relationships that could influence the findings or interpretations presented.

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