

# Combined effect of quantum pressure and rotation on stability of stratified viscoelastic Rivlin Ericksen fluid saturating a porous medium

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## **Abstract**

The stabilizing effect appearing in the presence of both quantum mechanism and kinematic viscoelasticity parameter may be physically interpreted such that a part of the kinetic energy of the waves has been absorbed, which leads to damping in the frequency of the waves. The quantum pressure dissipates the energy of any disturbance more than that carried out by the kinematic viscoelasticity parameter. In other words, the role of the kinematic viscoelasticity parameter helps the quantum effect to find more stability on the Rayleigh-Taylor instability problem, while the quantum pressure plays the fundamental role to generate the complete stability. It is found that, the critical point for the stability that occurs in the presence of quantum term remains unchanged by the addition of the other parameters of the problem. Both  $\omega$  and  $k$  point for the instability are unchanged by the addition of kinematic viscoelasticity parameter. All growth rates are reduced in the presence of porosity of the medium, the medium permeability, kinematic viscosity and kinematic viscoelasticity.

Keywords:

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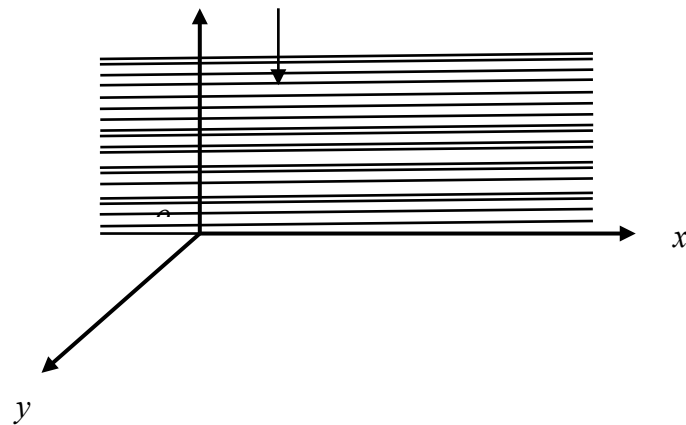
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## **I. INTRODUCTION**

The quantum hydrodynamic model was introduced by Gardner [1] for semiconductor physics to describe the transport of charge, momentum and energy in plasmas. Several studies were analysed both analytically and numerically in plasma with quantum corrections. For example, Haas et al. [2] studied a quantum multi-stream model for one and two stream plasma instabilities. It is well known that quantum effects become important in the behaviour of the charged plasma particles when the de-Broglie wavelength of the charged carriers become equal to or greater than the dimension of the quantum plasma system. It should be observed that there is a difference between a light-wave and the de-Broglie or Schrodinger wave associated with the light-quanta. Firstly, the light-wave is always real, while the de-Broglie wave associated with a light-quantum moving in a definite direction must be taken to involve an imaginary exponential. Quantum plasma can be composed of electrons, ions, positrons, holes and or grains and it plays an important role in ultra small electronic devices which has been given by Dutta and McLennan [3], dense astrophysical plasmas system has been given by Madappa et al. [4], intense laser-matter experiments has been investigated by Remington et al. [5] and non-linear quantum optics has been given by Brambilla et al. [6]. While naturally occurring plasma is relatively unusual on earth, it is playing a larger and increasingly important role in how we use and develop modern technology, for instance, producing compact chips developing of alternative energy sources, nuclear fusion and increasingly becoming part of the industrial area. The study of viscoelastic fluids has become important in recent years because of their many applications in petroleum drilling, manufacturing of foods and paper, etc. With the growing importance of the non-Newtonian fluids in modern technology, industries and astrophysics, the investigations of such fluids are desirable. Hoshoudy [7] has studied the effects of incompressible quantum plasma on Rayleigh-Taylor instability of Oldroyd model through a porous medium, where he has shown that both maximum  $\omega$  and critical point for the instability are unchanged by the addition of the strain retardation and the stress relaxation. Owing to the importance of quantum plasma and viscoelasticity, the present paper deals with effect of the quantum pressure and rotation on Rayleigh-Taylor instability for a finite thickness layer of incompressible viscoelastic fluid/plasma in a porous medium.

## **Mathematical formulation of the problem and perturbation equations**

The initial stationary state whose stability we wish to examine is that of an



**Figure 1:** Geometrical configuration of incompressible heterogeneous (heavy) and viscoelastic fluid saturating a porous medium.

incompressible, heterogeneous infinitely extending viscoelastic Rivlin Ericksen fluid of thickness bounded by the planes and ; of variable density, kinematic viscosity, kinematic viscoelasticity and quantum pressure, arranged in horizontal strata of electrons and immobile ions in a homogeneous, saturated, isotropic porous medium with the Oberbeck-Boussinesq approximation for density variation, so that the free surfaces are almost horizontal. The fluid is acted on by gravity force as shown in figure 1.

The relevant equations of motion, continuity (conservation of mass) and incompressibility are

$$\begin{aligned} & \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \nabla \cdot \mathbf{T} + \mathbf{g} \quad (1) \\ & \nabla \cdot \mathbf{v} = 0 \quad (2) \\ & \nabla \cdot \mathbf{v} = 0 \quad (3) \end{aligned}$$

where  $\mathbf{v}$  and  $\rho$  represent the filter velocity, density, pressure, viscosity, viscoelasticity, medium permeability and medium porosity, respectively.

$\mathbf{v}$  is represented by Bohm vector potential term (also, is called a quantum pressure).

It is a fact that, for many situations of interest in ICF (inertial-confinement fusion), unstable flow occurs at velocities much smaller than the local sound speed. This has the effect that accelerations in the flow are not strong enough to change the density of a fluid element significantly, so the fluid moves without compressing or expanding as is seen from equation (3).

The initial steady state whose stability we want to examine is characterized by  $\mathbf{v} = 0$  and  $\rho = \rho_0$  (4)

To investigate the stability of the hydrodynamic motion infinitesimal perturbations in the physical quantities describing the system (4) are superimposed on the steady state and let  $\mathbf{v}'$  denote respectively, the infinitesimally small perturbations in fluid velocity, density, pressure and quantum pressure .

Substituting these perturbations and using the linear theory, the equations (1) - (3) in the linearized form become

$$\begin{aligned} & \rho_0 (\mathbf{v}' \cdot \nabla) \mathbf{v}' = -\nabla p' + \nabla \cdot \mathbf{T}' + \mathbf{g}' \quad (5) \\ & \nabla \cdot \mathbf{v}' = 0 \quad (6) \\ & \nabla \cdot \mathbf{v}' = 0 \quad (7) \end{aligned}$$

where

The equations (5)-(7) in the Cartesian form yield

$$\begin{aligned} & \rho_0 (\mathbf{v}' \cdot \nabla) \mathbf{v}' = -\nabla p' + \nabla \cdot \mathbf{T}' + \mathbf{g}' \quad (8) \\ & \nabla \cdot \mathbf{v}' = 0 \quad (9) \\ & \nabla \cdot \mathbf{v}' = 0 \quad (10) \\ & \nabla \cdot \mathbf{v}' = 0 \quad (11) \\ & \nabla \cdot \mathbf{v}' = 0 \quad (12) \end{aligned}$$

where

and.

Since the boundaries are assumed to be rigid. Therefore, the boundary conditions appropriate to the problem are at  $z=0$  and  $z=h$ , on a rigid surface. (13)

Analyzing the perturbations into normal modes, it is assumed that all the physical quantities describing the perturbations are ascribed a dependence on  $x, y, z, t$  of the form

$$f(x, y, z, t) = f_0(x, y, z) e^{i(k_x x + k_y y + k_z z - \omega t)} \quad (14)$$

where  $k_x$  and  $k_y$  are wavenumbers along  $x$  and  $y$ -directions,  $k$  is the resultant wavenumber and  $\omega$  is the growth rate which is, in general a complex constant.

Now, using the expression (14), equations (8)-(12) become

$$\dots \quad (15)$$

$$\dots \quad (16)$$

$$\dots \quad (17)$$

$$\dots \quad (18)$$

$$\dots \text{ where } \dots \quad (19)$$

Multiplying equation (15) by  $\dots$  and equation (16) by  $\dots$ , adding and using equation (18) and (19), we obtain

$$\dots \quad (20)$$

Multiplying equation (15) by  $\dots$  and equation (16) by  $\dots$ , adding and using equation (4.2.18), we obtain

$$\dots \quad (21)$$

where  $\dots$  is the  $z$ -component of vorticity. Since  $\dots$ , therefore, equation (21) yields that  $\dots$ .

Eliminating variables  $\dots$  and  $\dots$  from the system of equations (17) and (20), a characteristic equation is obtained as

$$\dots \quad (22)$$

Now the case of incompressible continuously stratified viscoelastic plasma layer is considered in a porous medium and the density, viscosity, viscoelasticity and quantum pressure are assumed to vary exponentially with the vertical axis as follows:

$$\dots \quad (23)$$

where  $\dots$  and  $\dots$  are constants and so the coefficient of kinematic viscosity, and coefficient of kinematic viscoelasticity are constant everywhere.

Using the stratifications of the form given by (23), the characteristic equation (22) transforms to

$$\dots \quad (24)$$

where  $\dots$  represents the parameter accounting for quantum pressure.

Using the boundary conditions (13), the equation (24) implies that

$$\dots \quad (25)$$

The exact solution of the eigen-value problem (24) satisfying the boundary conditions (13) and (25), is chosen to be

$$\dots \quad (26)$$

Substituting the solution given by (26) in equation (24), we obtain

$$\dots \quad (27)$$

Equating the coefficients of  $\dots$  and  $\dots$  in equation (4.2.27) yield that

$$\dots \quad (28)$$

As  $\dots$ , therefore, equation (4.2.29) implies that

Substituting this value of  $\dots$  in equation (4.2.28), the dispersion relation so obtained is

$$\dots \quad (29)$$

Now introducing the non-dimensional quantities

where  $\dots$  is the plasma frequency, the equation (4.2.30) after dropping the astricts for our convenience yields

$$\dots \quad (30)$$

where  $\dots$  and  $\dots$  in the case of  $\dots$  (stable oscillations), then the equation (4.2.31) becomes

$$\dots \quad (31)$$

In the absence of  $\dots$ , equation (31) reduces to

$$\dots \quad (32)$$

which is in good agreement with the earlier results by Sunil et al. (2004).

Now some special cases are considered from equation (31) to clarify the different role of the parameters on the physical problem

**Case a):** When, the expressions (32) imply that and

So the classical normalized growth rate of equation (33) in the absence of quantum physics denoted by is given as

$$(35)$$

In the absence of viscoelastic parameter, the equation (.33) reduces to that of the result by Hoshoudy (2009).

**Case b):**When , we find that whileremains the same as in equation (32). So the quantum normalized growth rate from equation (33) is

$$(36)$$

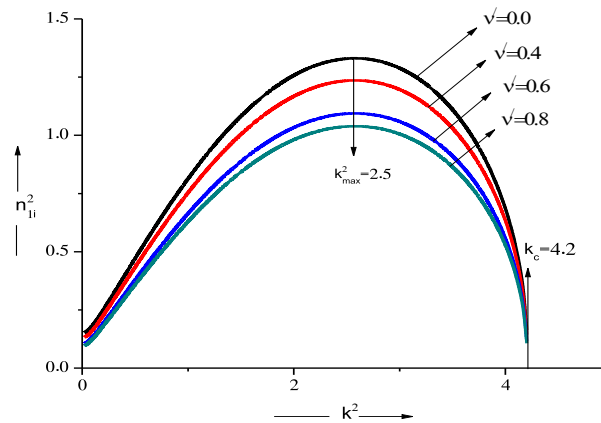
which is in good agreement with the earlier result of Hoshoudy (8).

It is clear from the comparison of expressions (35) and (36) that the quantum pressure has stabilizing effect on Rayleigh-Taylor instability problem.

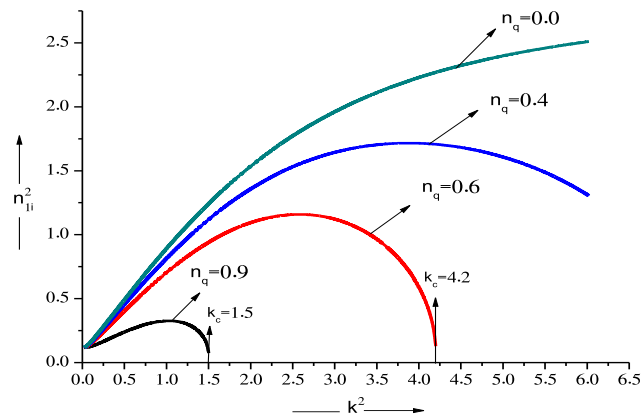
## II. Numerical Results and Discussion

To investigate the effects of variousparameters on the stability of the system under consideration, equation (33) has been solved numerically using the software Fortran 95. In these figures the fixed permissible values of parameters are and, respectively.

Figure 2 shows the variation of the square of normalized growth rate versus the square of normalized wavenumber satisfying the equation (33). The graph shows that the presence of kinematic viscoelasticity does not affect both the maximum point for the instability and corresponding critical pointfor the stability, where and at .However, the kinematic viscoelasticity has a stabilizing role on the considered system,as the growth rates decrease with the increase in kinematic viscoelasticity.



**Figure2:**Variations of the square of normalized growth rateversus the square normalized wavenumberfor four different values of kinematic viscoelasticity.



**Figure 3:** Variations of the square of normalized growth rate versus the square normalized wavenumber for four different values of quantum plasma.

In figure 3, the graph shows that  $n_{ii}^2$  takes different values at  $k_c^2$ , while the values of  $k_c^2$  at  $n_{ii}^2 = 0$ , respectively, approaching to complete stability. Also, it is observed that, no mode of maximum instability exists when  $n_q = 0$  as the square normalized growth rate usually increases by increase with the square normalized wavenumber values, while in the presence of quantum term, there is a mode of maximum instability, where the square normalized growth rate increases with increases through the range  $k_c^2$  (the square normalized growth rate arrives to maximum instability) and when  $k_c^2$ , the square normalized growth rate starts to decrease as  $k_c^2$  increases. This means that the quantum pressure parameter has a crucial capability to suppress the instability.

### III. Conclusions

The effect of quantum pressure on the Rayleigh-Taylor instability of stratified viscoelastic Rivlin Ericksen fluid / plasma saturating a porous medium has been studied. The effect of elasticity is revealed through the quantum pressure. It is found that, the critical point for the stability that occurs in the presence of quantum term remains unchanged by the addition of the other parameters of the problem. Both  $n_q$  and  $k_c^2$  point for the instability are unchanged by the addition of kinematic viscoelasticity parameter. All growth rates are reduced in the presence of porosity of the medium, the medium permeability, kinematic viscosity and kinematic viscoelasticity. These results indicate that quantum pressure plays a major role in approaching a complete stability.

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