International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 12 Issue 8 ǁ August 2024 ǁ PP. 186-191

Jordan Derivations and Jordan Triple Derivations on Banach Γ-algebras

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Abstract
Let M be a Banach Γ-algebra with a right identity e with conditions that eαM is commutative and semi-simple. We prove that the Jordan derivations and the product of any two Jordan derivations on M are derivations on M. We also prove that the Jordan triple left(right) derivations on a Banach Γ-algebra M having a right identity e are Jordan left(right) derivations on M. Furthermore, we prove that every Jordan right derivation on a Banach Γ-algebra M with a right identity e is a zero derivation when it acts on the annihilator of M.
***Mathematics Subject Classification**: 16Y30, 16W25, 16U80.*
***Keywords**: Banach Γ-algebras, right identity, Jordan derivations, derivations, Jordan triple left(right) derivations, Jordan left(right) derivations.*

Date of Submission: 14-08-2024 Date of acceptance: 30-08-2024

I. Introduction

Definition 1.1: Let M and Γ be two linear spaces over a field F. M is said it be a Banach Γ-algebra over F if M is a Banach space and the following conditions are satisfied:
(a) $m\alpha n \epsilon M$,
(b) $(m\alpha n)\beta p = m\alpha(n\beta p)$,
(c) $c(m\alpha n) = (cm)\alpha n = m(c\alpha)n = m\alpha(cn)$,
(d) $m\alpha(n + p) = m\alpha n + m\alpha p$,
(e) $m(\alpha+\beta)n = m\alpha n + m\beta n$,
(f) $(m+n)\alpha p = m\alpha p + n\alpha p$,
(g) $\|m\alpha n\| \leq \|m\|.\|\alpha\|.\|n\|$,
for all m, n, $p\epsilon M$, α, $\beta \epsilon \Gamma$, and $c\epsilon F$.
Example 1.2: Any Banach algebra can be regarded as a Banach Γ-algebra by suitably taking Γ.
Definition 1.3: Let M be Banach Γ-algebra, and d: M ⟶ M be linear mapping.
(a) d is said to be derivation if $d(m\alpha n) = d(m)\alpha n + m\alpha d(n)$, ∀ m, $n\epsilon M$, and $\alpha\epsilon\Gamma$.
(b) d is said to be Jordan derivation if $d(m\alpha m) = d(m)\alpha m + m\alpha d(m)$, ∀ $m\epsilon M$, and $\alpha\epsilon\Gamma$.
(c) d is said to be Jordan left derivation if $d(m\alpha m) = 2m\alpha d(m)$, ∀ $m\epsilon M$, and $\alpha\epsilon\Gamma$.
(d) d is said to be Jordan triple left derivation if $d((m\alpha)^3\alpha m) = 3(m\alpha m)\alpha d(m)$, ∀ $m\epsilon M$ and $\alpha\epsilon\Gamma$.
Jordan right derivations and Jordan triple right derivations can be defined similarly.
Definition 1.4: We denote the right annihilator of a Banach Γ-algebra M by ran(M) and is defined by ran(M) = $\{x\epsilon M: M\alpha x = \{0\}$ for all $\alpha\epsilon\ \Gamma\}$.
Definition 1.5: We denote the radical of a Banach Γ-algebra M by rad(M) and is defined by the intersection of maximal left ideals of M.
Definition 1.6: Let M be a Banach Γ-algebra. The linear mapping T: M ⟶ M is said to be spectrally bounded if there exists a non-negative number t such that $r(T(m)) \leq t\alpha r(m)$, ∀ $m\epsilon M$, and $\alpha\epsilon\Gamma$, where r(.) denotes the spectral radius.
Definition 1.7: Let M be a Banach Γ-algebra and Z(M) be the center of M. Then for an integer k, a linear mapping T: M ⟶ M is said to be k−centralizing if $T(m)\alpha((m\alpha)^k\alpha m) - ((m\alpha)^k\alpha m)\alpha T(m)\epsilon Z(M)$, ∀ $m\epsilon M$, and $\alpha\epsilon\Gamma$.

Y. Ceven [15] investigated the Jordan left derivations on completely prime Γ-rings. He showed that if a Jordan left derivation on a completely prime Γ-ring is non-zero with an assumption, then the Γ-ring is commutative. He also proved that every Jordan left derivation together with an assumption on a completely prime Γ-ring is a left derivation on it. In this paper, he gave an example of Jordan left derivations on Γ-rings.
Mustafa Asci and Sahin Ceran [9] studied on a nonzero left derivation d on a prime Γ-ring M with an ideal U and the center Z of M such that $d(U) \subseteq U$ and $d^2(U) \subseteq Z$ for which M is commutative. They also investigated that M is commutative with the nonzero left derivation d_1 and right derivation d_2 on M such that $d_1(U) \subseteq U$ and $d_1 d_2(U) \subseteq Z$.
A.C. Paul and Amitabh Kumer Halder [1] studied on the existence of a non-zero Jordan left derivation from a Γ-ring M into a 2-torsionfree and 3-torsionfree left ΓM-module X that makes M commutative. They also showed that if $X = M$ is a semiprime Γ-ring then the derivation is a mapping from M into its centre and if M is a prime Γ-ring then every Jordan left derivation d on M is a left derivation on M.

Nilakshi Goswami [12] worked on the characterizations of Jacobson radicals of Γ-Banach Algebras in different perspectives.
Nadia M. J. Ibrahem [13] studied on the full stable Banach gamma-algebra modules with the introduction of fully stable Banach gamma-algebra modules relative to ideal and some properties and characterizations of the classes of full stability.
M. J. Mehdipour, GH. R. Moghimi and N. Salkhordeh [8] studied on the types of Jordan derivations of a Banach algebra A with a right identity e. They proved that if eA is commutative and semi-simple, then every Jordan derivation of A is a derivation. They investigated that every Jordan triple left (right) derivation of A is a Jordan left (right) derivation. Furthermore, they investigated the range of Jordan left derivations and proved that every Jordan left derivation of A maps A into eA.
In this study, we generalize the results of M. J. Mehdipour, GH. R. Moghimi and N. Salkhordeh [8] in Γ version. We investigate that the Jordan derivations and the product of any two Jordan derivations on a Banach Γ-algebra M having a right identity e with conditions that e**α**M is commutative and semi-simple are derivations on M. We also show that every Jordan triple left(right) derivation on a Banach Γ-algebra M with a right identity e is a Jordan left(right) derivations on M. Finally, we prove that every Jordan right derivation on a Banach Γ-algebra M with a right identity e is a zero on ran(M).

II. Jordan Derivations on Banach Γ-algebras

Lemma 2.1: Let M be a Banach Γ-algebra with a right identity e such that d: M ⟶ M be a Jordan derivation. Then,
(a) ran(M) is invariant under d.
(b) If d: M ⟶ ran(M) is a mapping, then d is a derivation.

Proof: (a) Since d: M ⟶ M is a Jordan derivation, we get

$$d(m\alpha n + n\alpha m) = d(m)\alpha n + m\alpha d(n) + d(n)\alpha m + n\alpha d(m),\ \forall\ m, n\epsilon M, \text{ and } \alpha\epsilon\Gamma \quad (1)$$

Writing m = n = e in eq. (1), we get

$2d(e) = d(e\alpha e + e\alpha e)$
$= d(e)\alpha e + e\alpha d(e) + d(e)\alpha e + e\alpha d(e)$
$= 2d(e) + 2e\alpha d(e)$.

This yields $e\alpha d(e) = 0$, and so $d(e)\epsilon ran(M)$.
Replacing m by e in (1) to get

$d(e\alpha n) + d(n) = d(e\alpha n + n\alpha e)$
$= d(e)\alpha n + e\alpha d(n) + d(n)\alpha e + n\alpha d(e)$
$= d(e)\alpha n + e\alpha d(n) + d(n)$, $\forall\ n\epsilon M$, and $\alpha\epsilon\Gamma$.

Thus, $d(e\alpha n) = d(e)\alpha n + e\alpha d(n)$, (2)
$\forall\ n\epsilon M$, and $\alpha\epsilon\Gamma$.
Using (2), we have

$m\alpha d(p) = m\alpha e\alpha d(p)$
$= m\alpha\ (d(e)\alpha p + e\alpha d(p))$
$= m\alpha d(e\alpha p)$
$= 0$, $\forall\ m\epsilon M$, $p\epsilon ran(M)$, and $\alpha\epsilon\Gamma$, and so $d(p)\ \epsilon\ ran(M)$.

(b) Suppose d: M ⟶ ran(M) is a mapping. We apply equations (1) and (2) to get
$d(e\alpha m) = d(e)\ \alpha m$ and $d(p\alpha m) = d(p)\alpha m$, $\forall\ m\epsilon M$, $p\epsilon ran(M)$, and $\alpha\epsilon\Gamma$.
For every $m\epsilon M$, there exists $p\epsilon ran(M)$ such that $m = e\alpha m + p$, $\forall\ \alpha\epsilon\Gamma$, and so we have

$d(m\alpha n) = d(e\alpha m\alpha n + p\alpha n)$
$= d(e\alpha m\alpha n) + d(p\alpha n)$
$= d(e\alpha m)\ \alpha n + d(p)\ \alpha n$

= d(e**α**m + p)**α**n
= d(m)**α**n, ∀ nϵM, and **α**ϵΓ.
Now, m**α**d(n) = 0 yields that d(a**α**x) = d(m)**α**n + m**α**d(n) showing d is a derivation on M.

Theorem 2.2: If M is a Banach Γ-algebra with a right identity e such that e**α**M is commutative and semi-simple and d: M ⟶ M is a Jordan derivation, then d is a derivation. Moreover, d is spectrally infinitesimal and d(M)⊆ran(M).

Proof: Suppose d: M ⟶ M is a Jordan derivation. Due to Lemma 2.1 (b), d: M ⟶ M is a mapping. We define the Jordan derivation D: M/ran(M) ⟶ M/ran(M) by
D(m + ran(M)) = d(m) + ran(M), which is well-defined. It is notable that M/ran(M) and e**α**M are Banach Γ-algebras, and so they are isomorphic. Thus, using the Γ-version in Corollary 1.5.3 (ii) of [6] it can be inferred that M/ran(M) is commutative semi-simple Banach Γ -algebra since e**α**M is commutative, semi-simple, and isomorphic to M/ran(M). Then D is a derivation, and D is zero on M/ran(M) [by observing the Γ-versions in necessary parts in [7], [10], [11]]. This gives that d(m)ϵran(M), and so by Lemma 2.1(b), d is a derivation. We note that d(M) is nilpotent, and so d is spectrally infinitesimal.

Corollary 2.3: Let M be Banach Γ-algebra M with a right identity e such that e**α**M be commutative and semi-simple. If d, d_1, d_2 are Jordan derivations on M, then the following conditions are satisfied:
(a) The range of a Jordan derivation d of M is contained in rad(M).
(b) $d_1 d_2$ is a derivation.
(c) For any positive integer k, the zero map is the only k−centralizing Jordan derivation of M.

Proof: (a) The proof directly follows from Theorem 2.2.
(b) Since d_1 and d_2 are Jordan derivations on M, by Theorem 2.2, d_1 and d_2 are derivations on M, and d_1(M)⊆ran(M) and d_2(M)⊆ran(M). Then for any ∀ m, nϵM, and **α**ϵΓ, we have d_1(m)**α**d_2(n) + d_1(n)**α**d_2(m) = 0. This shows that $d_1 d_2$ is a derivation on M.
(c) Suppose d: M⟶M is a k-centralizing for some positive integer k. Then by Theorem 2.2, we have d(m)**α**$((m\alpha)^k \alpha m)$ = d(m)**α**$((m\alpha)^k \alpha m)$ - $((m\alpha)^k \alpha m)$**α**d(m)ϵran(M)∩Z(M). Then d(m)**α**$((m\alpha)^k \alpha m)$ = {0}, and so d(e) = 0. For any mϵM and **α**ϵΓ, let p = m - e**α**m. Then
p + e = $\left((p+e)\alpha\right)^k \alpha(p+e)$, and so d(p) = 0. This gives d(m) = d(p + e**α**m) = d(e**α**m) = d(e)**α**m = 0. Therefore, d =0.

Theorem 2.4: If M is Banach Γ-algebra together with M/ran(M) is commutative and semi-simple, then the following statements are satisfied:
(a) If d: M ⟶ M is a derivation, then d: M ⟶ ran(M)⊆rad(M).
(b) Every derivation d: M ⟶ M is spectrally infinitesimal.
(c) If d_1 and d_2 are derivations on M, then $d_1 d_2$ is a derivation of M.

Proof: (a) Since d is a derivation on M, d is a Jordan derivation on M, and so d(M) is invariant under d by Lemma 2.1 (a).
(b**)** Applying derivations instead of Jordan derivations to the proof of Theorem 2.2, we get the required result.
(c) $d_1 d_2$: M ⟶ M is a derivation due to (a).

III. Jordan triple derivations on Banach Γ-algebras

Theorem 3.1: If M is a Banach Γ-algebra with a right identity e such that d: M ⟶ M is a Jordan left derivation, then d: M ⟶ e**α**M is a mapping.

Proof: Since d: M ⟶ M is a Jordan left derivation,
d(e) = 2e**α**d(e), (3)
and so m**α**d(e) = 2m**α**d(e), ∀ mϵM, and **α**ϵΓ.
This gives d(e)ϵran(M), and by eq. (3), d(e) = 0.
We fix p ϵ ran(M) to get
d(p) = d(p) + d(e)
= d((p + e)**α**(p + e))
= 2(p + e)**α**d(p + e)
= 2p**α**d(p) + 2e**α**d(p), for any **α**ϵΓ,
Thus, m**α**d(p) = 2m**α**d(p), ∀ mϵM, and **α**ϵΓ, and so d(p)ϵran(M), ∀ pϵran(M).
Since d: M ⟶ M is a Jordan left derivation,
d(m**α**m) = 2m**α**d(m), (4)
∀ mϵM, and **α**ϵΓ.
Replacing m by m + e in eq. (4), we get Let us replace a by a + e in (4). Then
d(m + e**α**m) = 2m**α**d(e) + 2e**α**d(m) = 2e**α**d(m).
This implies that
d(m) = 2e**α**d(m) − d(e**α**m), (5)

∀ m∈M, and α∈Γ.
Applying the fact that m – eαm∈ran(M), we assume that
eαd(m − eαm) = 0, (6)
∀ m∈M, and α∈Γ.
By eq. (5) and eq. (6), we get
d(m − eαm) = 2eαd(m − eαm) − d(eα (m − eαm)) = 0, ∀ m∈M, and α∈Γ.
This yields d(m) = d(eαm), ∀ m∈M, and α∈Γ.
Applying the above relation and eq. (5), we obtain d(m) = d(eαm), which together with (5) shows that d(m) = eαd(m), (7)
∀ m∈M, and α∈Γ. Therefore, d: M ⟶ eαM is a mapping.

Corollary 3.2: If M is a Banach Γ-algebra with a right identity e such that d: M ⟶ M is a Jordan left derivation and d: M ⟶ ran(M) is a mapping, then d is a zero mapping.

Proof: Since d: M ⟶ ran(M) is a mapping, d(M)⊆ran(M). By Theorem 3.1, we have d(M)⊆ran(A)∩eαM = {0}, for any α∈Γ. Therefore, d is a zero mapping.

Theorem 3.3: If M is a Banach Γ-algebra with a right identity e such that d: M ⟶ M is a Jordan triple left derivation, then d is a Jordan left derivation.

Proof: Since d: M ⟶ M is a Jordan triple left derivation, we have
d($(m\alpha)^3\alpha m$) = 3mαmαd(m), (8)
∀ m∈M, and α∈Γ.
Writing m + e for m in eq. (8), we get
2d(mαm) + d(m) + 2d(eαm) + d(eαm2) = 3eαd(m) + 3mαd(m) + 3eαmαd(m), (9)
∀ m∈M, and α∈Γ.
We have that d(e) = 0. Replacing m by -m in eq. (9), we get
2d(mαm) - d(m) - 2d(eαm) + d(eαm2) = 3eαd(m) + 3mαd(m) + 3eαmαd(m), (10)
∀ m∈M, and α∈Γ.
We apply eq. (9) and eq. (10) to
d(m) + 2d(eαm) = 3eαd(m), (11)
∀ m∈M, and α∈Γ.
Putting m – eαm for m in eq. (11), we get
d(m) = d(eαm), (12)
∀ m∈M, and α∈Γ.
Applying eq. (11) and eq. (12), we get
d(m) = eαd(m), (13)
∀ m∈M, and α∈Γ.
Replacing m by e in eq. (8) to get eαd(e) = d(e) = 0, for any α∈Γ.
Then,
d($(m\alpha)^3\alpha m$ + 2mαm + m + eαmαm + 2eαm + $(e\alpha)^3\alpha e$) = 3mαmαd(m) + 3mαmαd(e) + 3eαd(m) + 3eαeαd(e) + 3mαeαd(m) + 3mαd(e) + 3eαmαd(m) + 3eαmαd(e), ∀ m∈M, and α∈Γ.
Since d:M⟶M is a Jordan triple left derivation, d($(m\alpha)^3\alpha m$) = 3mαmαd(m), ∀ m∈M, and α∈Γ, and so we have
2d(mαm) + d(m) + d(eαmαm) + 2d(eαm) = 3eαd(m) + 3mαd(m) + 3eαmαd(m), ∀ m∈M, and α∈Γ.
Appling eq. (12) and eq. (13) in the above relation, we have
3d(mαm) + 3d(m) = 3d(m) + 3mαd(m) + 3eαmαd(m), ∀ m∈M, and α∈Γ, and so
d(mαm) = mαd(m) + eαmαd(m), (14)
∀ m∈M, and α∈Γ.
Using eq. (13) and eq. (14), we have
d(mαm) = eαd(mαm) = 2eαmαd(m), (15)
∀ m∈M, and α∈Γ.
This gives 2mαmαd(m) = mα (2eαmαd(m)) = mαd(mαm), and so we have
2mαmαd(m) = mαd(mαm), ∀ m∈M, and α∈Γ.
As a consequence, we have
2d($(m\alpha)^3\alpha m$) = 6mαmαd(m) = 3(2mαmαd(m)) = 3mαd(mαm).
Thus,
2d($\left((m+e)\alpha\right)^3\alpha(m+e)$) = 3(m + e) αd((m + e)α(m+e))
= 3mαd((m + e) $\alpha(m+e)$) + 3eαd((m + e) $\alpha(m+e)$)
= 3mαd((m + e) $\alpha(m+e)$) + 3d((m + e) $\alpha(m+e)$),
which yields
2d($(m\alpha)^3\alpha m$ + 2mαm + 2eαm + eαmαm + m) = 3mαd(mαm + 2m) + 3d(mαm + 2m)
This shows that

3d(mαm) = 6mαd(m), and so d(mαm) = 2mαd(m), ∀ mϵM, and $\boldsymbol{\alpha}\epsilon\Gamma$.
Therefore, d is a Jordan left derivation on M.

Theorem 3.4: If M is a Banach Γ-algebra with a right identity e such that d: M ⟶ M is a linear mapping and d is a Jordan triple right derivation, then d is a Jordan right derivation.

Proof: Since d: M ⟶ M is a Jordan triple right derivation, d(e) = 0 and so we have

d($((m+e)\alpha)^3\alpha(m+e)$) = 3d(m + e)$\alpha$((m + e)$\alpha(m+e)$), ∀ m$\epsilon$M, and $\boldsymbol{\alpha}\epsilon\Gamma$.

Thus, we have

2d(mαm) + 2d(eαm) + d(eαmαm) = 6d(m)αm + 2d(m), (16)

∀ mϵM, and $\boldsymbol{\alpha}\epsilon\Gamma$.

We write -m for m in eq. (16) to get

2d($m\alpha$m) − 2d(eαm) + d(eαmαm) = 6d(m)αm − 2d(m), (17)

∀ mϵM, and $\boldsymbol{\alpha}\epsilon\Gamma$.

Using eq. (16) and eq. (17), we have

d(m) = d(eαm) and 2d(mαm) + d(eαmαm) = 6d(m)αm.

From above, we have

3d(mαm) = 6d(m)αm, which yields d(mαm) = 2d(m)αm, ∀ mϵM, and $\boldsymbol{\alpha}\epsilon\Gamma$.

Therefore, d is a Jordan right derivation on M.

Theorem 3.5: If M is a Banach Γ-algebra with a right identity e such that d: M ⟶ M is a Jordan right derivation, then d: ran(M) ⟶ ran(M) is a zero derivation.

Proof: Since d: M⟶M is a right derivation and M is a Banach Γ-algebra with a right identity e, we get d(e) = 0. Again, if we fix pϵran(M), we get (p + e)**α**(p + e) = p + e, ∀ $\boldsymbol{\alpha}\epsilon\Gamma$

This gives d(p) = d((p + e)**α**(p + e)) = 2d(p + e) **α**(p + e) = 2d(p). Therefore, d(p) = 0, ∀ pϵran(M). Therefore, d: ran(M) ⟶ ran(M) is a zero derivation.

IV. Discussion

We studied derivations such as Jordan left derivations, Jordan triple left(right) derivations on Banach Γ-algebras M whereas Nilakshi Goswami [12] worked on the characterizations of Jacobson radicals of Γ-Banach Algebras in different perspectives. Also, Y. Ceven [15] showed that every Jordan left derivation together with an assumption on a completely prime Γ-ring is a left derivation on it whereas we proved that Jordan derivations and the product of any two Jordan derivations on a Banach Γ-algebra M with certain conditions are derivations as well as Jordan triple left(right) derivations on a Banach Γ-algebra M with a right identity e are Jordan left(right) derivations on M

V. Conclusion

The Jordan derivations and the product of any two Jordan derivations on a Banach Γ-algebra M are derivations based on the right identity e and the conditions that e**α**M is commutative and semi-simple. The Jordan triple left(right) derivations on a Banach Γ-algebra M having a right identity e are Jordan left(right) derivations on M. Finally, Jordan right derivation on a Banach Γ-algebra M with a right identity e is a zero derivation on the annihilator of M.

Acknowledgment

The author would like to thank the anonymous referees for their valuable suggestions and comments.

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