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An Inflated Probability Model for the Out-Migration: A case study in Karnataka

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Abstract
Migrants play a vital role in the social and economic progress of their family especially in developing nations. Accordingly, out-migration from rural zones is a significant medium for socio economic and demographic change in their households. In this paper an effort is made to consider the distribution of rural out-migration through the probability model based on specific presumptions. The parameters involved in this model have been estimated by the estimation techniques. The model applied to the set of observed real data. The proposed model is fitted well on observed data.
Keywords: - *Probability Model, Household, Risk of Migrants* **…,**

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I. Introduction

The migration behavior of individuals (families, households) and intends to explain the decision-making process by potential migrants to remain in a current residence or to migrate to another one. Micro models of migration usually are based on disaggregated data (i.e. the characteristics of an individual) delivered from censuses or sociological surveys. As input variables micro models use both the characteristics of the origin and destination regions and the characteristics of individuals involved in migration processes. Output variables can describe migration behavior of a representative individual (i.e. who follows the patterns of migration behavior common for the observed territory) or, as in the case of macro models, aggregate indicators of migration processes. A number of probability models have been developed to describe the distribution of households according to the total number of migrants (Sharma,1984,1987; Singh, 1985,1987,1988,1992;Yadava,1993; Yadava and Yadava,1998; Yadava et al 1989,1994) for explaining the distribution of total migrants. They have not taken in to account the fact that some households are not subjected to the risk of migration due to their establishment in village and other consideration.

Human grouping is a vital event in the study of mechanism of out-migration system at the micro-level. The most common unit of human grouping is household. The characteristics of household are bound to play an active role in the decision of an individual to move or not move from a household. In this connection Yadav and Singh (1983), Ojha and Pandey (1991) and Pandey (1994) proposed various models under certain limitations.

II. The Probability Model

A probability model to describe the nature of distribution of households according to total number of migrants has been derived on the basis of the following assumptions.

i. Let α be the probability of a migration from a household and $(1-\alpha)$ be the probability of no migration from a household.

ii. Two types of household are observed at the survey. First type of households, only single adult migrants aged fifteen years and above. In the second type of households, migrants are with their family.

iii. Let π_1 and π_2 be the risk of respective proportions of two types exposure households to the risk of migration such that

$$\pi_1 + \pi_2 = \alpha$$

iv. The number of migrants occurred in first type of household follows a displaced geometric distribution with parameter p is

$$p[Y=k]=pq^{k-1}, \qquad k=1,2,3..., \ p=1-q \tag{1}$$

v. In second type of households migrants move with their family member and follows zero^th truncated Poisson distribution with parameter θ is given by

$$p[Z=k]=\frac{\theta^k}{(e^{\theta}-1)k!}, \qquad k=1,2,3..., \ \theta>0 \tag{2}$$

Where Z is the number of migrants from second type of household.
Let X be the total number of migrants from a household. Therefore from above assumptions (i) to (v), the inflated form of the distribution for the total number of migrants from a household is

$$p[X=0]=1-\alpha, \ k=0 \ and$$

$$p[X=k]=\pi_1 pq^{k-1}+\pi_2\left[\frac{\theta^k}{(e^{\theta}-1)k!}\right], \qquad k=1,2,3... \tag{3}$$

III. Estimation

Probability model consist of four parameters $\alpha, p, \lambda \ and \ \pi$. It is difficult to estimate all these four parameters simultaneously. Therefore supposing $\pi_1=\pi_2$, the remaining parameters are estimated by method of moment and method of maximum likelihood.

A. Method of Moments

The parameters $\alpha, p \ and \ \lambda$ are estimated by equating the zero^th and first cell, theoretical cell frequencies to the observed frequencies of the respective cell and theoretical mean equal to the observed mean as follows

$$(1-\alpha)=\frac{f_0}{f} \tag{4}$$

$$\pi_1 p+\pi_2\left[\frac{\theta e^{\theta}}{(e^{\theta}-1)}\right]=\frac{f_1}{f} \tag{5}$$

$$\frac{\pi_1}{p}+\pi_2\left[\frac{\theta e^{\theta}}{(e^{\theta}-1)}\right]=\bar{x} \tag{6}$$

From equation (4)we have

$$\hat{\alpha}=1-\frac{f_0}{f} \ and \ \hat{\pi}=\frac{\hat{\alpha}}{2} \tag{7}$$

Where f_0 denotes the number of observed zero^th cell, f_1 denotes the number of observed first cell, f denotes the total number of observations and $\bar{x}$ denotes the observed mean of distribution. Parameter θ can be obtained by solving equation (5) and (6) simultaneously in the following way.
From equation (5)

$$\pi \ p+\pi\left[\frac{\theta e^{\theta}}{(e^{\theta}-1)}\right]=\frac{f_1}{f} \qquad \pi_1=\pi_2=\pi$$

$$p+\left[\frac{\theta e^{\theta}}{(e^{\theta}-1)}\right]=\frac{f_1}{f\pi}$$

$$p=\frac{f_1}{f\pi}-\left[\frac{\theta e^{\theta}}{(e^{\theta}-1)}\right] \tag{8}$$

From equation (6) we have

$$\frac{\pi}{p}+\pi\left[\frac{\theta e^{\theta}}{(e^{\theta}-1)}\right]=\bar{x} \qquad \pi_1=\pi_2=\pi$$

$$\frac{1}{p}+\left[\frac{\theta e^{\theta}}{\left(e^{\theta}-1\right)}\right]=\frac{\bar{x}}{\pi}$$

$$\frac{1}{p}=\frac{\bar{x}}{\pi}-\left[\frac{\theta e^{\theta}}{\left(e^{\theta}-1\right)}\right] \tag{9}$$

Multiply equation (7) by (8), we get

$$1=\left(\frac{f_1}{f\pi}-\left[\frac{\theta e^{\theta}}{\left(e^{\theta}-1\right)}\right]\right)\left(\frac{\bar{x}}{\pi}-\left[\frac{\theta e^{\theta}}{\left(e^{\theta}-1\right)}\right]\right)$$

$$\frac{\theta^2 e^{\theta}}{\left(e^{\theta}-1\right)^2}-\frac{\theta e^{\theta}}{\pi\left(e^{\theta}-1\right)}\left[\bar{x}+\frac{f_1 e^{\theta}}{f}\right]+\left[\frac{f_1\bar{x}}{f\pi^2}-1\right]=0 \tag{10}$$

The above equation is nonlinear in θ and it can be solve using numerical method (Newton Raphson method)

B. Method of Maximum Likelihood

After taking the value of θ from the method of moments and the remaining parameters are estimated by this method. Let X be a random variable from a sample of f observation with the probability function (3) then the likelihood function of the given value can be expressed as

$$L=(1-\alpha)^{f_0}\left[\pi_1 p+\pi_2\left[\frac{\theta}{\left(e^{\theta}-1\right)}\right]\right]^{f_1}\left[\alpha-\pi_1 p-\pi_2\left[\frac{\theta}{\left(e^{\theta}-1\right)}\right]\right]^{f-f_0-f_1} \tag{11}$$

Let $\frac{\theta}{\left(e^{\theta}-1\right)}=A$

The likelihood function gets the following form

$$L=(1-\alpha)^{f_0}\left[\pi_1 p+\pi_2 A\right]^{f_1}\left[\alpha-\pi_1 p-\pi_2 A\right]^{f-f_0-f_1} \tag{12}$$

Taking log on both sides, we get

$$\log L=f_0\log(1-\alpha)+f_1\log\left[\pi_1 p+\pi_2 A\right]+\left(f-f_0-f\right)\log\left[\alpha-\pi_1 p-\pi_2 A\right] \tag{13}$$

Now partially differentiating equation (13) with respect to α and p respectively and equating to zero we get the following equation.

$$\frac{\partial}{\partial\alpha}\log L=-\frac{f_0}{(1-\alpha)}+\frac{f_1}{(\pi_1 p+\pi_2 A)}\frac{\partial}{\partial\alpha}(\pi_1 p+\pi_2 A)+\frac{\left(f-f_0-f_1\right)}{\left(\alpha-\pi(p+A)\right)}\frac{\partial}{\partial\alpha}\left(\alpha-\pi(p+A)\right)=0$$

$$\pi_1=\pi_2=\pi=\frac{\alpha}{2}$$

$$\textit{from}\quad eq(7)\,\textit{we have}$$

$$-\frac{f_0}{(1-\alpha)}+\frac{f_1}{\left(\frac{\alpha}{2}\right)(p+A)}\frac{\partial}{\partial\alpha}\left(\frac{\alpha}{2}\right)(p+A)+\frac{\left(f-f_0-f_1\right)}{\left(\alpha-\frac{\alpha}{2}(p+A)\right)}\frac{\partial}{\partial\alpha}\left(\alpha-\frac{\alpha}{2}(p+A)\right)=0$$

$$-\frac{f_0}{(1-\alpha)}+\frac{f_1}{\alpha}+\frac{\left(f-f_0-f_1\right)}{\alpha}=0$$

$$\textit{On simplification, we get}$$

$$\frac{\left(f-f_0\right)}{\alpha}=\frac{f_0}{(1-\alpha)}$$

Solving for α, we get

$$\alpha=1-\frac{f_0}{f} \tag{14}$$

$$\frac{\partial}{\partial p}\log L = \frac{f_1}{(\pi_1 p + \pi_2 A)}\frac{\partial}{\partial p}(\pi_1 p + \pi_2 A) + \frac{(f - f_0 - f_1)}{(\alpha - \pi(p + A))}\frac{\partial}{\partial p}(\alpha - \pi(p + A)) = 0$$

$$\pi_1 = \pi_2 = \pi = \frac{\alpha}{2}$$

from equation (7), *we get*

$$\frac{f_1}{(p + A)} = \frac{(f - f_0 - f_1)}{(2 - p - A)}$$

$$2f_1 - pf_1 - Af_1 = fp - pf_0 - pf_1 + Af - Af_0 - Af_1$$

$$2f_1 = p(f - f_0) + A(f - f_0)$$

$$\therefore \qquad \hat{p} = \frac{2f_1}{f - f_0} - A \qquad (15)$$

Table 2: Observed and Expected Distribution of Households according to the Total Number of Migrants from a Household in Rural and Urban Places

Total No. of Migrants from a Household	Type of Places: Urban Observed	Urban Expected: Method of Moments	Urban Expected: Method of Likelihood	Rural Observed	Rural Expected: Method of Moments	Rural Expected: Method of Likelihood
0	81	81.0000	81.0000	72	72.0000	72.0000
1	26	25.9992	25.9892	32	31.9974	31.9974
2	17	18.6349	18.6449	21	20.7637	20.7637
3	11	10.6216	10.6216	10	10.9188	10.9188
4	7	5.5245	5.5245	6	5.5376	5.5376
5	3			4		
6	3	5.8638	5.8638	2	6.1952	6.1952
7+	2			3		
Total	**150**	**147.6442**	**147.6442**	**150**	**147.4128**	**147.4128**
α		0.46	0.46		0.52	0.52
p		0.3191	0.3191		0.3214	0.3214
		0.23	0.23		0.26	0.26
θ		1.4863	-		1.2596	-
Chi-Square		5.029	5.029		1.3884	1.3884

IV. Application of the Model

The probability model has been applied to migration data taken from the survey. The primary survey is conducted in rural and urban places of Dharwad to understand the risk of migration form households.

Table 2 gives the distribution of observed and expected number of households according to the total number of out migrants in urban and rural places of Dharwad. The risk of migration in a household, α is highest ($\hat{\alpha}$ = 0.52) in the rural place and lowest ($\hat{\alpha}$ = 0.46) in urban place. The estimated values of p according to two types of places are found to be 0.3191 and 0.3214 respectively. Subsequently it can be concluded that the risk of migration of rural is greater than urban place.

Applying χ^2 test for goodness of fit, the last three cells have grouped for both rural and urban places. The calculated χ^2 is insignificant at 5% level of significance in two types of places. This suggests that the model under consideration is a better approximation to observed distribution of out migration at the micro level.

V. Conclusions

Migration at micro-level will play a critical role in the development of theories of migration and examination of factors affecting movement process, is best carried out with models. In micro- level studies migration has been studied at community level, village level, household level, or individual level depending on the objective and availability of data. The considered micro model in the study proves the fundamental assumption that rural places have higher probability of migration than the urban places.

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