

A Systematic Review of Fourier Transform in Computer Vision

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Abstract

Fourier transforms is one of the oldest and a well-known technique in field of mathematic and engineering mathematical work. Fourier transform method represents the variable as a summation of complex exponentials. Fourier analysis has been used in signal processing and digital image processing for the analysis of a single image as a two-dimensional wave form, and many other type of form like Quantum mechanics, Signal processing, Image Processing.

This analysis also represents filters, Transformation, representation, and encoding, Data Processing, Analysis and many more fields. The use of Fourier transform in various applications has increased in recent years. This transform is one of the simplest transforms among the other transformation method used in mathematics. The time consumption is lesser due to this method. It has vast use in power distribution system, mechanical system, industries and wireless networks. Mainly in power distribution system, it is the mitigation of power quality disturbance required fast, accurate and high noise immune method. The Fourier transform is very important for the modern world for the easier solution of the problems. This study reviews the strength of Fourier transform, in recent year demand of this method and its use in different field and their applications.

Keywords: Fourier transforms, signal processing, image processing, Image Resizing, Image Cropping, Image Denoising, Image Enhancement.

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I. INTRODUCTION

The Fourier transform is a mathematical operation that allows us to analyze a signal or function in terms of its frequency content. It was first proposed by the French mathematician and scientist Jean-Baptiste Joseph Fourier in the early 19th century, thus the name. In addition to its use in mathematics, physics, engineering, and signal processing, the Fourier transform has many other applications.[2] The basic idea behind the Fourier transform is that a signal or function may be represented as the sum of sinusoidal functions at various frequencies. With its help, you may dissect a signal down to its component frequencies and learn more about them. In mathematics, the Fourier transform

$F(\omega)$ of a function $f(t)$ defined over a continuous domain is given by:[3]

$$F(\omega) = \int_{-\infty}^{\infty} [f(t)e^{-i\omega t}]dt$$

where ω stands for angular frequency and i for the irrational (-1) unit. For every value of t , the integral holds true. The amplitude and phase of each frequency component of the original signal are represented by the Fourier transform $F(\omega)$. Since it has a complex value, it may be interpreted in terms of both magnitude and phase. However, only the magnitude spectrum, which is the absolute values of the complex Fourier coefficients, is taken into account in many real-world applications.[4]

With the use of the inverse Fourier transform, we may go back to the original function or signal by analyzing its frequency content. It's a present from:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [F(\omega)e^{i\omega t}]d\omega$$

The integral is performed across all possible values of ω , and $F(\omega)$ stands in for the Fourier transform of $f(t)$.

Discrete signals may also benefit from the Fourier transform, with the summation replacing the integral. The fast Fourier transform (FFT) technique is often used to do this, since it quickly computes the discrete Fourier transform (DFT).[5]

II. Connection between Fourier transform and computer vision

To analyze and process pictures or videos, the discipline of computer vision, which is concerned with analyzing and extracting meaningful information from visual input, employs a number of mathematical approaches. The Fourier transform is a staple of computer vision as one of the most basic mathematical methods. Pictures can be filtered, features can be extracted, pictures can be registered, and patterns can be recognized all with the help of the Fourier transform and the examination of their frequency content. Images are often processed by computer vision algorithms as two-dimensional pixel value matrices. With the use of the Fourier transform, we may examine the image's underlying frequency components by transforming it from the spatial domain to the frequency domain. To do this, the Fourier transform is performed separately on each row and column of the image matrix. Image filtering is what the Fourier transform is to computer vision. Noise and other abnormalities are common in digital images, reducing image quality and making further processing more difficult. By performing a Fourier transform on a picture, we may isolate key frequencies to reduce their impact. When an image is represented in the frequency domain, filtering operations may be applied, such as high-pass filters to bring out small features and low-pass filters to smooth out the picture and reduce noise. Inverse Fourier transform is used to acquire the filtered picture by transforming it back to the spatial domain.[7]

The Fourier transform is not only important for image filtering, but also for feature extraction, a vital job in computer vision. Features in an image are the unique patterns or structures that may be used to identify or categorize objects, or even to follow their movement over time. Through analysis of their frequency content, some sorts of features may be isolated with the use of the Fourier transform. For instance, the Fourier spectrum often displays distinctive peaks at certain frequencies in the presence of periodic patterns and textures. Finding and analyzing these peaks allows for the inference of texture or periodicity in picture areas. Texture analysis, form identification, and even motion detection are all areas where such frequency-based feature extraction strategies might be put to use. The Fourier transform is also used in computer vision for a process called image registration, which entails the alignment of a series of pictures or of patches within an image. Stitching, tracking objects, and fusing images are only possible with accurate picture registration. By operating in the frequency domain, as the Fourier transform does, registration may be done quickly and accurately. Image alignment transformations like translations, rotations, and scaling factors may be estimated by comparing the Fourier spectra of the pictures or image patches. Phase correlation is a method for registering images that is both precise and stable, even in the face of noise or geometric alterations. The Fourier transform is related to other common computer vision methods. The Fourier descriptor is one such technique that captures the amplitudes and phases of an object's Fourier coefficients to characterize its form. In order to recognize lines and shapes in pictures, many people turn to Fourier-based methods like the Radon transform and the Hough transform.[8]

III. Common preprocessing techniques in computer vision

In computer vision applications, preprocessing methods are essential because they improve the quality and applicability of raw input pictures prior to analysis. These methods include a broad spectrum of processes designed to improve pictures, lower noise levels, extract useful characteristics, and standardize data for efficient analysis. Common preprocessing approaches in computer vision will be discussed along with their underlying ideas and some examples of their use.[11]

i. Image Resizing: To resize a picture is to change its proportions. One way to do this is to use an aspect ratio goal. Images are often resized for a variety of reasons, including conforming to a consistent input size for models and algorithms, decreasing computing complexity, and meeting specialized needs. Images may be resized to the same dimensions for object identification tasks so that the model can learn robust characteristics independently of the original picture size.

ii. Image Cropping: Cropping a picture means removing unwanted parts while keeping the desired parts. It's helpful for zeroing down on certain details or areas of interest in a picture. Cropping can enhance the precision and efficiency of subsequent analytic processes by eliminating undesirable or distracting background elements. Cropping a picture to remove the background, as is done in face recognition, may improve the quality of feature extraction and comparison.

iii. Image Denoising: In order to deal with the noise in pictures that might result from sensor limits, compression, and other sources, noise reduction methods are utilized. There are several different denoising methods to choose from, such as the Gaussian blur, median filtering, and bilateral filtering. These algorithms smooth the picture while keeping the essential information intact, reducing noise and boosting the image's overall quality. Medical imaging, surveillance, and low-light photography are just a few examples of where denoising is crucial.

iv. Image Enhancement: The purpose of picture enhancement is to increase an image's clarity and aesthetic appeal. Changing the image's contrast, brightness, or colour balance may make it more aesthetically attractive or bring attention to certain details. Enhancing details, increasing visibility, and revealing previously concealed

information inside a picture may be achieved using techniques like contrast stretching, histogram equalization, or adaptive enhancement algorithms.

v. Color Space Conversion: Changing from one colour representation to another is what colour space conversion is all about. RGB, HSV, and LAB are just a few of the many colour spaces available, each with its own set of benefits and applications. Image conversions like grayscale simplify processing and minimize computing demands, whereas LAB colour space conversions may improve color-based segmentation.[12]

IV. Application of Fourier Transform

i) In signal processing, the typical use of Fourier transform is to decompose the signal into amplitude and phase components. At present, in the field of signal processing and communication, the most active use of MATLAB in the mathematical application of software applications in the numerical calculation of second to none, and the current Fourier transform in the field of communication applications is based on this mathematical software, and in addition to digital signal processing, the excellent graphics processing capabilities enable it to solve the problem of specific types of Fourier transforms in these applications in digital image processing techniques, so that Fourier transform in the communication to be better application and development.

ii) Fourier transform is used in image analysis, image filtering, image reconstruction and image compression. The Fourier Transform is an important image processing tool which is used to decompose an image into its sine and cosine components.

iii) Radiology uses the Fourier transform extensively, as it is a basic mathematical tool for signal analysis and is essential to the creation of contemporary MR images. Fourier transform spectroscopy (FTS) has emerged as a cutting-edge, robust, and ultrasensitive tool for investigating a wide range of physiologically significant systems, from single molecules to complex samples like live cells and tissues.

V. Comparative study of various transform.

It will not be an exaggeration to assert that digital image processing came into being with introduction, in 1965 by Cooley and Tukey, of the *Fast Fourier Transform* algorithm (FFT, [1]) for computing the *Discrete Fourier Transform* (DFT).

The second wave in this process was inspired by the introduction into communication engineering and digital image processing, in the 1970s, of Walsh-Hadamard transform and Haar transform and the development of a large family of fast transforms with FFT-type algorithms whereas Walsh-Hadamard and Haar transforms have already been known in mathematics, other transforms, for instance, quite popular at the time Slant Transform, were being invented “from scratch. “This development was mainly driven by the needs of data compression, though the usefulness of transform domain processing for image restoration and enhancement was also recognized very soon. This period ended up with the introduction of the Discrete Cosine Transform (DCT, which was soon widely recognized as the best choice among all available at the time transforms and resulted in JPEG and MPEG standards for image, audio, and video compression.

The third large wave of activities in transforms for signal and image processing was caused by the introduction, in the 1980s, of a family of transforms that was coined the name “*wavelet transform*”. The main motivation was achieving a better local representation of signals and images in contrast to the “global” representation that is characteristic to Discrete Fourier, DCT, Walsh-Hadamard, and other fast transforms available at the time. During 1980s–1990s a large variety of discrete wavelet transforms were suggested for solving various tasks in signal and in image processing.

TABLE I
Comparative study of various transform

Transforms	Relevance to Imaging optics	Main Characteristic features	Main application areas
Discrete Fourier Transforms	Represents optical Fourier Transform	Cyclic shift invariance vulnerable to boundary effects	(i) Analysis of periodicities. (ii) Fast convolution and correlation.
Discrete Cosine Transform	Represents optical Fourier Transform	Cyclic shift invariance (with double cycle); virtually not sensitive to boundary effects	(iii) Fast and accurate image resampling. (iv) Image compression. (v) Image denoising and deblurring. (vi) Numerical reconstruction of holograms.
Discrete Fresnel Transforms	Represent optical Fresnel	Computable through DFT/DCT	Numerical reconstruction of holograms

	Transform		
Walsh-Hadamard Transform	No direct relevance	Binary basis functions. Provides piece-wise constant separable image band-limited approximations	(i) Image compression (marginal). (ii) Coded aperture imaging.
Haar Transform and other Discrete Wavelet Transforms.	Sub band decomposition	Binary basis functions. The fastest algorithm. Multiresolution. Shift invariance in each particular scale.	(i) Signal/image wideband noise denoising. (ii) Image compression.

VI. Uses of image recognition in our daily routine

1. Modern smart phones uses facial recognition as feature for unlock phones and this feature is highly useful in case phone is stolen or missing. This feature not only unlock phones but very helpful in protecting data and secure data from being accessed.
2. Investigation agencies uses image recognition to collect data regarding history of suspected, they take photograph and search with their database. Investigation agencies also scanned and add photographs to carry out another search.
3. Image recognition technique is also used in international aerodromes and ports for travelers having biometric passports for facial recognition, it not only saves times but also improve safety and security.
4. Image recognition method is also used in reverse image search on web. In combination with artificial intelligence software, we can search and identify image features and compare with the similar images available on web, able to find out the origin of the image, ownership of the image and all the relevant information of the image available from different sources.
5. Image recognition is also very helpful in e commerce industry. Online customers are more willing to search products by their image instead of using text search for their purchases.
6. Attendance is very important and compulsory process in each and every organization. Automatic attendance system uses image recognition technique and generate data accurate and real time.

VII. Challenges and Constraints

The Fourier Transform is limited to analyzing signals that are continuous in time, so it cannot be used to analyze signals that are discrete in time. It is sensitive to noise, so signals with high levels of noise may not be accurately represented by the Fourier Transform. The Fourier Transform plays a critical role in a broad range of image processing applications, including enhancement, analysis, restoration and compression.

VIII. Conclusion

The proposed Fourier transform has a wide range of help in various domains like power distribution system, wireless, signal processing, cell phone manufacturing, mechanical and industrial application. In power system, proposed method easily analyzes the faults, harmonics and disturbance. In wireless system, they identify the noise and easily calculating the losses in same way. The Fourier transform provides the world an easy and most comfortable method for solution of questions. Fourier transformation is also frequency domain which represent time series analysis proved its application in Quantum mechanics, Signal processing, Image Processing and filters, Transformation, and encoding, Data Processing and Analysis and other fields. In this study showed that Fourier transform can successfully applied in the field of medical engineering hence some more research yet to be explored.

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