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Analysis of Optimal Strategies under Multiple Auction Mechanisms

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***Abstract**: This paper analyzes the selection and effects of optimal strategies based on four auction mechanisms: bidding auction, Dutch auction, first-price sealed auction, and second-price sealed auction. This study compares various auction mechanisms through literature review, model building, and analysis of their characteristics, advantages, disadvantages, and scope of application. It also examines the impact of these mechanisms on the distribution of benefits between sellers and buyers and reveals optimal decision-making strategies for participants. The study analyzes the optimal strategies of buyers and sellers under each auction mechanism. In bidding auctions, buyers should aim to bid the highest price within the real valuation range, while sellers should focus on ensuring an open and transparent auction process. In Dutch auctions, buyers should bid as soon as the price drops to a level they are willing to accept, while sellers should set a reasonable starting price to attract bidders. The strategies that are optimal for buyers and sellers in first-price sealed auctions and second-price sealed auctions differ and need to be adjusted based on the characteristics of the auction mechanism. The research findings presented in this dissertation are significant in guiding auction participants and organizers to formulate optimal strategies that can lead to more effective trading results.*
***Keywords**: auction mechanism, game theory, NASH equilibrium, and the trapezoidal distribution.*

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I. Introduction

Auctions are a crucial market mechanism with various applications in selling goods and services, allocating resources, and gathering information. The design of an excellent auction mechanism to achieve market efficiency and fair distribution has been the focus of research in fields such as economics, mathematics, and computer science. In recent years, the emergence of new auction mechanisms due to the development of information technology and market competition has presented challenges to researchers and requirements for practical application in activities such as government procurement and disposal of state-owned assets.

Various auction mechanisms can have different impacts on bidders' choices and results. Therefore, it is more meaningful to study different auction mechanisms. This thesis will examine optimal strategies and differences in effects under various auction mechanisms from the perspective of game theory. This will aid in the evaluation and selection of appropriate auction mechanisms, improve market efficiency, enhance fairness, and promote economic development. Specifically, the significance of this thesis is reflected in the following aspects:

Provide theoretical analysis of optimal strategies under different auction mechanisms to provide guidance for bidders and auction mechanism designers.

It provides guidance for bidders and auction mechanism designers by comparing the advantages and disadvantages of different auction mechanisms, analyzing the economic logic behind the choice of a particular mechanism, and providing decision-making support and reference for different aspects of application.

Empirical analysis provides verification and assessment of auction mechanisms in practical applications, supporting practical activities and policy formulation with theoretical evidence.

The objective of this paper is to examine optimal strategy selection across various auction mechanisms, compare their characteristics, advantages, and disadvantages, and provide guidance for sellers and buyers participating in auctions. The specific objectives of this paper are as follows:

This text aims to explain the principles and characteristics of four common auction mechanisms: bidding auction, Dutch auction, first-price sealed auction, and second-price sealed auction. The similarities and differences between these mechanisms will also be compared.

The text will explore which auction mechanism allows the seller to maximize profits and the buyer to minimize costs. Additionally, it will evaluate and analyze the effects of different auction mechanisms.

This paper aims to analyze the distribution of benefits and strategy selection between the seller and buyer under different auction mechanisms. The optimal strategy will be identified to provide a decision-making basis for the auction.

The research will use a combination of literature review and mathematical modeling analysis to achieve its purpose. By reviewing the existing literature, this study will analyze the characteristics, advantages, and disadvantages, as well as the scope of application of four auction mechanisms: bidding auction, Dutch auction, first-price sealed auction, and second-price sealed auction. Secondly, mathematical models are established to solve and analyze optimal strategies under different auction mechanisms. This reveals the impact of various auction mechanisms on the distribution of benefits between sellers and buyers. Finally, the research results are comprehensively analyzed and summarized, and shortcomings are identified to provide reference for future research directions.

II. Literature Review

Bidding auctions, Dutch auctions, first-price sealed auctions, and second-price sealed auctions are common auction mechanisms, and they all have their own advantages and disadvantages in different contexts. In this paper, we will overview the analysis of optimal strategies under multiple auction mechanisms and cite relevant literature to explain them.

Bidding auction is a common auction method, which is also known as English Auction . In a bidding auction, the seller publicly displays the item and accepts progressively higher offers from buyers until no one else is willing to bid higher, and the buyer with the highest bid ultimately obtains the ownership of the item.Bjarne Brendstrup, Harry J. Paarsch(2006) studied the optimal strategy in English Auction, and the results of the study showed that the bidder needs to consider his own valuation and the bids of other bidders in order to determine his highest bid. In addition, the effects of the auction rules and the randomness of the number of bidders need to be considered. Characteristics include an open and transparent bidding process, gradual price increases, and longer time periods. The advantage is that the price can rise gradually, which is favorable to the seller to obtain a higher price; the disadvantage is that it takes a long time, and may lead to the phenomenon of "bidder's curse". Applicable to the transaction of high-value items, such as art, real estate and so on.

Dutch auction is a closed-door auction mechanism in which the auctioneer starts with a high price, then keeps lowering the price until a bidder is willing to accept it, and finally the bidder gets the lot. In a Dutch auction, a bidder needs to consider the speed of price change and the bids of other bidders in order to determine his or her maximum bid.Mukun Cao, Qing Hu, Melody Y. Kiang & Hong Hong. (2020) investigated the optimal bidding strategy in a Dutch auction. The results showed that bidders need to consider their own valuation and other bidders' bids to determine their own maximum bid. In addition, the effects of auction rules and uncertainty about the value of the lot need to be considered. Features include a gradual decline in price, a relatively short period of time, and the ability of sellers to sell items quickly. Advantages are that the auction process is relatively quick and prices are automatically adjusted to market-acceptable levels; disadvantages are that they can lead to underpricing. It is suitable for high demand, price-sensitive commodities, such as agricultural products.

First-price sealed-bid auction is a closed-door auction mechanism, the bidders submit their bids within a specified time, the highest bidder gets the lot, but the price paid is the price of their own bids.Deng, Fei;Liu, Sifeng;Fang, Zhigeng(2021)studied the optimal bidding strategy in the first-price sealed auction, the results show that the discrete bidding strategy is the same as the discrete bidding strategy in the first-price sealed auction. The results show that discrete bids affect the fairness and efficiency of the auction results, so it is necessary to consider discrete bids when designing the auction mechanism. The features include bid secrecy, the highest bidder wins, and the demander has to bear his bid. The advantage is that it encourages buyers to quote their true valuations and avoids the "bidder's curse"; the disadvantage is that it may lead to undervalued bids. Suitable for situations where the demander has a clear valuation of the item, e.g., business acquisitions.

Second-price sealed-bid auction is a closed-door auction mechanism in which bidders submit their bids within a specified period of time, and the highest bidder is awarded the lot, but the price paid is that of the second-highest bidder.Christian Barrot ， Sönke Albers ， Bernd Skiera & Björn Schäfers （2010） studied the optimal bidding strategy in second price sealing auctions.The results show that discrete bids affect the fairness and efficiency of the auction results, so it is necessary to consider discrete bids when designing the auction mechanism. The features include buyers' bidding confidentiality, the highest bidder wins, and paying the next highest price. Advantages include encouraging buyers to quote their true valuations and not lose out by overbidding; disadvantages include the potential for strategic bidding by bidders. The disadvantage is that it may lead to strategic bidding. It is suitable for situations where the demanders have a clearer valuation of the items and do not want to lose money by overbidding.

In summary, the analysis of optimal strategies under multiple auction mechanisms is an important part of network auction research, which needs to be analyzed and designed in the context of the actual situation.

III. Assumptions and notation

Bidders' assessment of the value of the lot: the auction presupposes the existence of some assessment of the value of the lot by the bidders. Each bidder has a private valuation that indicates the maximum price he or she is willing to pay for the lot. This private estimate is an important reference for the bidders when deciding how much to bid.

Auction Rules: Auction rules refer to the starting price, markup, and time of the auction. These rules affect the bidding strategy of the bidders, because the bidders must consider the impact of these rules on their bidding strategy. For example, if the starting price is low, a bidder may decide to adopt a conservative bidding strategy and wait for other bidders' bids before following up; if the markup is small, a bidder may choose to bid higher to avoid being overtaken by other bidders.

Bidder Strategies: Bidders develop bidding strategies based on their own assessment of the value of the lot and the rules of the auction. These strategies include initial bids, follow-the-bid strategies, and exit strategies. For example, if the bidder's assessment of the value of the lot is high, he may use a higher initial bid; if the bidder believes that the current price has exceeded his own assessment of the value of the lot, he may choose to withdraw from the auction.

Competition: Competition refers to the interactions between bidders, including the number, strategy and behavior of competitors. The number and strategy of competitors affects a bidder's bidding strategy because the bidder must consider the impact of other bidders' bids on his or her own. For example, if the number of competitors is low, a bidder may adopt a higher bidding strategy to ensure winning the lot; if the number of competitors is high, a bidder may adopt a conservative bidding strategy and wait for other bidders' bids before following up. Different types of auctions have different modeling assumptions, and in order to build the auction model needed for this paper, we need to make the following assumptions:
Bidding auctions: it is assumed that all bidders want to acquire the good or service at the lowest price. It is also assumed that all bidders know their respective valuations and make their decisions independently.
Dutch Auction: Assume that all bidders know their respective valuations and make decisions independently.
First Price Sealed Auction: Assume that all bidders want to buy the good or service at the lowest price. It is also assumed that all bidders know their respective valuations and make decisions independently.
Second Price Sealed Auction: Assumes that all bidders know their respective valuations and make their decisions independently.
The symbols used in the auction model developed in this paper include:

v_i : Participant i 's private expected valuation of the item, i.e., what he/she thinks the item is worth.

b_i : The bid quoted by participant i , i.e., the price quoted by the participant in determining the bid in each round.

p_i : The bid paid by participant i , i.e., the price to be paid if the item is successfully auctioned.

p_t : The high price at the beginning of a Dutch auction.

$p_{(2)}$: The second highest bid in a second price sealed auction.

u_i : Benefit function.Participant i 's net benefit from participating in the auction, i.e., the value of the item he receives minus the bid he pays.

U_i : Expected utility function.Participant i 's expected utility, i.e., its weighted estimate of its expected return based on the participant's private expected valuation of the item and risk attitude.

$F_i(v_{-i})$: Probability Distribution Function.Used to describe participant a's uncertainty about the value of the item, i.e., his or her probability distribution function for the item's price.

IV. Modeling and Solving

The Bidding Auction mechanism involves bidders submitting their bids to the auctioneer, with the highest bidder winning the auction, given the expected utility function of the participants.
Where the expected utility function of the participant i :

$$U_{\mathrm{i}}(b_i)=\begin{cases} v_i-b_i & b_i>\max\{b_1,\ldots,b_{i-1}\} \\ 0 & b_i\le\max\{b_1,\ldots,b_{i-1}\} \end{cases}$$

That is a participant i 's expected utility is the net benefit of subtracting the value of the good v_i from the price he or she quotes b_i, where his or her expected utility is zero if he or she quotes a price lower than the current maximum bid.

The auctioneer's payoff function $R = \sum_{i=1}^{n} p_i$ is the sum of the bids of all successful participants. The Nash equilibrium solution for bidding auctions can be proved using the inverse method.

Suppose at this point there is a participant i whose bid is $U_i(b_i^{'}) = v_i - b_i^{'}$. He will be the highest bidder and win the item. However, his expected payoff is $b_i^{'} = v_{(n)}$, whereas if he bids is $v_{(n)}$, his expected payoff is $U_i(v_{(n)}) = v_i - v_{(n)}$. This is because $b_i^{'} > v_{(n)}, U_i(b_i^{'}) < U_i(v_{(n)})$ violates his optimal strategy of bidding at. Thus, we show that the b_i optimal strategy is $b^* = v_{(n)}$, i.e., all participants bid according to their values for the item, and the highest bidder wins the item and pays his bid. This is a Nash equilibrium because if each participant matches his bid to his private valuation, there is no other bidding strategy that allows him to realize a higher expected return. This is because it maximizes their expected utility. The auctioneer's payoff maximization strategy revolves around the dual requirement of maximizing the auctioneer's payoff if the highest bidder wins: first, the auctioneer must auction the item to the highest bidder, which means that the auctioneer must advertise and publicize the item extensively prior to the auction in order to maximize the number of potential competitors. Second, the auctioneer must maximize the sum of the bids of all winning bidders, which can be achieved either by raising the starting price or by expanding the number of bidding participants.

The expected revenue of the highest bidder is $U_{(n)} = v_{(n)} - v_{(n+1)}$, where $v_{(n+1)}$ is the valuation of the $n+1$th highest bidder. At this point, the auctioneer's payoff is also maximized, i.e., $R_{\max} = \sum_{i=1}^{n} v_{(i)}$.

In a bidding auction, the optimal bidding strategy is to bid the expected value of the bidders, i.e., v_i. This is because in a bidding auction based on a uniform distribution, where the probability of each possible bid is equal, participants must bid the value of the product in terms of their expectation of its value in order to maximize their expected revenue. Under this optimal strategy, the auctioneer's payoff is the expected value of the highest bid, $E[p]$.

This bidding auction is suitable for competitive environments where the situation is simple, i.e., all participants are willing to pay similar prices and the competitive environment is relatively stable. If the competitive environment is very intense, where each participant wants the product and there are significant differences among them, this optimal strategy may no longer be applicable. In addition, we must consider other factors such as market demand and the number of competitors. If the market demand is high, the auctioneer's revenue may be higher, whereas if the number of competitors is too high, it will lead to a saturated market with competitors constantly raising their bids. In summary, choosing an appropriate auction strategy requires a comprehensive consideration of a variety of factors, and if the situation is relatively simple, it is a good choice to refer to the Bidding Auction Model's optimal bidding strategy.

A Dutch auction is an auction mechanism of gradual price reductions, also known as a declining auction or a public service auction, in which the starting price of an item is gradually lowered until one bidder accepts the current price. Bidders can make offers at any time, and the auction ends when a bidder accepts the current price.

The auctioneer divides the auction into T equal time periods (e.g., T hours or T days), and then at the beginning of each period announces the current bid p_t, and in the next period announces a lower bid p_{t+1}, until there is a bidder with a price equal to p_t.

At each moment t, the expected return of participant i is:

$$U_i(b_i, t) = \begin{cases} v_i - p_t, & b_i = p_t \\ 0, & b_i \neq p_t \end{cases}$$

The auctioneer's revenue R is the sum of the bidders' payments, i.e., $R = \sum_{i=1}^{n} b_i$.

The equilibrium strategy of a Dutch auction can be determined using a dynamic strategy. In a Dutch auction, each participant must maximize his utility by choosing the right time to stop bidding and pay. This is because, at any given moment, a bidder may be overtaken by other bidders and thus cease to be the highest bidder. Therefore, participants must consider whether to exit or continue bidding at the current stage and when to exit. Suppose that at time t, the utility of bidder i is maximized by $U_t^*(p_t)$. Using the maximization principle, we can obtain the relationship between the maximum utility of bidder i and the exit threshold:

$$U_t^*(p_t) = \max_{b_i \ge p_t} U_i(b_i, t)$$

A bidder's exit threshold is his or her true valuation of the item, and the bidder will exit if the price falls below this threshold.We assume that bidder i's exit threshold is v_i, i.e., he exits the bidding when his actual valuation of the item is not higher than v_i. This means that bidder i exits the bidding when the price is . That is, when the price is $p_t \le v_i$, bidder i exits the bidding, paying his maximum bid b_i.

$$b_i(p_t) = \begin{cases} p_t, & p_t > v_i \\ \max\left\{ p_t, U_i^*(p_t) + p_t \right\}, & p_t \le v_i \end{cases}$$

In this case, for bidders with $p_t > v_i$, their bids depend on whether there is competition. If there is competition, they must pay a price of p_t to continue the race. For bidders with $p_t \le v_i$, they compete with other bidders until the price reaches their exit threshold.
Therefore, we can use the above function to calculate the auctioneer's revenue:

$$R = \sum_{i=1}^{n} b_i(p_t)$$

Next, we use dynamic programming methods to determine the optimal strategy for the bidders. Specifically, we start at T and recur forward, considering what strategies the bidders will use to maximize their returns at each point in time.

Suppose that at time $t+1$, bidder i's optimal strategy is to exit at the current price p_{t+1}, with payoff $U_t^*(p_{t+1})$. At this point, it's a past price, and if $p_t > p_{t+1}$, then for bidder i the current price p_{t+1} is irrelevant, and the bidder's payoff is $U_t^*(p_{t+1})$. On the other hand, if the current price $p_t \le p_{t+1}$, the bidder must consider both the utility of continuing to bid at the current price and the utility of exiting. This gives us the optimal strategy for bidder i:

$$U_i^*(p_t) = \max\left\{ U_i^*(b_i(p_t), t), U_i^*(p_{t+1}) \right\}$$

Dynamic programming methods can be used to recurse from T forward until stopping at $t=1$to obtain the optimal strategy for each bidder and the maximum revenue R^* for the auctioneer. A complex model of a Dutch auction was built and dynamic programming was used to obtain the optimal strategy for each bidder and the maximum revenue for the auctioneer. This model provides a practical auction mechanism that can be widely used in many practical applications. The optimal strategy for bidding in a Dutch auction is for the bidder to offer decreasing prices until the bid matches their own valuation or someone else's offer. This allows bidders to approach the auction's equilibrium price gradually and maximize their utility, increasing the likelihood of purchasing the good. The final bidder is the first to place a bid at a given price. The maximum revenue outcome of a Dutch auction depends on the lowest winning price, which is the lowest of the highest bids among all bidders, i.e., $p_{(1)}$. This price can also be thought of as the lowest winning bid among all bidders. Dutch auctions typically generate higher auction proceeds than other types of auctions, particularly when the buyer pool is relatively small and the ratio of supply to demand for the commodity is slightly lower.

It is important to note that Dutch auctions are better suited for highly competitive markets. This is because bidders feel more comfortable lowering their offers, which can effectively prevent overbidding and reduce market volatility. Additionally, Dutch auctions can encourage more bidders to participate, thus increasing the market coverage of the auction and the stickiness of the participants. However, in a highly competitive market environment, this auction method may result in lower auction returns when the supply-to-demand ratio is low. Therefore, when choosing an auction strategy, it is necessary to consider the impact of many factors, such as market demand, the competitive environment, and the participants.

In a uniformly distributed first-price sealed auction based on a uniform distribution, the auctioneer sells a good by offering an item to multiple bidders, and the final price of the auction is determined by the bidders' bids. In this auction mechanism, each bidder's bid is private information, the auctioneer only knows each bidder's offer, and the final auction decision is based on the maximum value of that bid. We will model the first price-sealed auction based on uniform distribution and compute its Nash equilibrium solution. We will model the uniformly distributed first-price sealed auction using the following notation:

The expected utility function of participant i is:

$$U_i(b_i, p) = \begin{cases} v_i - b_i, & b_i = \max_j b_j \\ 0, & otherwise \end{cases}$$

Assuming that the bidders' quotes are uniformly distributed random variables and that each individual's quote is a mutually independent event, the probability of a participant winning by choosing quote b can be calculated. If there are n participants, each with a quote generated uniformly at random from 0 to V,The probability that a participant wins by choosing quote b_i is:

$$F(v_{-i}) = \frac{b_i^n}{V^n}$$

In this case, the maximum bid is represented by $\max_j b_j$. Therefore, bidder i's expected payoff can be calculated as follows:

$$E[U_i(b_i, p_i)] = \frac{b_i^{n-1}}{V^n} \cdot (v_i - b_i)$$

To solve the Nash equilibrium, it is necessary to identify an offer strategy that meets the needs of each participant to maximize their expected return. The maximum expected return is:

$$\max_{b_i} \left\{ \frac{b_i^{n-1}}{V^n} \cdot (v_i - b_i) \right\}$$

To calculate the optimal price, we must find its first and second derivatives, set the first derivative to 0, and ensure that the second derivative is less than 0 to achieve a maximum value.

Examine the first-order derivatives of $\frac{b_i^{n-1}}{V^n} \cdot (v_i - b_i)$.

The value of which is set to 0 can be calculated:

$$b_i = \frac{(n-1)V + v_i}{n}$$

The second-order derivatives can be calculated to determine if the stationary points obtained here are extremely large.The value of $\frac{\partial^2}{\partial b_i^2} \frac{b_i^{n-1}}{V^n} \cdot (v_i - b_i)$ after computation is:

$$\frac{\partial^2}{\partial b_i^2} \frac{b_i^{n-1}}{V^n} \cdot (v_i - b_i) = -\frac{(n^2-1)}{n^2} \cdot \frac{b_i^{n-4} V^{n-2}}{V^{2n}} \cdot (n \cdot b_i - (n-2)V - 2v_i) < 0$$

Thus, $b_i = \frac{(n-1)V + v_i}{n}$ is the optimal strategy for participant i. Under this equilibrium strategy, the expected return for each participant is:

$$E[U_i(b_i, p_i)] = \frac{(\frac{(n-1)V + v_i}{n})^{n-1}}{V^n} \cdot \frac{Vv_i}{n}$$

The optimal strategy for participant i is $b_i = \frac{(n-1)V + v_i}{n}$. Under this equilibrium strategy, each participant can expect a return of:

$$E[U_i(b_i, p_i)] = \frac{(\frac{(n-1)V + v_i}{n})^{n-1}}{V^n} \cdot \frac{Vv_i}{n}$$

At this point, the revenue of the auctioneer is represented by $R=\frac{(n-1)V+v_{(n)}}{n}$, where $v_{(n)}$ represents the maximum value of the bidders' offers.

Note that this Nash equilibrium solution is based on the assumption that bidders are independent of each other's offers and their own values are sampled independently from the same uniform distribution. However, in practice, there are many factors that may affect the bidders' bids, such as information asymmetry, the degree of competition, and market demand. Therefore, it is important to identify and consider these factors when choosing an appropriate auction mechanism.

In the first-price sealed auction model, bidders should quote their true valuation of the commodity as the optimal bidding strategy. This requires evaluating the value of the good and making a bid based on that value. The seller may have more information due to information asymmetry, while the buyer needs to estimate the value of the good. Bidders may offer a price higher than their estimate to ensure they can obtain the good. The bidding strategy that is optimal ensures that each bidder maximizes their expected revenue.

The auctioneer's maximum gain is the income from the highest bidder's offer, which is the value that the highest bidder is willing to pay. As the auction model can only have one winner, the auctioneer can only receive revenue from the highest bidder. In summary, when practicing auction strategies, it is important to consider various factors such as market size, auction environment, and number of competitors. If there are more bidders and a complex market environment, it may be necessary to choose other auction strategies, such as laboratory auctions or Dutch auctions.

The trapezoidal distribution is a function that describes bidders' valuations of items. Bidders' valuations are uniformly distributed within a specific interval, and those with relatively higher valuations are more likely to win the bid. We will model the second-price sealed auction based on the trapezoidal distribution and compute its Nash equilibrium solution.

The expected utility function of bidder i is:

$$U_i(b_i,p)=\begin{cases} v_i-p_{(2)}, & b_i=\max_j b_j \\ 0, & b_i\neq\max_j b_j \end{cases},$$

where $p_{(2)}$ denotes the second highest of all offers. It is assumed that the bidders' offers are trapezoidally distributed random variables and that each bidder chooses their offer independently. If there are n participants, each with a trapezoidal uniformly randomly generated quote ranging from T_a to T_b, the probability that participant i wins by choosing quote b_i is:

$$F(v_{-i})=\begin{cases} \frac{2}{\alpha T_a+T_b-T_a}\cdot(v_i-T_a), T_a\leq v_i\leq T_a+\frac{T_b-T_a}{2} \\ \frac{T_b-T_a}{2}\cdot(T_b-v_i), \quad T_a+\frac{T_b-T_a}{2}<v_i<T_b \\ 0, \quad otherwise \end{cases}$$

In this case, the second highest bid corresponding to b_i is $\max_{j\neq i} b_j=b_2$. Thus, bidder i's expected payoff is:

$$E[U_i(b_i,p)]=F(v_{-i})\cdot(v_i-b_2)$$

To solve the Nash equilibrium, we need to find the optimal strategy for each participant that maximizes his expected return. The optimal strategy for each participant is to choose an offer that maximizes his expected return. We must assume that each participant will set his bid to maximize his expected return, and also assume that the other participants will choose bids that maximize their returns. Therefore, the optimal strategy for bidder i is to choose b_i such that its expected return is maximized. Maximizing $E[U_i(b_i,p)]$ yields a first-order derivative:

$$\frac{\partial E[U_i(b_i,p)]}{\partial b_i}=\frac{2}{(\alpha T_a+T_b-T_a)^2}(v_i-T_a-T_b+\alpha T_a+\alpha b_2), T_a\leq v_i\leq\frac{T_a+T_b}{2}$$

$$\frac{\partial E[U_i(b_i,p)]}{\partial b_i}=\frac{2}{(T_b-T_a)^2}(T_b-\alpha T_a-T_a+v_i+\alpha b_2),\frac{T_a+T_b}{2}<v_i<T_b$$

Set the derivative equal to 0 to obtain the optimal bid of bidder i :

$$b_i=\begin{cases}\dfrac{2\alpha b_2+(\alpha T_a+T_b-T_a)\cdot v_i-(n-2)T_\alpha}{2\alpha},T_a\le v_i\le\dfrac{T_a+T_b}{2}\\ \dfrac{2b_2+v_i+T_a+\alpha T_a-T_b}{2}, \qquad \dfrac{T_a+T_b}{2}<v_i<T_b\end{cases}$$

Under this optimal strategy, the expected payoff for each bidder is:

$$E[U_I(b_i,p)]=\begin{cases}\dfrac{(b_2-T_a)^2}{(\alpha T_a+T_b-T_a)^2}(v_i-T_a),T_a\le v_i\le\dfrac{T_a+T_b}{2}\\ \dfrac{(b_2-T_b)^2}{(T_b-T_a)^2}(T_b-v_i), \qquad \dfrac{T_a+T_b}{2}<v_i<T_b\end{cases}$$

The auctioneer's revenue at this point is $R=b_2$. Note that the larger the sample space, the more complex the formula for the optimal policy. Note that the Nash equilibrium solution we obtain is based on some assumptions, such as the independence of bids among bidders and the trapezoidal distribution of private values. In practice, we need to consider a variety of uncertainty factors and choose an appropriate auction mechanism. Second price closed auctions can be viewed as a special case of Dutch auctions, where the highest bidder wins the item, but only has to pay the price of the second highest bid. Thus, the optimal bidding strategy is also known as the Vickrey price, where participants bid a price equal to their own valuation of the item. The other parts are the same as in a Dutch auction.

The optimal bidding strategy can be derived from game-theoretic analysis and computation. In a second-price closed auction, the optimal bidding strategy is the price at which the bid is equal to one's own valuation of the item, which is based on the same principle as the Nash equilibrium, where participants are unrelated and independent of each other, and which maximizes the utility of such a strategy under all combinations of strategies.

In a second-price closed auction, if the optimal bidding strategy is violated, participants pay a higher cost and do not receive a higher payoff. If the participant's bid is higher than his or her valuation, the participant pays more than the actual value of the item; if the participant's bid is lower than his or her valuation, the participant loses the opportunity to win the auction. For auctioneers, the optimal bidding strategy actually ensures that they can maximize their revenue. This is because in a second-price closed auction, the auctioneer receives the additional price difference from the second-highest bid. If participants bid more than the true value of the item, the auctioneer will receive a higher return, and if more participants participate in the auction, the auctioneer will receive a higher total return.

In summary, the optimal bidding strategy and the maximum return results for the bidders and the auctioneer in the second-price closed auction model are evidence of theoretical calculations and empirical proofs that confirm the maximum utility of rational participants using this strategy and confirm that the auctioneer is able to maximize the return in such auctions.

V. Analysis of results

5.1 Comparison of Auction Mechanisms

Auction model	Equilibrium Results	Specificities
Bidding Auction	$b^*=v_{(n)}$ $U_{(n)}=v_{(n)}-v_{(n+1)}$ $R_{\max}=\sum_{i=1}^{n}v_{(i)}$	Auction participants submit bids and the auction item goes to the highest bidder. Bidding auctions are appropriate for small quantities of high-value items because of the need for fast, real-time decision making and buying power.
Dutch auctions	$U_i^*(p_t)=\max\left\{U_i^*(b_i(p_t),t),U_i^*(p_{t+1})\right\}$	The auction starts with a high price and then gradually decreases, and when a bidder feels that the current price is within their budget, or when they feel that the price will not go any lower, they bid and win the lot.

First Price Sealed Auctions	$E[U_i(b_i,p_i)]=\frac{(\frac{(n-1)V+v_i}{n})^{n-1}}{V^n}\cdot\frac{Vv_i}{n}$ $R=\frac{(n-1)V+v_{(n)}}{n}$	All bidders, after being informed of the details of the auction, individually prepare their own bid book, which contains the bid price and details of the auction. The auctioneer collects all bids and selects the bidder with the highest bid to successfully purchase the lot.
Second Price Sealed Auction	$b_i=\begin{cases}\frac{2\alpha b_2+(\alpha T_a+T_b-T_a)\cdot v_i-(n-2)T_\alpha}{2\alpha}, T_a\le v_i\le\frac{T_a+T_b}{2}\\ \frac{2b_2+v_i+T_a+\alpha T_a-T_b}{2}, \quad \frac{T_a+T_b}{2}<v_i<T_b\end{cases}$ $E[U_I(b_i,p)]=\begin{cases}\frac{(b_2-T_a)^2}{(\alpha T_a+T_b-T_a)^2}(v_i-T_a), T_a\le v_i\le\frac{T_a+T_b}{2}\\ \frac{(b_2-T_b)^2}{(T_b-T_a)^2}(T_b-v_i), \quad \frac{T_a+T_b}{2}<v_i<T_b\end{cases}$	Bidders place their own bids, and the highest bidder wins the lot, but they only pay the price of the next highest bid if they actually pay the price, so this auction is also known as a second highest bid auction.

Bidding auctions, Dutch auctions, first-price sealed auctions, and second-price sealed auctions are four common auction mechanisms, each with its own advantages and disadvantages, and you can choose the appropriate auction mechanism based on the characteristics of the items being auctioned.

A reverse-bid auction mechanism is one in which all bidders submit a single bid to the auctioneer and the highest bidder wins the auction. The advantage of the bidding auction mechanism is that bidders have a psychological cue as to their own valuation and can play to their advantage as much as possible. However, there can be a "winner's curse" where the winning bidder overbids. Bidding is appropriate for items with obvious qualities or utility characteristics, such as flowers and works of art.

In the Dutch auction mechanism, the price is adjusted downward at the beginning of the auction until a bidder is willing to bid. If more than one bidder bids, the highest bidder wins the auction, and his or her bid is equal to the reserve price of the auction. The advantage of the Dutch auction mechanism is that it allows the true value of the auctioned item to be determined and prevents the "winner's curse". However, the disadvantages of the Dutch auction mechanism are that it takes a long time and is not very efficient. Dutch auctions are suitable for large numbers of items with little variation among them.

The sealed-first-price auction mechanism means that each bidder submits a sealed bid, and the highest bidder wins the auction, with his or her bid being the price of the auction. This mechanism has the advantage of ensuring that the winning bidder pays the actual valuation. However, it can also lead to biased valuations of items by auction participants and is prone to unfair competition. The first-price sealed-auction mechanism is suitable for situations where there is a high degree of consistency in bidders' valuations of the items.

In the second-price sealed auction mechanism, each bidder also submits his or her own bid in a sealed auction, but the winning bidder is required to pay the second-highest bid. The advantage of this mechanism is that it avoids the "winner's curse" and ensures that bidders submit their own true valuations. However, the disadvantage of the second-price sealed auction mechanism is that bidders may have to submit complex strategies to determine their bids. The second-price sealed auction mechanism is suitable for situations where bidders need to be highly incentivized to bid.

Overall, the choice of auction mechanism should be based on the characteristics and purpose of the auction in order to fully exploit the competitive aspects of the auction.

5.2 Analyzing the Advantages and Disadvantages of Different Auction Mechanisms

Bidding auctions, Dutch auctions, first-price sealed auctions, and second-price sealed auctions are several commonly used auction mechanisms, each of which has its own advantages and disadvantages. The following is an analysis of their main advantages and disadvantages:

1. Competitive Bidding Auctions

Advantages: A competitive bidding auction mechanism allows bidders to determine their own valuation and potentially obtain a better price. In addition, due to the simplicity of the reverse-bid auction mechanism, the entire auction process can be completed relatively quickly.

Disadvantages: Auctions can suffer from the "winner's curse," where the winning bidder may overbid. In addition, it is not suitable for certain items, such as valuable or unique items, where the auction may result in the nominal sale of a very highly valued item.

2. Dutch Auction Mechanism

Advantages: The Dutch auction mechanism allows the market value of an item to be determined while preventing the winning bidder from overbidding. In addition, the Dutch auction mechanism is suitable for bulk sales of a class of goods.

Disadvantages: The full process of the Dutch auction mechanism is lengthy, especially when there are a large number of bidders, which can lead to some iterations and delays. In addition, because sellers are required to quote a reserve price from the outset, individual buyers may respond by purchasing goods at unreasonable prices.

3. First-price sealed-bid auction mechanism

Advantages: The first-price sealed auction mechanism is relatively simple. In addition, it allows the winning bidder to bid the same amount as their valuation, which means that bidders will not be uncomfortable with the level of their bids.

Disadvantages: The first-price sealed-bid auction mechanism suffers from errors in item valuation. In addition, this mechanism has fairness problems, i.e., since all bids are not known to the bidders, it may leave some bidders with insufficient information.

4. Second Price Sealed Auction Mechanism

Advantages: The second price sealed auction mechanism prevents overbidding and avoids the problem of "winner's curse" of overbidding for goods that bidders want to acquire. It is also fair and reliable.

Disadvantages: Second price sealed-bid auctions require bidders to have a specific bidding strategy or a detailed understanding of other bidders' strategies in order to make an optimal bid. In addition, since bidders are required to submit more than one bid, this can affect bidder performance.

Overall, different auction mechanisms have their own characteristics and scope. In practice, the most appropriate auction mechanism should be selected based on the characteristics and purpose of the items being auctioned.

VI. Visualizing Participants' Revenues Under Different Auction Mechanisms

6.1 Bidding Auctions

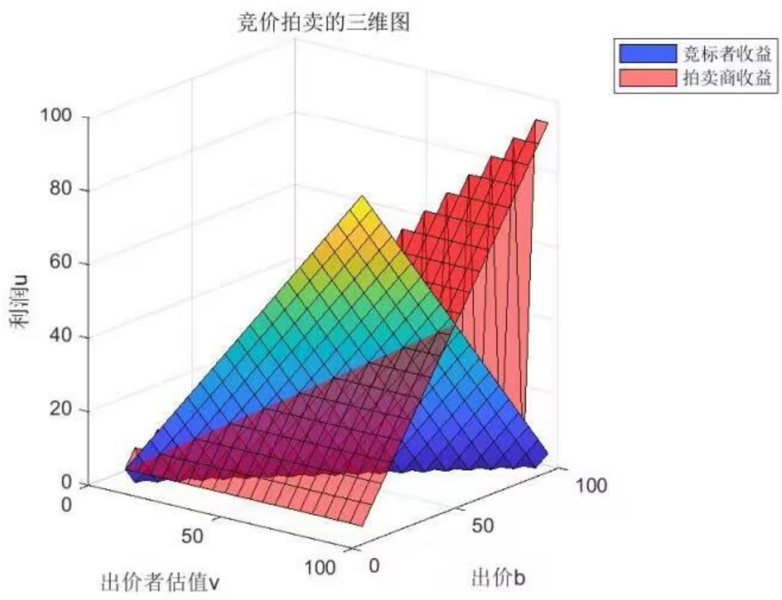

Figure 1: Three-dimensional view of a bidding auction

The 3D graph output above is a visual representation of the relationship between bidders' valuations, bids, and the profit of the bidding auction. The horizontal axis of this image is the bidder's valuation of the product, the vertical axis is the bid price, and the price paid by the winner of a reverse auctions is equal to the next highest bid, so the vertical axis of this image cannot exceed the horizontal plane at the top. Looking at the graph, we can roughly see that the entire three-dimensional graph is divided into two parts; the blue area represents the bidder's payoff, which is an area with a conical amplitude line, and the auctioneer's payoff is the red area, a plane that runs from left to right.

By analyzing this graph, we can draw the following conclusions and observations: As the bidder's valuation (horizontal axis) increases, both the bidder's and the auctioneer's payoffs increase, indicating that the item being bid on is of high value to the bidder. As the number of bids (vertical axis) increases, the return to the bidder decreases while the return to the auctioneer increases, suggesting that the bidder must consider the balance between his own return and the auctioneer's interest. In this setting, the auctioneer can earn higher returns while the bidders' returns are relatively low, so bidders need to carefully consider their bidding strategy in order to minimize their costs and win the competition. Because the relationship between bidder valuation and bid is not linear, the shape of the "skinny" triangle is created, indicating that small changes in bids can have large effects in certain valuation ranges.

In summary, this three-dimensional graph provides a realistic view of the impact of various factors (e.g., bidder valuation, bids, etc.) on the returns to bidders and auctioneers in a reverse auction, and the graphical information is useful for better understanding the characterization and analysis of reverse auctions.

6.2 Dutch Auction

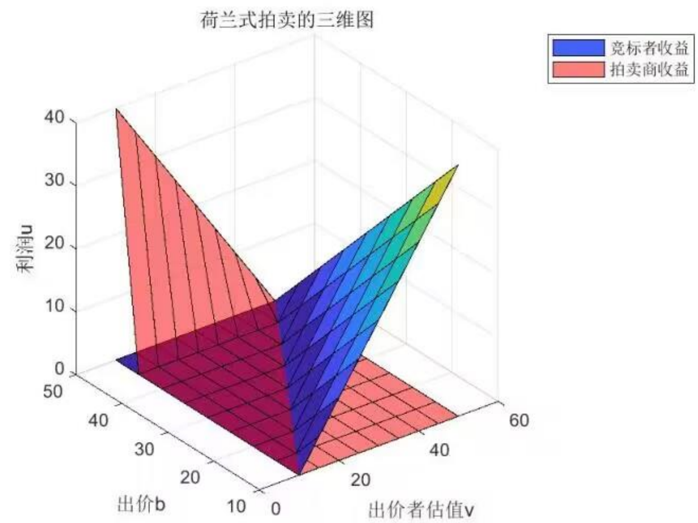

Figure 2 : Three-dimensional view of a Dutch auction

The 3D graph output above is a visual representation of the relationship between bidders' valuations, bids, and profits in a Dutch auction. The horizontal axis of this image is the bidders' valuations of the product, the vertical axis is the bids, and the price is always decreasing in a Dutch auction, so it cannot exceed the top horizontal plane on the vertical axis of this image. Looking at the graph, we can see that the entire 3D graph slopes gently from the upper right corner to the lower left corner.

By analyzing this graph, we can make the following conclusions and observations: As the bidder's valuation (horizontal axis) increases, both the bidder's and the auctioneer's returns increase, indicating that the item being bid on is of high value to the bidder. As the price (vertical axis) decreases, the return to the bidder decreases while the return to the auctioneer gradually increases, suggesting that in a Dutch auction, the bidder needs to win as quickly as possible before it is perceived as exceeding the market equilibrium price.

In this setting, auctioneers are more likely to earn higher returns and bidders relatively lower returns over time, so bidders must carefully consider their bidding strategy to minimize costs and win the competition.

6.3 First Price Sealed Auction

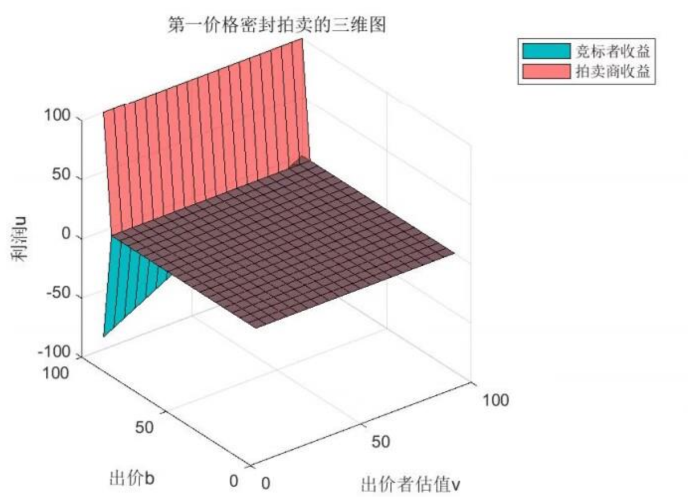

Figure 3: Three-dimensional view of the first-price sealed auction

The following conclusions and observations can be drawn from this figure: As the bidder's valuation (horizontal axis) increases, both the bidder and the auctioneer receive higher returns, indicating that the item being bid on is of high value to the bidder. In the first-price sealed-bid auction, the highest bidder wins, so the peak of the blue area is above the diagonal in this figure, and the rate of decline is closely related to price. The auctioneer's payoff is equal to the highest bid, so the red area is a flat horizontal area above the horizontal line of the highest bid.

In summary, this 3D graph shows the relationship between different factors (e.g., bidders' valuations, different bids, etc.) in the first price-fixing auction, and its graphical information will be useful in analyzing the impact of bidders' and auctioneers' payoffs.

6.4 Second Price Sealed Auction

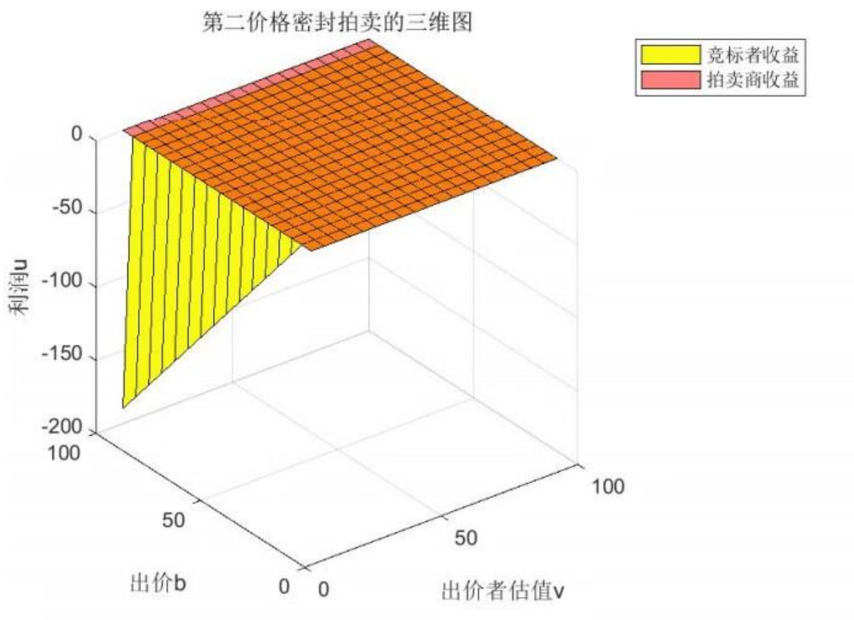

Figure 4: Three-dimensional view of the second-price sealed auction

The following conclusions and observations can be drawn from this figure: As the bidder's valuation (horizontal axis) increases, both the bidder and the auctioneer receive higher returns, indicating that the item being bid on is of high value to the bidder. In the second-price sealed-bid auction, the highest bidder wins and purchases the product at the next highest price, so the peak of the blue area is above the diagonal in this figure, and the peak quickly drops to the next highest value. In this setting, the auctioneer's payoff is equal to the next highest bid, so the red area is a flat horizontal area above the horizontal line of the next highest bid. In summary, this 3D graph shows the relationship between different factors (e.g., bidder valuations, different bids, etc.) in the second price-sealing auction, and its pictorial information will be useful for analyzing the impact of bidder and auctioneer payoffs.

VII. Conclusion

In this thesis, the researchers revealed the impact of different auction mechanisms by analyzing several auction models in depth, and provided an analysis of participants' optimal strategies under different auction mechanisms.In general, the research results of this thesis can be summarized as follows:

1. In the bidding auction model, the highest bidder wins if the valuation is correct. However, the bidder can gain additional revenue by obtaining consumer surplus if the bid is lower than the competitors' valuation or if the bid is higher than the actual value of the good.

2. In the Dutch auction model, participants can gradually reduce their bids based on previous bids and bid according to the competition. In this model, the auctioneer's pricing strategy and each bidder's bidding strategy can affect the final transaction price and revenue.

3. In the first-price sealed auction model, the highest bid from a particular bidder is considered the most viable strategy. However, if the bid is too high, the bidder may lose its comparative advantage and pay an unnecessarily high price. 4.

4. In the second price sealed auction model, the highest bidder still wins, but pays the price of the next highest bidder. This mechanism eliminates the need for bidders to misrepresent their estimates, thereby reducing risk and improving efficiency.

Taken together, the choice of an appropriate auction method is critical to both the auction objectives and the market environment, and given the pursuit of competitors, bidders need to choose an appropriate strategy. In addition, risks and unknown information need to be properly managed and appropriately publicized during the auction process to ensure the fairness and efficiency of the auction. Overall, the study provides valuable theoretical support for analyzing the auction mechanism and participants' strategies, as well as an effective reference for making auction decisions in practical situations.

The innovative points of this paper mainly focus on the following aspects:1. The optimal strategies under multiple auction mechanisms (bidding auction, Dutch auction, first-price sealed auction, second-price sealed auction) are comprehensively analyzed and compared, and the optimal bids and transaction results under different auction methods are discussed in depth.2. This paper uses the trapezoidal distribution to approximate instead of the uniform distribution to establish the relevant model, and analyzes the classical second-price sealed auction game model, and concludes that the optimal strategy of bidders maximizes the expected returns.3. This paper not only analyzes the auction mechanism from the perspective of game theory, but also explains the assumptions and symbols in detail; in addition, this paper also discusses and analyzes the transaction results under the auction mechanism in depth, which has certain guiding significance for market participants to understand the transaction costs and benefit distribution under the auction mechanism. In conclusion, this paper comprehensively analyzes and compares the optimal strategies under different auction mechanisms from different perspectives, and puts forward a series of innovative research results, which provide valuable references for decision makers in the auction market.

In this dissertation research, although the authors provide in-depth analysis of different auction mechanisms and participants' strategies, there are still the following shortcomings:

1. some complex factors in real-world situations are not fully considered. For example, information asymmetry among participants, changes in the number of competitors, fluctuations in market demand, and other factors, all of which may affect auction strategies and transaction prices, have not been adequately considered in this paper.
2. Only four types of auction mechanisms are analyzed, so other different auction mechanisms and their influencing factors may not have been explored in depth, for example, other auction models such as VCG.
3. The research mainly focuses on the strategy analysis of rational participants, which to some extent cannot fully represent the real situation, because in fact the participants' thinking and behavior are still influenced by many factors, which may not be obtained by purely rational analysis.

References

[1]. Zou Bin. A review of auction theory[J]. Science Consulting(Science and Technology-Management),2013,No.346(11):19-20.

[2]. LI Sanxi,WANG Taiming. A review of auction theory research[J]. China Science Foundation,2021,35(01):2-3.DOI:10.16262/j.cnki.1000-8217.2021.01.002.

[3]. C. Zhou, H. Li, L. Zhang, et al. Optimal recommendation strategies for AI-powered e-commerce platforms: A study of duopoly manufacturers and market competition. Journal of Theoretical and Applied Electronic Commerce Research, 2023, 18(2): 1086-1106.

[4]. Tian Miaozhu. Research on auction algorithm and its application [D]. Qingdao University,2015.

[5]. C. Zhou, X. Li, Y. Ren, et al. How do fairness concern and power structure affect competition between e-platforms and third-party sellers? IEEE Transactions on Engineering Management, 2023, DOI: 10.1109/TEM.2023.3262318.

[6]. Ma Guoshun,Yang Liying,Liu Wenwen. Game and equilibrium analysis of primary seal price auction based on triangular distribution[J]. Industrial Technology Ji,2010,29(02):74-76.

[7]. DU Li, HU Qiying. A Class of Online British Auctions:A Study of Customer Bidding Behavior[J]. Journal of Management Science,2006(03):31-38.

[8]. Zhu Wuxiang. Overview of auction theory development Auction theory development[J]. Chinese Auction,2022,No.190(05):65+64.

[9]. ZHAO Fengrong,WU Jun. First-price and second-price auction model based on inverse commission rate constraint[J]. Development Research,2021,No.214(03):121-128.DOI:10.13483/j.cnki.kfyj.2021.03.017.

[10]. DING Meisheng,GUO Suwen. Economic analysis of reduced-price auctions[J]. Business Times,2005(18):87-89+32.

[11]. C. Zhou, M. Leng, Z. Liu, et al. The impact of recommender systems and pricing strategies on brand competition and consumer search. Electronic Commerce Research and Applications, 2022, 53: 1-15.

[12]. Theoretical analysis of the secondary sealed auction mechanism[J]. Lin Rong, Zhang Yizhen. Zhejiang Social Science,1999(03)

[13]. MA Guoshun, WANG Ting. Game and equilibrium analysis of primary seal price auction based on trapezoidal distribution[J]. Journal of Southwest University(Natural Science Edition),2016,38(01):78-84.DOI:10.13718/j.cnki.xdzk.2016.01.012.

[14]. J. Yu, C. Zhou, G. Leng. Is it always advantageous to establish self-built logistics for online platforms in a competitive retailing setting? IEEE Transactions on Engineering Management, 2024, 71: 1726-1743.

[15]. HAN Jiulin,MA Li. Research on optimal bidding price based on game theory[J]. China Business Journal,2022,No.854(07):106-109.DOI:10.19699/j.cnki.issn2096-0298.2022.07.106.

[16]. Cheng Ran,Wu W S,Sun Zhuorui. Study on the improvement of bidding mode of government investment projects under the theoretical framework of second-price sealed auction[J]. Industrial Technology and Economics,2011,30(03):137-141.

[17]. Gao Guangxin, Fan Zhiping. Optimal bidding strategy for one-level sealed auction considering bidder's regret[J]. Management Science,2016,29(01):1-14.

[18]. Zeng Xianke,Feng Yuqiang. Reverse multi-attribute British auction model and optimal bidding strategy based on asymmetric bidders[J]. Systems Engineering Theory and Practice,2012,32(04):769-775.

[19]. C. Zhou, N. Ma, X. Cui, Z. Liu. The impact of online referral on brand market strategies with consumer search and spillover effect. Soft Computing, 2020, 24(4): 2551-2565.
[20]. YANG Yingmei, WANG Wenju. Game analysis of optimal reservation price and quotation strategy in first price sealed bidding[J]. China Price,2009,No.244(08):16-18+22.
[21]. J. Yu, J. Zhao, et al. Strategic business mode choices for e-commerce platforms under brand competition. Journal of Theoretical and Applied Electronic Commerce Research, 2022, 17(4): 1769-1790.
[22]. LI Jianbiao,WANG Minda,WANG Pengcheng et al. Risk aversion and finite corruption in primary sealed auctions-a model and experiment[J]. Management Science,2011,24(04):95-104.
[23]. An Xin,Chen Jiancheng. Generalized segmented pseudo-maximum likelihood estimation based on first-price auctions[J]. Systems Engineering,2015,33(11):101-106.
[24]. TIAN Guoqiang,LIU Chunhui. Finite corruption in sealed-price auctions or bidding[J]. Economic Research,2008,No.481(05):116-12
[25]. J. Yu, Z. Song, et al. Self-supporting or third-party? The optimal delivery strategy selection decision for e-tailers under competition. Kybernetes, 2023, 52(10): 4783-4811.
[26]. HUANG Zhangyou,CHENG Shuixiang,RUAN Lianfa. Research on bidding model based on second price sealed auction theory[J]. Construction Economy,2004(05):39-41.
[27]. CHENG RAN, LI QI, HAN FENG. Research on Bidding Mode of Government Investment Projects-Based on the Theory of Second Price Sealed Auction[J]. Technology and Innovation Management,2011,32(02):143-147.DOI:10.14090/j.cnki.jscx.2011.02.013.
[28]. Lv Huijuan. A brief discussion on second-price sealed auctions[J]. Contemporary Economy,2007,No.185(05):63-64.
[29]. Mukun Cao, Qing Hu, Melody Y. Kiang & Hong Hong. (2020) A Portfolio Strategy Design for Human-Computer Negotiations in e-Retail. International Journal of Electronic Commerce 24:3, pages 305-337.
[30]. Bjarne Brendstrup, Harry J. Paarsch,Identification and estimation in sequential, asymmetric, English auctions,Journal of Econometrics,Volume 134, Issue 1,2006,Pages 69-94
[31]. X. Cui, et al. Interaction between manufacturer's recycling strategy and e-commerce platform's extended warranty service. Journal of Cleaner Production, 2023, 399: 1-16.
[32]. Deng, Fei;Liu, Sifeng;Fang, Zhigeng Grey Systems : Theory and Application, 19 Oct 2021, Vol. ahead-of-print, Issue ahead-of-print, pages 741 – 753
[33]. C. Zhou, W. Tang, R. Zhao. Optimal consumption with reference-dependent preferences in on-the-job search and savings. Journal of Industrial and Management Optimization, 2017, 13(1): 503-527.
[34]. C. Zhou, Y. Xu, J. Yu. Logistics sharing mode selection and pricing strategies in platform supply chain. 2023 IEEE 7th Information Technology and Mechatronics Engineering Conference, 2023: 606-610.
[35]. Christian Barrot ， Sönke Albers ， Bernd Skiera & Björn Schäfers （2010） Vickrey vs. eBay： Why Second-Price Sealed-Bid Auctions Lead to More Reali-Demand Functions， International Journal of Electronic Commerce， 14：4， 7-38， DOI：10.2753/JEC1086-4415140401