International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 12 Issue 11 ǁ November 2024 ǁ PP. 134-138

The nature of the change in the lifting force acting on a porous cylinder

U. Dalabaev
University of World Economy and Diplomacy, Tashkent, Uzbekistan
Corresponding Author udalabaev@mail.ru

ABSTRACT
The paper investigates the nature of the lifting force acting on a porous cylindrical particle. The porous cylinder is located perpendicular to the flow and is flown by a viscous liquid in a flat channel. The calculation of the lifting force acting on the cylinder is made for different values of the Reynolds number, porosity and their location in the flow.
KEYWORDS Flow, *Darci law, Rakhmatulin model, porous media, numerical method*.

Date of Submission: 13-11-2024 Date of acceptance: 26-11-2024

I. INTRODUCTION

In many technological processes, viscous fluid flows are observed in areas with and without porous inclusions. In this regard, there is a need to study such phenomena [1-3].

The experimental results of Segre-Silberberg [4] on particle migration gave impetus to many researchers to search for an analytical expression for the lift force acting on a particle from the flow. An analytical expression for the lift force for a sphere in an unlimited linear flow was obtained in the work of Saffman [5]. There were attempts to expand Saffman's formula by many authors [6-12]. In [6,7], a correction to Saffman's lift force is given. In the work of Asmolov [8], the formula for the transverse force acting on a spherical particle in a laminar boundary layer was expanded using the method of matched asymptotic expansions. The magnitude of the force is expressed through a multiple integral. In [10], using Asmolov's corrections [8,11], the motion of a particle in the Couette layer was investigated. Work [11] is devoted to the effect of inertia and the presence of a boundary on the lift force. The effect of inertia on the lift force in a linear velocity field was investigated in [11] using the numerical Fourier method. In [13], the flow around a cylinder by an infinite two-dimensional flow was numerically investigated based on the non-stationary Navier-Stokes equation in the variables of the stream function and vortex.

Important elements of such studies are the mathematical description of such phenomena. The complexity of such flows lies in the consideration of the flow in two subdomains: the free zone and the porous subdomain. In the free zone, the flow is modeled by the Stokes/Navier-Stokes equations, and in the porous subdomain there are various approaches to modeling the flow in the porous region [1-3].

There are two directions of modeling such flows in the literature. The first direction is characterized by modeling the flow with its own equations in each subdomain. The flow in the free zone is modeled using the Navier-Stokes/Stokes equation systems. There are many equations to describe the filtration flow through a saturated porous layer [1-3]. In particular, the classical Darcy equation [1] is widely used to describe the flow through a porous layer. The Brinkman filtration equation is similar in type to the Navier-Stokes equation. When using a two-region model, a choice of interregional boundary arises. The Beavers-Joseph or Saffman conditions are used as the interboundary condition [14].

The second direction is characterized by the use of a single differential equation for the entire region [15,16]. In this case, the use of the interphase boundary is eliminated. In this case, the use of the equation in the free zone turns into the Navier-Stokes equation.

In this article, we use a single equation for the entire region, and the model is obtained on the basis of a two-velocity continuum, first described by H.A. Rakhmatulin [18]. The study of filtration motion based on a two-speed model was also carried out in works [19,20], in which the forces of interaction between the phases corresponded to Darcy's law, while observing the Kozeny-Carman relation.

.

II. MATHEMATICAL MODEL

Let a porous cylinder be located in the region of a viscous flow (Fig. 1). To describe the flow inside and outside the porous cylinder, we proceed from the interpenetrating model of two-phase media. If in the two-velocity model for describing the flow of two-phase media we assume that 1) there is no motion of the discrete phase, 2) the deformation of the discrete phase can be neglected, 3) there is no heat and mass transfer between the phases,

4) the flow is incompressible, then the equations of motion and continuity in dimensionless Cartesian coordinates have the form:

$$\varepsilon u\frac{\partial u}{\partial x}+\varepsilon v\frac{\partial u}{\partial y}=-\frac{\varepsilon}{\mathrm{Re}}\frac{\partial p}{\partial x}+\frac{4}{3\mathrm{Re}}\frac{\partial}{\partial x}\left(\varepsilon\frac{\partial u}{\partial x}\right)+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial y}\left(\varepsilon\frac{\partial u}{\partial y}\right)-$$
$$-\frac{2}{3\mathrm{Re}}\frac{\partial}{\partial x}\left(\varepsilon\frac{\partial v}{\partial y}\right)+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial y}\left(\varepsilon\frac{\partial v}{\partial x}\right)-\frac{D^2}{\mathrm{Re}}\frac{(1-\varepsilon)^2}{\varepsilon^3}u, \tag{1}$$

$$\varepsilon u\frac{\partial v}{\partial x}+\varepsilon v\frac{\partial v}{\partial y}=-\frac{\varepsilon}{\mathrm{Re}}\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x}\left(\varepsilon\frac{\partial v}{\partial x}\right)+\frac{4}{3\mathrm{Re}}\frac{\partial}{\partial y}\left(\varepsilon\frac{\partial v}{\partial y}\right)-$$
$$-\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x}\left(\varepsilon\frac{\partial u}{\partial y}\right)+\frac{2}{3\mathrm{Re}}\frac{\partial}{\partial y}\left(\varepsilon\frac{\partial u}{\partial x}\right)-\frac{D^2}{\mathrm{Re}}\frac{(1-\varepsilon)^2}{\varepsilon^3}v, \tag{2}$$

$$\frac{\partial \varepsilon u}{\partial x}+\frac{\partial \varepsilon v}{\partial y}=0, \tag{3}$$

where u and v are the longitudinal and transverse components of velocity, p is the pressure, x, y are the Cartesian coordinates; ε is the porosity of the layer, $\mathrm{Re}=hU\rho/\mu$ is the Reynolds number, U is the average volumetric velocity, h is the channel width, ρ and μ is the density and viscosity of the flow, $D=\sqrt{\alpha}h/d$ is a dimensionless number, d is the characteristic size of the porous medium, α is the proportionality coefficient.

In the equations of motion (1), (2), the dimensionless variables are related to the dimensional ones as follows:

$$x=x'/h,\ \ y=y'/h,\ \ u=u'/U,\ \ v=v'/U,\ \ p=\mathrm{Re}\,p'/\rho U^2 .$$

Here the variables with primes are dimensional.

Note that if we set $\varepsilon=1$ in equations (1) - (3), we obtain the Navier-Stokes equation for an incompressible fluid. This circumstance allows us to study the problem based on equations (1) - (3) in the entire region occupied by the porous layer and the fluid. We take the value ε to be equal to one at the points of the region occupied by the fluid, and at the points of the porous region we set it equal to the porosity of the medium.

We direct the axis x along the bottom of the channel, and the axis y perpendicular to it. At the entrance to the channel, a symmetrical velocity profile is specified. We take the channel height as the characteristic dimension, and the average volumetric velocity over the channel cross-section as the characteristic velocity.

In [21], the flow around a solid cylinder is numerically investigated using the Navier-Stokes equation.

Here we investigate the nature of the change in the lifting force for a porous cylinder located in a flat channel and check that the above conclusion holds for a porous cylinder.

Boundary conditions. No-slip conditions are set on the channel walls:

$u=v=0.$

At the pipe inlet, we set a Poiseuille flow:

$u=6y(1-y),\ v=0.$

At a sufficient distance from the inlet section, we set a soft condition:

$$\frac{\partial u}{\partial x}=0,\ \frac{\partial v}{\partial y}=0.$$

III SOLUTION METHOD

To solve the systems of equations (1) - (3) under the corresponding boundary conditions, the SIMPLE algorithm [22] with the corresponding generalizations is used. A non-uniform grid, condensed near the cylinder, is used.

The calculation is performed in the 3x1 region for different values of the Reynolds number: Re 1:500, and for a cylinder radius of r = 0.1 and for different porosities. The particle center is located at points $(1,\ y_s)$, where us varies from 0.1 to 0.5 with a step of 0.05.

Calculations were made for Re = 1,10,50,70,80,100,200,300,400,500. For a specific Reynolds number and cylinder location, calculations were made with the required accuracy 10^{-4}. In all calculations, $\delta/r_h=0.1$ was assumed (r_h is the characteristic size of the porous cylinder).

The dimensionless lifting force per unit length of the cylinder was calculated using the formula:

$$F_y = \int_0^{2\pi} (1-\varepsilon) \left[\left(-p - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial \vartheta}{\partial x}\right) + 2\frac{\partial \vartheta}{\partial y} \right) \sin\theta + \left(\frac{\partial u}{\partial y} + \frac{\partial \vartheta}{\partial x} \right) \cos\theta \right] r d\theta$$

The dimensionless force is related by the ratio: $F_y = \frac{F'_y}{\mu U L}$. Here μ is the flow viscosity, L is the cylinder length, F'_y is the dimensional lifting force.

IV RESULTS AND DISCUSSIONS

Figure 1 shows the change in the lift force for different porosities.

Curve 1 in Figure 2 is obtained based on the Navier-Stokes equation. The nature of the change in lift forces for a porous cylinder, for the porosities considered, coincides with the nature of the solid cylinder. A similar nature of the change in lift force is observed for other Reynolds numbers. In Figure 2, the nature of the change in lift force

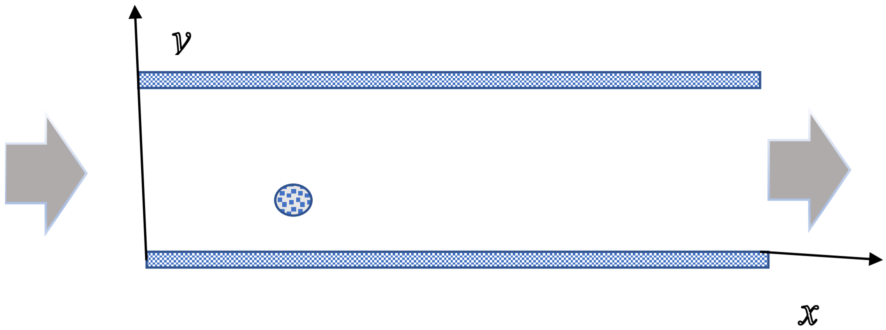

Fig.1 Flow region

is given for half the cross-section of the channel, since the change in lift force over the cross-section is an odd function relative to the channel axis. Therefore, Figure 1 shows the results for the lower part of the channel. It is evident from the graph that the porosity of the cylinder significantly affects the lift force, while with increasing porosity, the lift force decreases. If we place the center of the cylinder at points with coordinates (1; 0.5), due to the symmetry of the flow, the lift force is equal to the fill (by the way, this condition is necessary for the correctness of the calculation). Note that the influence of the Poiseuille flow is maximum when the cylinder is located near a solid boundary, regardless of the porosity of the cylinder.

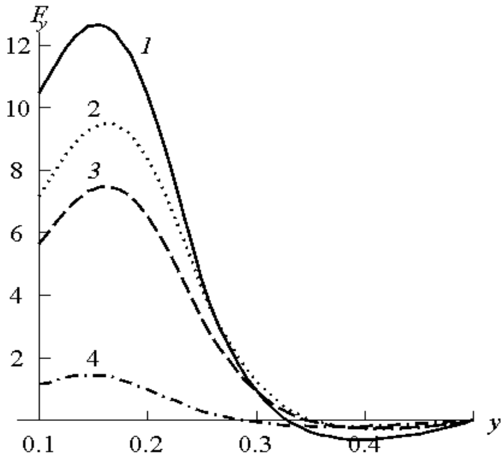

Fig. 2 Change in lift force at different porosities. 1-ε=0; 2- ε=0.1; 3- ε=0.3; 4-ε=0.7; Re=100.

Fig. 3 shows the change in lift force at different Reynolds numbers. The behavior of the lift force for a porous cylinder coincides with the behavior of the lift force for a solid cylinder [21], although the range of change for a porous cylinder is reduced. An increase in the Reynolds number will lead to an increase in the range of change of the lift force. From Fig. 3 it is clear that the lift force vanishes not only on the channel axis (due to the symmetry of the flow), but also at other specific points changing with a change in the Reynolds number. Fig. 3 shows that the location of the point where the lift force is equal to zero approaches the wall with increasing Reynolds number. On the other hand, with increasing Reynolds number, the maximum point of the lift force approaches the wall. Starting from Re 70, for all the considered sizes of the porous cylindrical particle ($0.1 \le \varepsilon \le 0.7$), another point appears where the lift force is equal to zero (Fig. 3).

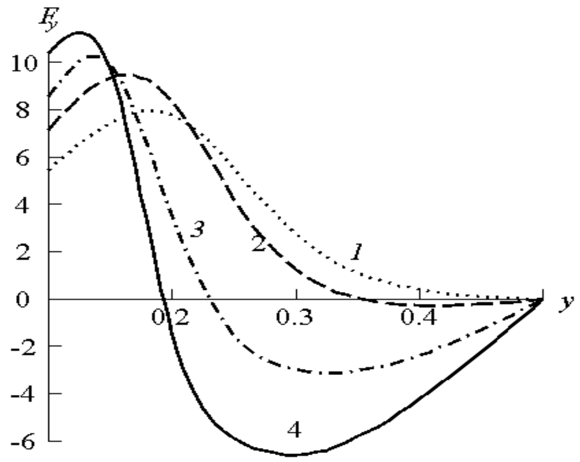

Fig. 3 Change in lift force at different Reynolds numbers.
1-Re=70, 2-Re=100, 3-Re=200, 4-Re=300, ε=0.3.

However, for porosity ε=0.8 and 0.9 up to Reynolds numbers Re=500, no equilibrium point was observed (Fig. 4). In this case, the maximum value of the lift force does not approach the wall, but is in the middle between the channel axis and the channel edge.

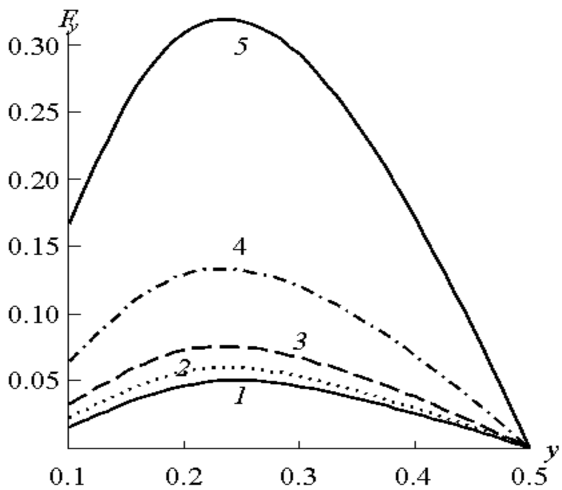

Fig. 4 Change in lift force at different Reynolds numbers, 1-Re=50, 2-Re=70, 3-Re=100, 4-Re=200, 5-Re=500, ε=0.9

Fig. 5 shows the change in the ratio of forces $Z = F_y / F_x$ (F_x is the drag force), which shows that in the boundary zones, at low porosities, the lift force is ≈ 50% of the drag force. With increasing porosity, although the contribution of the lift force decreases, this factor should still be taken into account.

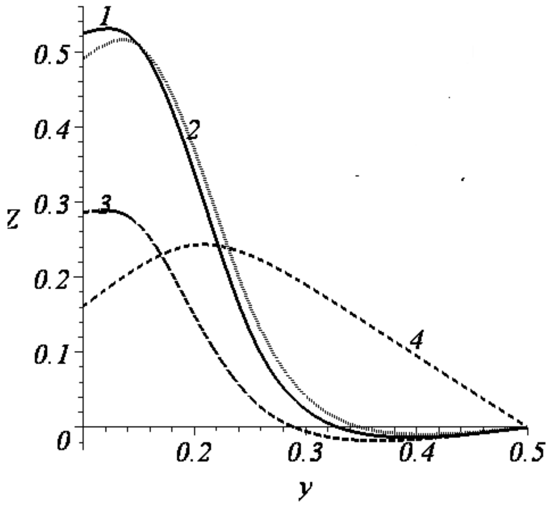

Fig. 5. Change in the ratio of resistance forces.

III CONCLUSION

The maximum value of the lifting force with increasing Reynolds number approaches the wall at ε ≤ 0.7. In this case, this maximum value is the closer to the wall, the smaller the particle radius. Starting with Re□70, additional equilibrium points are formed symmetrically located relative to the channel axis at ε ≤ 0.7. Additional equilibrium points with increasing Reynolds number approach the walls. Within the framework of this model, at Re ≤ 500 for porosities ε ≥ 0.8, no additional equilibrium points are observed. Near the wall, for small values of cylinder porosity, the lifting force is 50% of the drag force.

REFERENCE

[1]. J. Bear, Dynamics of Fluids in Porous Media (Elsevier, Amsterdam, 1972).
[2]. P.Y. Polubarinova-Kochina, Theory of Groundwater Movement (Princeton University Press, Princeton, N.J., 1962).
[3]. D. A. Neild and A. Began, Convection in Porous Media (Springer International Publishing AG (2017) pp. 1-35
[4]. Segre G., Silberberg A. Behavior of macroscopic rigid spheres in Poiseuille flow . J. Fluid Mech. 1962. Vol.14. pp. 115-135.
[5]. Saffman P. G. The lift on a small sphere in a slow shear flow./ J. Fluid Mech., 1965. Vol. 22.pp. 385-400.
[6]. Huber N. and Sommerfeld M. Modeling and numerical calculation of dilute-phase pneumatic conveying in pipe system./ Powder Technology. 1998. Vol. 99. pp. 90-101.
[7]. Mei R. An approximate expression for the shear force on a spherical particle at finite Reynolds number./ Int. J. of Multiphase flow, 1992. Vol. 18(1). Pp. 145-147.
[8]. Asmolov E. S. On the transverse force acting on a spherical particle in a laminar boundary layer./ Izv. AN SSSR. Mechanics of liquid and gas. 1989. No. 5. P. 66-71.
[9]. Asmolov E. S. On the motion of a dispersed impurity in a laminar boundary layer on a flat plate./ Izv. RAS. Mechanics of liquid and gas. 1992. No. 1. P. 66-73.
[10]. Naumov V. A. Effect of Saffman lifting force on particle motion in the Couette layer // Engineering Physics Journal. 1995. Vol. 68, No. 5. P. 840-844.
[11]. Cherukat P. and McLaughlin J. B. Inertial lift on a rigid sphere in a linear flow field near a flat wall . J. Fluid Mech. 1993. Vol. 263. Pp. 1-18.
[12]. Cherukat P., McLaughlin J. B. and Dandy D. S. A computational study of the inertial lift on a sphere in a linear shear flow field // Int. J. Multiphase Flow. 1999. Vol. 25. p. 15-33.
[13]. Mazo A. B., Morenko I. V. Resistance and rotational properties of gratings of circular cylinders at low and moderate Reynolds numbers // IFZ. 2004. Vol. 77, No. 2. P. 75-79.
[14]. Beavers G. S. and Joseph D. D., Boundary Conditions at a Natural Permeable Wall, J. Fluid Mech., vol. 30, pp. 197-207, 1967
[15]. S.V. Polyakov, M.A. Trapeznikova, A.G. Churbanov, N.G. Churbanova. Calculation of incompressible flows in the porous body-free flow system / Preprints of the Institute of Mechanical Engineering named after. M.V.Keldysh. 2021. No. 71.19 p.m.
[16]. Industrial and Systems Engineering, 2020 Vol. 34 No. 3, pp. 283 – 300,
[17]. F. J. Valdes-Parada and D. Lasseux. A novel one-domain approach for modeling flow in a fluid-porous system including inertia and slip effects. Phys. Fluids 33, 022106 (2021).
[18]. H.A.Rakhmatulin. Fundamentals of gas dynamics of mutually propagating liquids: PMM. 1956. v. 20. Vyp. 2
[19]. Dalabaev U. Flow structure during flow through a stationary granular layer. Engineering physical journal. 1997. Vol. 70, No. 3. P. 390-393.
[20]. Dalabaev U. On an approach for viscous flow modeling about porous media. Thermophysics and Aeromechanics. 1998. Vol. 5. No 1. Pp. 45-50.
[21]. Dalabaev U. Numerical study of the nature of the lifting force acting on a cylindrical particle in a Poiseuille flow of a flat channel. IFZ. 2011. Vol. 84, No. 6. Pp. 1288 – 1292
[22]. Patankar S. Numerical methods for solving heat transfer problems and fluid dynamics. M. Energoatomizdat. 1984.