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Application of Dynamic Programming in Capacity Allocation

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Abstract: *This work demonstrates the application of dynamic programming decomposition method in capacity allocation using line four of Seven Up Bottling Company as a case study. On daily basis, the processing capacity is kept constant and orders of different priorities were made randomly and decisions had to be made in order to maintain viability. Jobs waiting to be processed accumulate holding cost due to the level of priority. The main objective was to obtain the optimal expected revenue over a finite planning horizon of 200 days- 100 days for lean period and 100 days for peak period of production. The constraints are taken into consideration using the approximate dynamic programming decomposition method that breaks down the dynamic program into peak and lean periods of production. Results obtained show that the dynamic programming decomposition method indicates the optimization of the total expected cost while considering the peak period of production and performs better than the First Come, First Served approach with a reasonable performance gap.*
Keywords*: Dynamic Programming, Decomposition, Capacity Allocation, Optimal Revenue, Peak period and Lean Period.*

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I. Introduction

The major challenge of any company is to maintain viability at any point in time, whilst reducing cost and increasing productivity. This will ultimately increase profit[1]. The approach of first come first served, which has been in place, does not produce optimal result. Orders placed by the customers are not properly attended to. There exists the need for a more optimal method, aimed at optimizing the use of available resources[2]. Most organizations encounter the problem of allocating limited processing capacity to jobs that arrive randomly over time. This is found in production environments where a company utilizes a fixed production resource to serve customers demand with different profit margins. The major trade-off in these is between using the available processing capacity for a low-priority job that is currently available and delaying the processing of low-priority job with the hope that a high-priority job can use the available capacity later. This trade-off is further complicated by the fact that the processing capacity is sharable. If the processing capacity that is available on a day is not used by the end of that day, then it ends up being wasted [3]. During summer (dry Season), the overall product demands usually exceed the allowable production capacity. Based on the past production records of the company of the company Aba always lead in the demand of this selected product, while Isialangwa has the lowest demand within the major distribution centers, this results in different profit margins from the centers, thus, priority is given to their job orders accordingly. Production activities are scheduled to meet time duration of forty eight hours with a maximum duration of fourteen days subject to available daily production capacity [4]. Priority is given to different job orders based on profit margin, distance, need/urgency level.

Job orders should be scheduled on which days due to the limited production resources; the company has an option of either delaying an order with a low priority level that is currently available with the hope that a job with higher priority level can use the available capacity or processing less the demanded quantity. Jobs that are waiting to be processed incur holding cost while rejected/delayed job incur penalty cost[5]. Production, inventory and supply chain problems are optimization problems and much consideration is required to make sure that the sub optimization contributes to the overall optimization of the activities of which they form a part. Constrained optimization is a procedure for determining optimal allocation of resources. Dynamic programming decomposition method have found practical application in most facet of business that involves constraints like time, random orders and various job scheduling priorities. Dynamic programming is a useful mathematical technique for making a sequence of interrelated decisions. It provides a systematic procedure for determining the optimal combination of decisions.[6]. Applying proper optimization technique, an organization will continue to be viable at any point in time or season. Dynamic programming decomposition strategy which ultimately provides separable approximation that gives a lower bound on the optimal total expected cost and make the job of scheduling simpler over time. An approximate dynamic programming formulation strategy that is based on decomposing the different days in the planning horizon is used to address the capacity allocation problem. The approach starts with a deterministic linear programming formulation which is constructed under the assumption that the job arrivals take their expected values and can be scheduled in fractional number of jobs. In linear programming, there is one capacity constraint for each day in the planning horizon, ensuring that jobs scheduled on a day cannot exceed the daily available capacity [7]. Lagaragian relation is used to relax these capacity constraints for every day in planning horizon except for one particular day, so that deterministic linear program for a capacity only on one day will be obtained. The latter deterministic linear programming, in turn, indicates how to decompose the original dynamic programming formulation of the problem by the different days in the planning horizon.[8] Protection policies can be used to solve volume allocation limitations. This approach share some similarities with preferential first come first served Algorithm. When there are piles of jobs queuing up to be attended to, these polices set up function in the following manner: the volume is computed starting from the highest of them to the least. The work of the protection level here is to dully ascertain the volume which shouldn't be attended to and the ones to be left in order to be worked on later. This approach is employed in considering the choices of different customers in relation to the unfavorable factors which tend to limit their capacity [7]. Erdelyi and Topaloglu [7] proffered a solution in network management using dynamic programming. This approach is an advancement of lagragian system which can be applied overbooking problems in flights and hotel reservations. In this work, this approach is adopted in developing a model in a production plant (Seven Up Bottling Company).

When Dynamic Decomposition method is harnessed, it can be employed in raising the overall revenue in production of goods and services [10]. This work demonstrates how dynamic programming can be used to solve allocation problems whilst considering the constraints involved. A rectilinear programming system can give an optimum routine check of the scheme so as to devaluate the frequency of the constituents Asgarpoor (2004). Optimization model can be developed using an ideal and generic method which can be applied in based on the constraints and parameters involved. Duarte et al. (2006), Canto (2006). Tam et al (2006). Alardhi et al. (2007).

A job scheduling decision in capacity allocation model with job priorities can be developed using the available constraints and variables. When the model is decomposed as a problem in different days in the planning horizon and solved sequentially using dynamic programming method, an optimum we be achieved. This optimum will be in terms of maximum revenue and most efficient and effective use of resources coupled with appropriate timing[7].

II. Materials and Methods

i. Data is obtained from Seven Up Bottling Company Aba Plant.

ii. The mathematical model was formed while observing the constraints and conditions in order to show the problem to be solved

iii. The model is solved into objective value and total expected cost.

iv. The data is analyzed using MATHLAB by supplying the required values in the model.

v. The results are varied and used to plot graphs with the aid of MS Excel.

vi The results are compared with an existing technique and deductions are made.

Table 2.5: Data for the base problem

Variables	Description	Quantity
T	Total number of days	100

P	Total number of Priority level	3
S	maximum target duration	2
ß	Utility	4
Φ	Probability Distribution density	2
CV	Coefficient of variation	0.1

Considering a given set of days the set of possible priority levels for the jobs is P. The number of priority P jobs that arrive on day t is given by the random variable, so that D, (:P ϵ p) captures the job arrivals for day t. If a priority p job that arrives on day t is scheduled for day j, then a holding cost of h^p_{jt} will be incurred. The penalty cost of rejecting a priority p job that arrives on day t is $r,^p$. The convention h^p_{jt} ∞ used whenever it is infeasible to schedule a priority p job arriving on day t for day j. since it is infeasible to schedule a job for a day in the past, naturally h^p_{jt} ∞ Whenever j < t. similarly, if it is not feasible to schedule a job more than S days into the nature, then the constraint is captured by letting h^p_{jt} ∞ whenever j- t > or S, once a job is scheduled. For notational brevity, it is assumed that all jobs consume one unit of processing capacity, Given that it is day t, let$_{jt}$be the remaining capacity on day j so that $_t$ = captures the state of the remaining capacities as observed at the beginning of day t.

Given that it is on day t, ẋ = is the state variable. However, is for notational uniformity so that the state variable on different days ends up having the same number of dimensions.

Let be the number of priority p jobs that is scheduled for day j on day t so that captures the decisions that is taking on day t. in this case, the set of feasible decisions on day t is given by:

(1)

Where the first set of constraints ensure that the decisions made do not violate the remaining capacities and the second set of constraints ensure that the decisions that is made do not consulate the job arrivals. On the other hand, the total cost that is incurred on day t is given by:

(2)

Where the first term corresponds to the holding cost for the jobs that is scheduled for different days in the planning horizon and the second term corresponds to the penalty cost for the jobs that are rejected. It is assumed that denotes the minimum possible total expected cost over days and letting, be the dimensional unit vector with one in the element corresponding to the value function can be computed by solving the optimality equation.

2.1 Base Problem Formation Using Deterministic Linear Programming

The optimality equation in (2) involves solving a deterministic linear program that is formulated under the assumption that the job arrivals take on their Expected values and it is possible to schedule fractional numbers of jobs for different days. Using the decision variable we can solve the problem as follows

To approximate the optimal total expected cost over the planning horizon, constraints (4) in the problem above ensure that the decisions made on each day do not violate the expected numbers of job arrivals. Problem (3)-(6) occurs when it necessary to obtain lower bounds on the optimal total expected cost is the optimal objective value of problem (3)-(6). Where

2.2 Formation of Model Using Dynamic Programming Decomposition Method for the Manufacturing Environment

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Let, it = time

P = priority, j = the particular day.

Parameters:

The probability that nth item uses the qth unit of available capacity.

T Set of days in the planning horizon
P Set of possible priority level from the jobs
P Priority jobs that arrive on day t
Number of priority job
t Days under consideration
Remaining capacity on day t
No job arrival
j Days a priority jobs are scheduled
Holding cost for a priority job
Penalty cost for priority job
State of remaining capacity of a job
Number of priority jobs scheduled on day j
Decision taken that day
Holding cost equal to zero

2.2.1 Decision Variables:

Minimum possible total expected cost
LP optimal objective value

2.2.2 Assumptions

The processing capacity is fixed.
The planning horizon is 100 days for each period (lean and peak periods)
Priorities of jobs are graduated in the form of profit margin, urgency and time.
Jobs or orders not attended to incur penalty cost (equal zero)
Holding cost is taken into consideration.

Decomposing the optimality equation in (2) into a sequence of optimality equations with scalar state variables, to construct separable approximations to the value functions withas the optimal values of the dual variables associated with constraints (4) in problem (3)-(6) can be constructed.
Problem 3-6 is same as

The optimal total expected cost equation is given by:
(9)

Computing the optimal total expected cost equation by sort separation and the result obtained solved as a knapsack problem we have.

Where;

Where

Is an approximation to
Job scheduling decision is taken using the dynamic programming decomposition (DPD) decision rule giving in equation below.

2.2.3 Solution to the Model
The parameter equation:

Compute the average job arrivals for the different priority level (Appendix A)
Compute the coefficient of variation (CV)
Compute and

Compute ,
Compute
Compute
Compute
Apply MATLAB in solving the parameter equation (Appendix A)

III. Results and Discussion

3.1 Results

The results were computed using Excel spread sheet by adopting the raw data collected from the company (see tables 3.1, 3.2, 3.3, 3.4, 3.5) as a base problem and modification of some of the attributes to obtain a test problem with different characteristics and is listed in Table 4.1. Results were obtained by solving the mathematical models as developed by Erdelyi and Topaloglu (2015). Definition of variables and the solution script are shown in chapter three and the appendix.

Table 3.1 Result for the base problem

Lower Bound (Z^*_{LP})	Optimal Total expected cost		Gap with DPD	Cost for 2days scheduling plan
	DPD	FC	FC	
22,909	**24,857**	**29,000**	**4143**	**333.047**

Table 3.2 Results for the varying holding cost.

Φ (naira)	**Total expected cost (naira)**
1.5	23987
2.0	24857
2.5	24947
3.0	25048
3.5	25181
4.0	25324

Validation of result

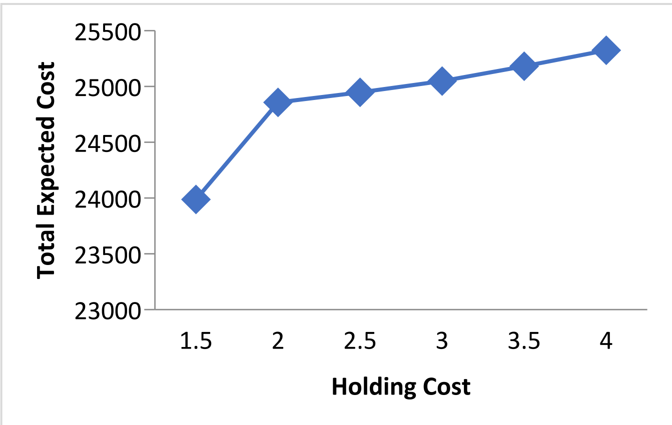

Figure 3.1 Total expected Cost Against Holding Cost (computed)

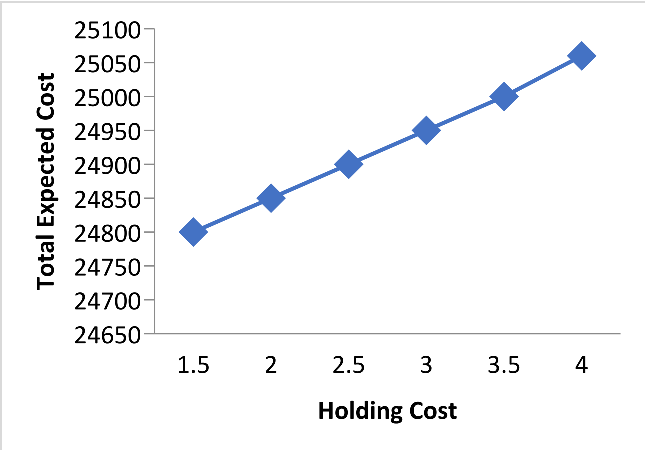

Fig. 3.2 Total Expected Cost against Holding Cost (Derived)

Table 3.3 Results for varying $_{j1}$

$_{j1}$	**Total expected cost(naira)**
1500	45747
2000	31347
2500	25067
3333	24857
3900	24843
3000	24991

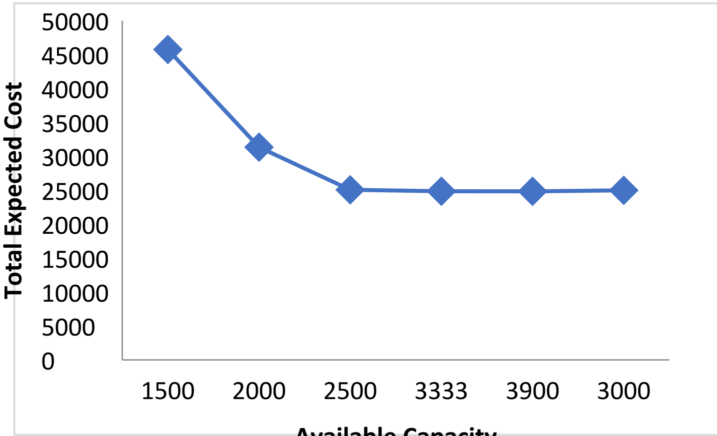

Figure 3.3 Total Expected Cost against Available Capacity (computed)

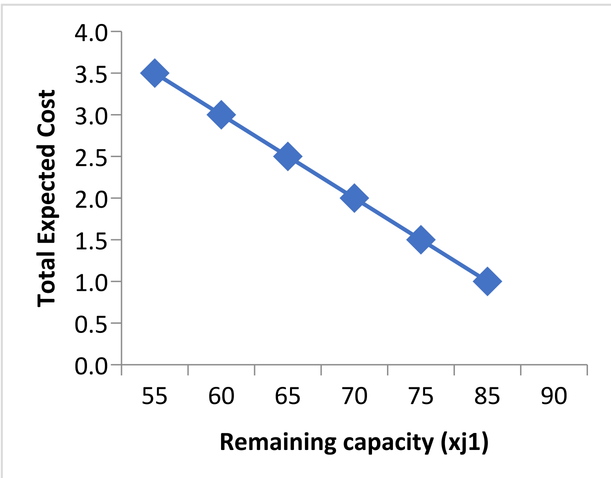

Figure 3.4 Total Expected Cost against Remaining Capacity (Derived).

Table 3.4 Results for varying S

S	Total expected cost(Naira)
2	24857
4	26257
6	27417
8	28577
10	32377
14	37377

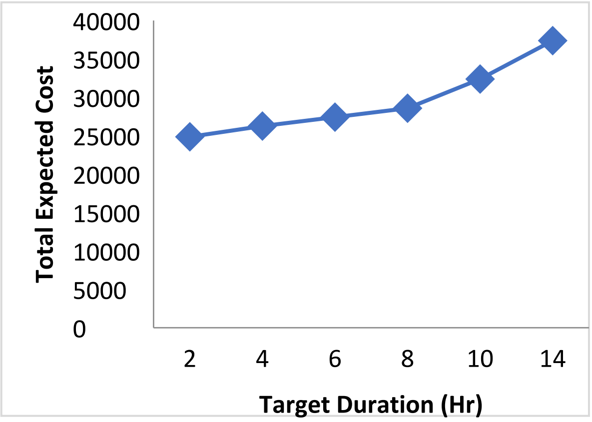

Figure 3.5 Total Expected Cost against Target Duration (computed)ss

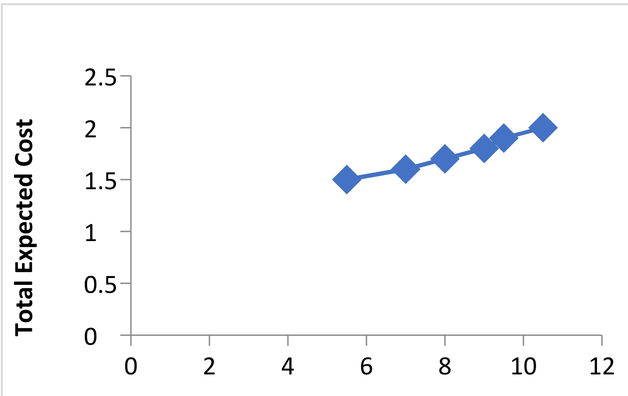

Figure 3.6 Total Expected Cost against Target Duration (Derived)

IV. Discussion

Table 2.5 shows the data for the base problem. The first column gives the variables as defined in chapter 3, while the second column gives their respective values. The first row gives the total number of days (T) in the planning horizon. The second row gives the total number of priority (p) levels. The third row has the maximum target duration a priority job can be scheduled (S) in the future. The fourth and fifth rows give the value for the probability distribution density (ϕ) and (dis)utilities (β) of items respectively. The sixth row contains the coefficient of variation for the number of daily job arrivals. The seventh row gives value for the daily processing capacity ($_{ji}$) while rows eight to ten gives the expected number of daily job arrivals for the first, second, and third priority level respectively.

Table 3.1 shows the results that were obtained by using Matlab programming algorithm for the base problem. The first column gives the lower bound on the optimal total expected cost provided by the optimal objective value of the problem by adopting equations (3) to (6). The second columns gives the optimal total expected cost incurred by the dynamic programming decomposition (DPD) by adopting equation (8) and FC decision rule as provided by the company respectively. The third column shows the cost for two days scheduling plan provided by solving equation (10) as a linear programming. The fourth column shows the gap between the total expected cost incurred by the DPD and FC decision rule. Base on collected data the total expected cost associated with FC policy is the carrying cost (naira) for hundred days: this is the holding cost associated with inventory majorly the cost of storage space since the product does not have any cost associated with obsolescence, spoilage, pilferage, insurance and taxes on the items in the inventory. The results show that DPD decision rule performs substantially better with a performance gap of four thousand one hundred and forty three naira. The performance indicates an approximate of fourteen percentage decrease to total expected cost of the company if implemented. This also implies an increase on the overall total expected revenue of the company by fourteen percentage. Different attributes of the base problem were varied to generate test problems with different characteristics. The first columns in tables (3.2, 3.3 and 3.4) indicate the parameter that was modified and its value while the second column gives the optimal total expected cost. In table 3.2 holding cost was varied. The outcome gave an increase in the total expected cost as the holding cost increases. The total incurred cost as given in the model is directly proportional to the total expected cost thus an increase in its value would relatively give an increase in the total expected cost. The first row gives the minimum of the optimal value on the total expected cost.

Figure 3.1 is plotted with data in table 3.2 while figure 3.2 is plotted with data from the work of Edelyi and Topaloglu (2015) as given in their tables. In both figures, the relationship between holding cost and total expected cost is similar in terms of linearity with significant increase in total expected cost as holding cost increases. The standard deviation(std) for the holding cost in both figures are 144 and 850 respectively while that of total expected cost is approximately 1.0 respectively. The difference is std of both figure show that the values of holding cost in figure 4.1 are much more spread out than the values in figures 3.2, this difference might account for the little difference between them in term of shape.

In table 3.3, the daily available capacity was varied. The total optimal expected cost decreases as the daily capacity increases. The reason is that as the capacity increases it accommodates the high priority jobs, thus the problem becomes less challenging resulting in the decrease of the total expected cost. Figure 3.3 and figure 3.4 are plotted with data in table 3.3 and data from the work of Edelyi and Topaloglu (2015) as given in table 6. Both graphs shows a decrease in total expected cost as the available capacity increases. Both figures also show similarity in their trend. The standard deviation (std) for the available capacity in both figures in 974 and 12 respectively while that of total expected cost is approximately 9034 and 10000 respectively. The difference in std of both figure show that the values of in figure 3.3 are much more spread out than the values in figures 3.4, these differences might account for the difference between them in terms of shape. Table 3.4, shows how far we can project into the future when scheduling the jobs in varied manner. The outcome also gave an increase in the optimal total expected cost. This is because as the target duration increases, the holding cost increases as a result the total expected cost increases.

Figure 3.5 is plotted with data in table 3.4 while figure 3.6 is plotted with data from the work of Edelyi and Topaloglu (2015) as given in table 4. In both figures, the relationship between the target duration and total expected cost is similar in terms of linearity with significant increase in total expected cost as target duration increases. The standard deviation (std) for the target duration in both figures are five (5) and three (3) respectively while that of total expected cost is approximately 5062 and 3039 respectively. The difference in std of both figure show that the data used in figure 4.5 are much more spread out than the data in figure 4.6, these differences might account for the difference between them in term of shape.

V. Conclusion

A capacity allocation problem that involves allocating a fixed amount of daily processing capacity to jobs of different priority levels arriving randomly over time has been solved using Dynamic Programming Decomposition (DPD) method. Results show that DPD decision rule performs substantially better than First-come-First-serve decision rule with a performance gap of four thousand one hundred and fifty three naira. It also implies that if DPD decision rule is implemented by the company, a total of four hundred and fourteen three hundred and seventy three thousand naira approximately would be saved quarterly on the total production cost. Thus, Approximate Dynamic Programming policy provides optimization to the total expected revenue and performs significantly better than First-come First Serve Policy as applied by the company. Finally, findings are concurrent with findings given in the work of Erdelyi and Topaloglu (2015).

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