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Application for NANO Regular b – Closed Sets in NANO Topological Spaces

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ABSTRACT
The significance of this paper is to apply the concept of Nano regular b – closed sets in nano topological spaces in some real-life application. And also to obtain the results by using an algorithm of nano regular b-closed sets in the application.
Keywords: *Lower approximation space, Upper approximation space and Boundary region, Nano regular b-closed sets (Nrb-closed sets).*
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I. INTRODUCTION

The concept of generalized closed sets in topological space was introduced by Levine [4] in 1970. Andrijevic [1] introduced a new class of generalized open sets namely, b-open sets in 1996. A. Narmadha and N. Nagaveni [5] introduced regular b-closed sets in 2012. In 2013, A. Narmadha, N. Nagaveni and T. Noiri [6] introduced regular b-open sets. The notion of nano topology was introduced by Lellis Thivagar [2] in 2013 and also he established certain weak forms of nano open sets such as nano α -open sets, nano semi-open sets and nano pre open sets. In 1991 Pawlak Z introduced new set theory called rough set theory [7]. And also he discovered some application in rough set theory in 2002 [8]. Later Lellis thivagar introduced a new topology rough topology in terms of lower, upper approximations and boundary. And then the concepts of nano topological basis was applied for finding the deciding factors in data analysis [3]. In 2022 P. Srividhya and T. Indira [9] introduced Nano regular b-open sets and Nano regular b-closed sets**.** In this paper, an application based on the Nano regular b-closed sets (Nrb-closed sets) was discussed.

II. PRELIMINARIES

Definition: 2.1 [2] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space.

Let X $\subseteq$ U. Then

- The lower approximation of X with respect to R is the set of all objects which can be for certainly classified as X with respect to R and is denoted by $L_R(X)$.

$$L_R(X) = \bigcup_{x\in U} \{R(x): R(x) \subseteq X\}$$

- The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.

$$U_R(X) = \bigcup_{x\in U} \{R(x):R(x) \cap X \neq \phi\}$$

- The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not –X with respect to R and is denoted by $B_R(X)$.

$$B_R(X) = U_R(X) - L_R(X).$$

Definition: 2.2 [2] Let U be the universe, R be an equivalence relation on U and $\tau_R(X)$ ={U, ϕ , $L_R(X)$, $U_R(X)$, $B_R(X)$} where X $\subseteq$ U. Then $\tau_R(X)$ satisfies the following axioms:

- U and $\phi \in \tau_R(X)$.
- The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$. Then $\tau_R(X)$ is called the nano topology on U with respect to X, (U, $\tau_R(X)$) is called the nano topological space.

Elements of the nano topology are known as nano open sets. The complement of elements of the nano open sets are called as nano closed sets.

Definition: 2.3 [9] A subset A of a nano topological spaces (U, $\tau_R(X)$) is said to be "nano regular b-closed" (briefly nano rb-closed) if Nrcl(A) $\subset$ G whenever A $\subset$ G and G is nano b-open in U.

Definition: 2.4 [9] The union of all nano regular b-open sets contained in A is nano regular b-interior of A and it is denoted by Nrbint(A).

The intersection of all nano regular b-closed sets containing A is called the nano regular b-closure of A and it is denoted by Nrbcl(A).

III. APPLICATION FOR NANO REGULAR B-CLOSED (NRB-CLOSED) SETS

Definition: 3.1 [3] Let (U, A) be an information system, where A is divided into a set C of condition attributes and a set D of decision attribute. A subset R of C is said to be a core, if Nrb-closed sets of R = Nrb-closed sets of C and Nrb-closed sets of C $\neq$ Nrb-closed sets of C – {r}for all r $\in$ R. That is, a core is a minimal subset of attributes which is such that none of its elements can be removed without affecting the classification power of attributes.

TO FIND THE KEY FACTORS FOR A HEALTHY LIFE:

Here we discussed an application based on Nano regular b-closed sets. We get the main aspect for getting satisfaction in the choice of higher studies. The general algorithm for identify the key factors is given below.

ALGORITHM:

Step 1: Let U be the finite universe, A be the finite set of attributes which is divided into two classes such that condition attributes and decision attributes where condition attributes are denoted by C and decision attributes are denoted by D. Then R is the equivalence relation on U corresponding to the condition attributes C and a subset X of U. The given tabular column was represented by the datas , in which columns are the attributes and rows are the objects. The entries of the table are known as the attribute values.

Step 2: Find the lower approximation $L_C(X)$, upper approximation $U_C(X)$ and the boundary region $B_C(X)$ of X with respect to R.

Step 3: Find the nano topology $\tau_C(X)$ on U and the nano regular b-closed sets corresponding to the conditional attribute set C.

Step 4: Remove an attribute x from C and find the lower and upper approximations and the boundary region of X with respect to the equivalence relation on C - {x}.

Step 5: Find the nano topology $\tau_{C-\{X\}}$ (X) on U and the nano regular b-closed sets for C - {x}.

Step 6: Repeat steps 4 and 5 for all attributes in C.

Step 7: Those attributes in C for which Nano regular b-closed sets of C $\neq$ Nano regular b-closed sets of C – {x} form the CORE.

APPLICATION:

In everyone's life choosing the higher studies is the most important part. It depends upon their interest, Scope & Job opportunities of the subject, Guidance of their parents & well wishers and their marks etc.,

Whatever the reason behind their choice, some are satisfied and some are not satisfied. From some final year students, we collect the reasons of choosing their higher studies.

Based on these datas, we apply the algorithm for Nano regular b-closed sets and find out the key factors for the self satisfaction.

In the following table 3.1, the set of students denoted by 1,2,3,4,5,6,7,8 and A = {Interest, Scope, Guidance, Scored Marks} are the set of attributes. " ✓ " and " x " are the entries in tables which is known as attribute values. The attributes in A are the condition attributes and is denoted by C = {I, S, G, SM} and SATISFIED / NOT SATISFIED be the decision attribute and is denoted by D = {S / NS}.

STUDENTS	INTEREST	SCOPE	GUIDANCE	SCORED MARKS	SATISFIED / NOT SATISFIED
1	✓	✓	✓	✓	S
2	X	✓	X	✓	NS
3	X	X	X	X	S
4	✓	✓	X	✓	S
5	X	✓	✓	✓	NS
6	✓	X	X	✓	S
7	X	X	✓	✓	NS
8	✓	✓	X	✓	NS

Here the family of equivalence classes, U/C corresponding to C is given by U/R(C) = {1},{2},{3},{4,8},{5},{6},{7}.

Case 1 : (The students who are satisfied)

Let X = {1,3,4,6}, the set of students who are satisfied. Then $L_C(X)$ = {1,3,6}, $U_C(X)$ = {1,3,4,6,8} and $B_C(X)$ = $U_C(X) – L_C(X)$ = {4,8}. Then $\tau_C(X))$ = {U, ∅,{1,3,6},{1,3,4, 6,8},

{4,8}}. Then the Nano regular b-closed sets = {U, ∅, {2,4,5,7,8},{2,5,7}, {1,2,3,5,6,7}}.

Step 1:

(i) Remove the attribute "Interest" from C, U/R (C – I) = {1,5},{2,4,8},{3},{6},{7}}} then the nano topology $\tau_{C - \{I\}}(X))$ = {U, ∅, {3,6},{1,2,3,4,5,6,8},{1,2,4,5,8}} and the Nrb-closed sets of C – {I} = {U, ∅, {1,2,4,5,7,8},{7},{3,6,7}} ≠ Nrb-closed sets of C.

(ii) Remove the attribute "Scope" from C, U/R (C – S) = {{1},{2},{3},{4,6,8},{5,7}} Then the nano topology $\tau_{C\text{-}\{S\}}(X))$ = {U,∅,{1,3},{1,3,4,6,8},{4,6,8}} and the Nrb-closed sets of C – {S} = {U, ∅, {2,4,5,6,7,8},{2,5,7}, {1,2,3,5,7}} ≠ Nrb-closed sets of C.

(iii) Remove the attribute "Guidance" from C, U/R (C – G) = {{1,4,8},{2,5},{3},{6},{7}}. Then the nano topology $\tau_{C\text{-}\{G\}}(X))$ = {U, ∅,{3,6},{1,3,4,6,8},{1,4,8}} and the Nrb-closed sets of C – {G} = {U, ∅, {1,2,4,5,7,8},{2,5,7},{2,3,5,6,7}} ≠ Nrb-closed sets of C.

(iv) Remove the attribute "Scored Marks" from C, U/R (C – SM) = U/R(C) = {1},{2},{3},{4,8},{5},{6},{7}. Then the nano topology $\tau_{C\text{-}\{SM\}}(X))$ = {U, ∅,{1,3,6},{1,3, 4, 6,8},{4,8}}and the Nrb-closed sets of C – {SM} = {U, ∅, {2,4,5,7,8},{2,5,7}, {1,2,3,5,6,7}} = Nrb-closed sets of C.

Step 2:

If R = C – {SM} = {I, S, G}, Then τ_R(X)) = {U, ∅, {1,3,6},{1,3,4,6,8},{4,8}} and Nano regular b-closed sets of R = {U, ∅,{2,4,5,7,8},{2,5,7},{1,2,3,5,6,7}} = Nano regular b-closed sets of C.

From step 1, we get , the Nrb-closed sets of C – {I}, Nrb-closed sets of C – {S}, Nrb-closed sets of C – {G} ≠ Nrb-closed sets of C.

From this we get, Nano regular b-closed sets of R = Nano regular b-closed sets of C and Nano regular b-closed sets of C ≠ Nano regular b-closed sets of C-{r} for every r in R.

∴ **CORE** = {Interest, Scope, Guidance}

Case 2 : (The students who are not satisfied)

Let X = {2,5,7,8}, the set of students who are satisfied.

Then L_C(X) = {2,5,7}, U_C(X) = {2,4,5,7,8} and B_C(X) = U_C(X) – L_C(X) = {4,8}. Then τ_C(X)) = {U,∅ ,{2,5,7},{2,4,5,7,8},{4,8}}. Then the Nano regular b-closed sets = {U, ∅,{1,3,4, 6,8}, {1,3,6}, {1,2,3,5,6,7}}.

Step 1:

(i) Remove the attribute "Interest" from C, U/R (C – I) = {1,5},{2,4,8},{3},{6},{7}}} then the nano topology $\tau_{C - \{I\}}$(X)) = {U, ∅, {7},{1,2,4,5,7,8},{1,2,4,5,8}} and the Nrb-closed sets of C – {I} = {U, ∅, {1,2,3,4,5,6,8},{3,6},{3,6,7}} ≠ Nrb-closed sets of C.

(ii) Remove the attribute "Scope" from C, U/R (C – S) = {{1},{2},{3},{4,6,8},{5,7}} Then the nano topology $\tau_{C\text{-}\{S\}}$(X)) = {U,∅,{2,5,7},{2,4,5,6,7,8},{4,6,8}} and the Nrb-closed sets of C – {S} = {U, ∅, {1,3,4,6,8},{1,3}, {1,2,3,5,7}} ≠ Nrb-closed sets of C.

(iii) Remove the attribute "Guidance" from C, U/R (C – G) = {{1,4,8},{2,5},{3},{6},{7}}. Then the nano topology $\tau_{C\text{-}\{G\}}$(X)) = {U, ∅,{2,5,7},{1,2,4,5,7,8},{1,4,8}} and the Nrb-closed sets of C – {G} = {U, ∅, {1,3,4,6,8},{3,6},{2,3,5,6,7}} ≠ Nrb-closed sets of C.

(iv) Remove the attribute "Scored Marks" from C, U/R (C – SM) = U/R(C) = {1},{2},{3},{4,8},{5},{6},{7}. Then the nano topology $\tau_{C\text{-}\{SM\}}$(X)) = {U, ∅,{2,5,7},{2,4,5, 7,8},{4,8}}and the Nrb-closed sets of C – {SM} = {U, ∅, {1,3,4,6,8},{1,3,6}, {1,2,3,5,6,7}} = Nrb-closed sets of C.

Step 2:

If R = C – {SM} = {I, S, G}, Then τ_R(X)) = {U, ∅,{2,5,7},{2,4,5,7,8}, {4,8}}and Nrb-closed sets of R = {U, ∅,{1,3,4,6,8},{1,3,6}, {1,2,3,5,6,7}} = Nrb-closed sets of C.

From step 1, we get , the Nrb-closed sets of C – {I}, Nrb-closed sets of C – {S}, Nrb-closed sets of C – {G} ≠ Nrb-closed sets of C.

From this we get, Nano regular b-closed sets of R = Nano regular b-closed sets of C and Nano regular b-closed sets of C $\neq$ Nano regular b-closed sets of C-{r} for every r in R.

$\therefore$ **CORE** = {Interest, Scope, Guidance}

From the core of the above two cases we conclude that the main aspect for self satisfaction for students in choosing the higher studies are **" Interest ", " Scope " and " Guidance ".**

IV. Conclusion

In this paper, an application based on the Nano regular b-closed sets (Nrb-closed sets) was obtained by using general algorithm for Nano regular b-closed sets.

References

[1]. Andrijevic. D, On b-open sets, Mat. Vesnik, 48 (1996) 59-64.
[2]. Lellis Thivagar and Carmel Richard, On nano forms of weakly open sets , International Journal of Mathematics and Statistics Invention, 1(1), (2013) 31-37.
[3]. Lellis Thivagar. M, Carmel Richard, Mathematical innovations of a Modern topology in Medical Events, International Journal of Information Science 2(4) (2012) 33-36.
[4]. Levine, Generalized closed sets in topology, Rendiconti del circolo Matematico di Palermo, 19(2) (1970) 89-96.
[5]. Narmatha. A and Nagaveni. N, On rb-closed sets in topological spaces, Heber International Conference on Applications of Mathematics and Statistics, (2012) 81- 87.
[6]. Narmatha. A, Nagaveni. N and Noiri. T, On rb-open sets, International Journal of Mathematical Analysis, 7 (2013) 937-948.
[7]. Pawlak Z., Rough sets, Theoretical aspects of reasoning about data, Kluwer Academic Publishers, London,(1991).
[8]. Pawlak Z., Rough set theory and its applications, Journal of telecommunications and Information Technology, 3 (2002), 7-10.
[9]. Srividhya. P & Indira. T, Nano regular b-closed sets and Nano regular b-open sets, Mathematical Statistician and Engineering Applications, 71(4) (2022) 892-911.