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Nonlinear Dynamics of Coupled Neurons

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Abstract: *The current paper has the objective of understanding the nonlinear dynamics of coupled neurons. Fitzhu-Nagumo neuron model being a simplified form of the original Hodgkin Huxley model, has been employed in the current study. The basic dynamical system is represented by a set of two coupled nonlinear first order, ordinary differential equations. One of the differential equations represents the time variation of the membrane voltage of the neuron while the other denotes the time evolution of the recovery variable of the neuron potential. The threshold input current which activates the neuron is determined by solving the differential equations and representing the solutions graphically. Realizing that "noise" plays an important role in understanding theories of learning as well as memory, dynamics of neuron membrane voltage is explored with an added random noise. With the increase in the strength of the noise, the threshold input decreases. Further, the effect of adding neurons to this dynamical system is explored. Though in reality the brain is a complex system involving network of interacting neurons, in this paper the coupling between the neurons is considered as the linear type. Though this model is highly simplified but it is useful for understanding the basic dynamics of the neuron membrane voltage and the recovery variable. Dynamics under coupling of neurons has been explored first for two neurons. The threshold input current value increases with the addition of a neuron. Another feature that emerges in the dynamics for two coupled neurons is that after the two neurons are synchronized , there is an optimum value of coupling constant beyond which the neurons do not fire for a given input signal. As expected, if the dimension of the neuronal model is increased by the addition of another neuron, the dynamics gets more complex. For linear coupling, the synchronization of the three neurons is not perfect. The cut off coupling constant beyond which the neurons stop firing has been found to increase with the increase in the number of neurons. The current investigation provides information about the variation of neuronal membrane voltage. This study also emphasizes that in neuronal communications, for the neuronal system to remain stimulated there is an optimum value of the strength of coupling.*

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I. Introduction

The information processing system of the brain involves a complex-dynamics. About 10^{11} nerve cells, the neurons, are involved in this process. An investigation of the effect of interactions, links and communications between neurons, therefore has a great significance for an in-depth understanding of the factors affecting the learning process as well as for memory theories. Furthermore, with the emerging applications of neural networks in diverse fields, it becomes absolutely essential to understand the dynamics of the interconnected neurons. In the simplest form the neuronal membrane if stimulated by a pulse of current, generates an output signal. This process is referred to as the firing of a neuron. However, a threshold input current is essential for the neuron to get activated [1-4]. After the neuron fires, it reaches a recovery phase where the neuron is not sensitive to the input pulse [5]. This phase is referred to as the refractory stage. To understand the mechanism of information processing and impulse transmission in the brain, it is important to first understand the dynamical system of single neuron in neuronal model. Hubel and Wiesel [6] determined a relationship between the sensory inputs and the rate of firing of the neurons. Freeman, Singer, Gay et al. [7,8] found that there could be a correlation between the firing of different neurons ultimately leading to synchronization. The neurons in the brain represent a complex dynamical system [9] and possess the nonlinear dynamical properties, exhibiting bifurcations and a wide range of dynamics depending on parameters characterizing the system [10]. Some of these bifurcations are switching from the rest mode to the firing mode, where short pulses can be generated. These could be firing at a constant frequency or there could be chaotic firing. Chaotic regime has the special property of dependence on initial conditions. Hence when the firing of neuron reaches a chaotic state it implies that the pattern does not repeat itself. Different initial conditions lead to different evolutions of the dynamics of the neurons. This means that a chaotic dynamical system can produce different outputs at different evolution times. The current work uses the simplified Fitzhu- Nagumo (FN) model to determine the variation of neuronal membrane voltage and the recovery variable for the membrane potential.

II. Description of the problem

The FN model is one of the mathematical representations of the excitability of the neurons. This model considers variation on two-time scales. The neuronal membrane voltage has a fast evolution while the recovery variable is on a slower time scale. The solutions to the simultaneous ordinary non-linear differential equations will describe the dynamics of the membrane voltage of a single neuron and the objective is to obtain the threshold value of input current required for firing of the single neuron. The next step will be to add a Gaussian noise with varying strengths and explore the impact on the dynamics of the system. To understand the interaction between two neurons, a second neuron is introduced in the original simplified model [11]. The dynamics is studied as a function of the coupling constant with the objective of detecting the threshold current for the second neuron to start firing. To increase the dimension of the system a third neuron is added to the 2neuron model. The effect of addition of a neuron on the threshold input will be explored. Further, the dynamics of all the 3 neurons and their synchronization will be investigated.

III. Mathematical formulation

The differential equations for membrane voltage variation of neuron1, neuron 2 and neuron 3 are respectively represented by eqn.(1),eqn.(3) and eqn.(5) . Their corresponding recovery variable dynamics are represented by eqn(2),eqn(4) and eqn. (6) respectively

$$\frac{dx_1}{dt} = c[x_1 - \frac{1}{3}{x_1}^3 + y_1 + I + k(x_2 - x_1)] \quad (1)$$

$$\frac{dy_1}{dt} = -\frac{1}{c}[x_1 - a + by_1] \quad (2)$$

$$\frac{dx_2}{dt} = c[x_2 - \frac{1}{3}{x_2}^3 + y_2 + I + k(x_1 - x_2) + k(x_3 - x_2] \quad (3)$$

$$\frac{dy_2}{dt} = -\frac{1}{c}[x_2 - a + by_2] \quad (4)$$

$$\frac{dx_3}{dt} = c[x_3 - \frac{1}{3}{x_3}^3 + y_3 + k(x_2 - x_3)] \quad (5)$$

$$\frac{dy_3}{dt} = -\frac{1}{c}[x_3 - a + by_3] \quad (6)$$

The constants have the following values $a = 0.75$; $b = 0.8$; $c = 3.0$, [12]. I is the input current injected in neuron 1 and k is the coupling constant between the neurons. Variables x_1, x_2, x_3 represent the neuronal membrane voltages of neurons 1,2 and 3 respectively. The recovery variables respectively of neurons 1,2 and 3 are represented by y_1, y_2 ,y_3.

These differential equations are solved using the MATLAB software (online access provided by University of Delhi) [13,14].

IV. Results

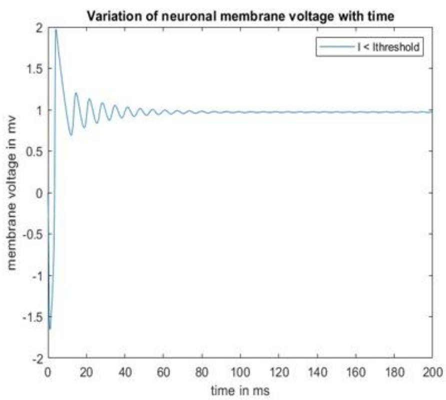

Figure 1a

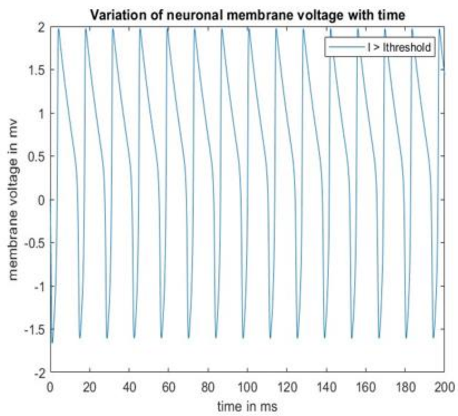

Figure 1b

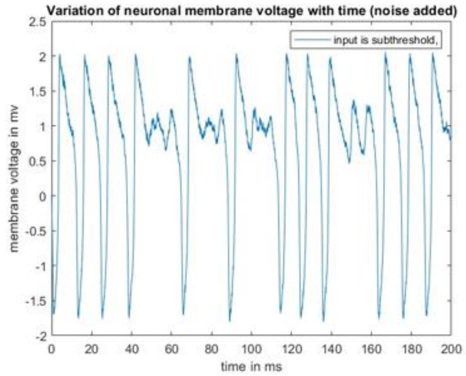

Figure 2a

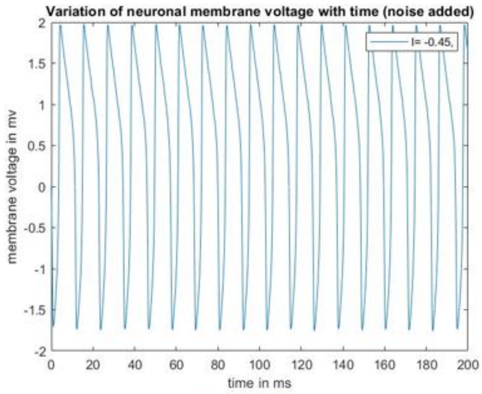

Figure 2b

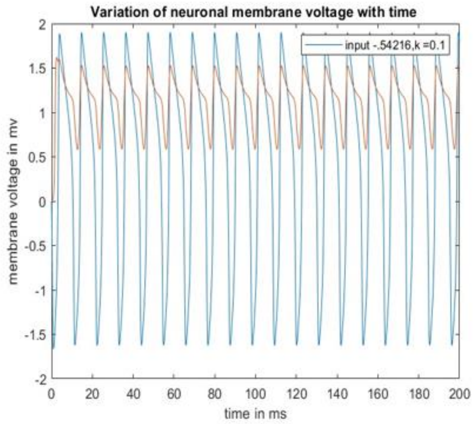

Figure 3a

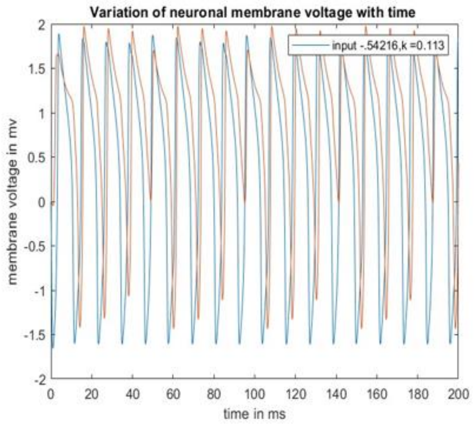

Figure 3b

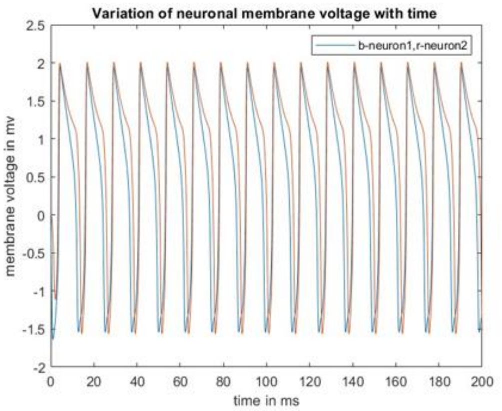

Figure 4a

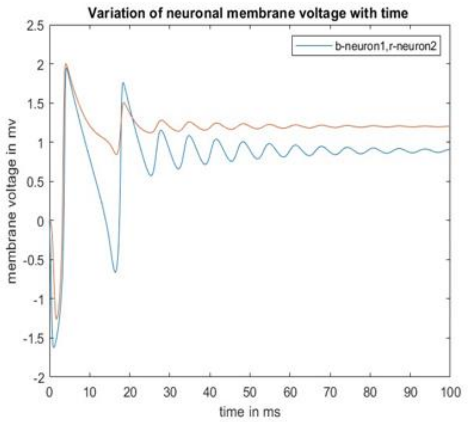

Figure 4b

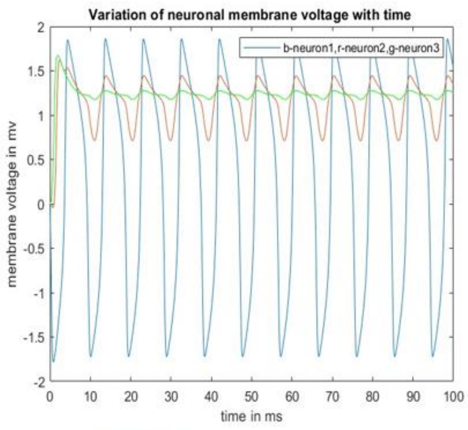

Figure 5a

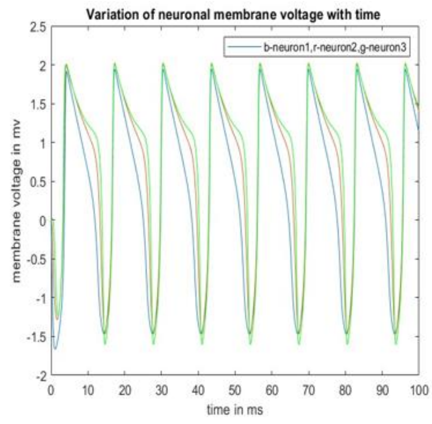

Figure 5b

V. Conclusions:

From Fig. 1a and Fig. 1b, it is obvious that the neuron starts firing only after the input is above a certain threshold value. For the parameters used in the present calculations, this value is -0.39. Fig.2a and Fig.2b is the representation of the dynamics of neuronal membrane voltage with the addition of Gaussian noise. Addition of random noise leads to the increase in the threshold input for firing of the neurons. Fig.3a and Fig. 3b represent the dynamics when two neurons are coupled. Addition of a neuron leads to an increase in the threshold input. The strength of the interaction between the neurons is denoted by the coupling constant. As the coupling constant is increased, neuron 2 gradually gets almost synchronized with neuron1 at k=0.15 (Fig.4a). However, for higher values of k, neuron 2 voltage value starts decreasing. there is a critical coupling constant beyond which the neurons stop firing. In the present work this has been determined to be 0.2. For k > 2.001, the neurons slowdown in firing(Fig. 4b). The effect of the addition of another neuron is investigated. As the neuron number increases, it is found that the synchronization value of the coupling constant increases. However, when the input is higher than the threshold, in this case 0.75, stronger coupling leads to higher kcut off. Figure 5a is representative of neuron voltage dynamics of all three neurons at coupling constant k = 0.1. From this figure it is clear that all three neurons have started firing but strength of voltage is lesser for neuron 2 than neuron 1 and least for neuron 3. With the increase in the coupling and the same input current, all the three neurons start firing almost in synchrony. This is represented by Figure 5b. with the coupling constant k =0.4. The final conclusion is that the interaction between neurons plays a very important role in the neuronal membrane voltage dynamics. For weak coupling, the triggering of the third neuron is at a much higher threshold in comparison to a two neuron system. As the coupling strength increases, the dynamics of the neuronal membrane voltage is altered till the critical coupling constant is reached, beyond which the neurons stop firing. The synchronization of the neurons plays a very important role in understanding theories of learning. Investigating the dynamics with addition of noise, is a big step towards understanding the effect of random input variation on memory.

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