

A Review on Dynamic Amplification Factor Applicable for Transient Load on MDOF System

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Abstract

Design of a structure against a given dynamic load is usually based on an equivalent static load approach, wherein the equivalent load is estimated by scaling the peak value of the dynamic load by an appropriate Dynamic Amplification Factor (DAF). Initial estimate of DAF is usually based on the response spectrum of the given dynamic load. A response spectrum is a graph of the maximum dynamic response (displacement, stress or acceleration response to the given dynamic load) of different single-degree-of-freedom (SDOF) systems, plotted, generally, with respect to the undamped natural frequency of the SDOF system. Ratio between the response spectrum and the static response under peak value of the given dynamic load gives the DAF as a function of undamped natural frequency of the SDOF system. For a SDOF system subjected to a step load, the maximum possible value of DAF is 2. Hence, many design guidelines specify 2 as an upper bound for the DAF even for the design of multiple-degree-of-freedom (MDOF) systems against dynamic load. However, quite a few published literature report that for MDOF systems subjected to step loads, DAF can exceed 2 and hence, the assumption of 2 as the upper bound of DAF can lead to unsafe structural designs.

Keywords: Dynamic Amplification Factor

I. INTRODUCTION

Design of a structure against a given dynamic load is usually based on an equivalent static load approach, wherein the equivalent load is estimated by scaling the peak value of the dynamic load by an appropriate Dynamic Amplification Factor (DAF). Initial estimate of DAF is usually based on the response spectrum of the given dynamic load. A response spectrum is a graph of the maximum dynamic response (displacement, stress or acceleration response to the given dynamic load) of different single-degree-of-freedom (SDOF) systems, plotted, generally, with respect to the undamped natural frequency of the SDOF system [**Error! Reference source not found.**]. Ratio between the response spectrum and the static response under peak value of the given dynamic load gives the DAF as a function of undamped natural frequency of the SDOF system. For a SDOF system subjected to a step load, the maximum possible value of DAF is 2. Hence, many design guidelines specify 2 as an upper bound for the DAF even for the design of multiple-degree-of-freedom (MDOF) systems against dynamic loads [**Error! Reference source not found.**]. However, quite a few published literature report that for MDOF systems subjected to step loads, DAF can exceed 2 [3-4] and hence, the assumption of 2 as the upper bound of DAF can lead to unsafe structural designs.

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1.1 Dynamic Analysis

Dynamic analysis is concerned with the behavior of the continuum under prescribed boundary conditions and dynamically applied loads; dynamic loads are applied as a function of time. This time-varying load application induces time-varying response (displacements, velocities, accelerations, forces, and stresses). These time-varying characteristics make dynamic analysis more complicated and more realistic than static analysis. When an action is applied to a structure slowly, i.e. over a time that is more than twice as long as the main vibration period of the structure, the response of the structure is practically the same as its static response. However, if the action is applied more rapidly, the structure shows a dynamic response. In this case for the given structure and the given action we can define, for each section and for each movement or internal force, a dynamic amplification factor (DAF), which is the ratio of the maximum dynamic response to the static response[7].

1.2 Dynamic Amplification Factor

The dynamic response of a single degree of freedom (SDOF) system under the action of different load types have been developed and can be found in many classical references as [1,7]

The D.A.F. is defined as the dimensionless ratio of the dynamic to the static response of a structure when it is subjected to dynamic loading. For SDOF systems, the maximum D.A.F. (D.A.F.max) that can be reached under the action of a pulse load is equal to 2.0, which makes it necessary for the ratio of the pulse duration to the natural period of the system to be equal to or larger than 0.5.

1.2.1 Response of Undamped SDOF Systems to Rectangular Pulse Loadings

Consider a viscous-damped SDOF system be subjected to an ideal step input[1].

The equation of motion is

$$m\ddot{u} + c\dot{u} + ku = P_0 \quad \text{for } t \geq 0$$

Let the system be at rest at $t = 0$, that is, $u(0) = \dot{u}(0) = 0$

Particular solution; $u_p = \frac{P_0}{k}$

Complementary solution; $u_c = e^{-\zeta\omega_n t} (A_1 \cos \omega_d t + A_2 \sin \omega_d t)$

Then $u = \frac{P_0}{k} + e^{-\zeta\omega_n t} (A_1 \cos \omega_d t + A_2 \sin \omega_d t)$

Using the initial conditions, step response of an underdamped SDOF system:

$$u(t) = \frac{P_0}{k} \left[1 - e^{-\zeta\omega_n t} \left(\cos \omega_d t + \frac{\zeta\omega_n}{\omega_d} \sin \omega_d t \right) \right]$$

Response ratio is the ratio of dynamic response to static deformation. For the ideal step input, $R(t)$ is given by;

$$R(t) = 1 - e^{-\zeta\omega_n t} \left(\cos \omega_d t + \frac{\zeta\omega_n}{\omega_d} \sin \omega_d t \right)$$

For an undamped system,

$$u(t) = \frac{P_0}{k} (1 - \cos \omega_n t)$$

1.2.2 Response of Undamped SDOF Systems to Rectangular Pulse Loading

Case 1: Forced-Vibration Era ($0 < t < t_d$) For this case, $R(t)$ is the same as for an ideal step[1],

$$R_1(t) = 1 - \cos \omega_n t \quad 0 \leq t \leq t_d$$

Case 2: Residual-Vibration Era ($t_d < t$)

$$R_2(t) = R_1(t_d) \cos \omega_n (t - t_d) + \frac{\dot{R}_1(t_d)}{\omega_n} \sin \omega_n (t - t_d)$$

Maximum Response of an undamped SDOF system to step inputs is shown in figure1

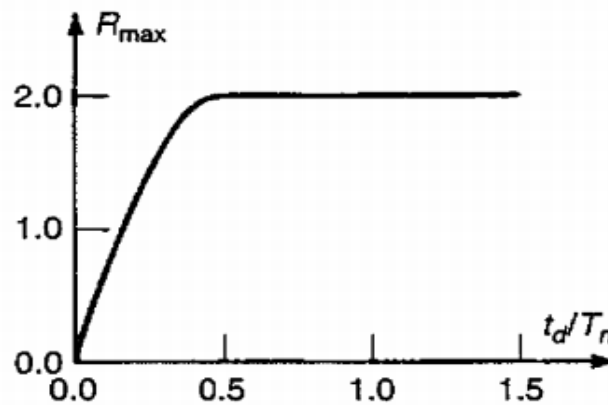


Figure1: Maximum Response of an undamped SDOF system to step inputs[1]

It can be seen that any pulse of duration longer than $t_n/6$ will cause a displacement larger than the static displacement, p_0/k , and for any pulse longer than $t_n/2$, the maximum displacement will be twice the static value.

1.2.3 Response of Undamped SDOF Systems to Ramp Loading

Consider a ramp input with rise time t_r , applied to an undamped SDOF system that is at rest prior to application of the load[1].

The equation of motion and initial conditions are

$$m\ddot{u} + ku = \frac{t}{t_r}P_0, 0 \leq t \leq t_r$$

$$P_0, t_r \leq t$$

For $0 \leq t \leq t_r$, particular solution is $u = \frac{t}{t_r} \frac{P_0}{k}$

Then $u = \frac{t}{t_r} \frac{P_0}{k} + e^{-\zeta\omega_n t} (A_1 \cos \omega_d t + A_2 \sin \omega_d t)$

Using the initial conditions, total response for, $0 \leq t \leq t_r$

$$u(t) = \frac{P_0}{k} \left(\frac{t}{t_r} - \frac{1}{\omega_n t_r} \sin \omega_n t \right)$$

For $t \geq t_r$

$$u(t) = \frac{P_0}{k} \left\{ 1 + \frac{1}{\omega_n t_r} [\sin \omega_n (t - t_r) - \sin \omega_n t] \right\}$$

Maximum Response of an undamped SDOF system to step inputs is shown in figure1

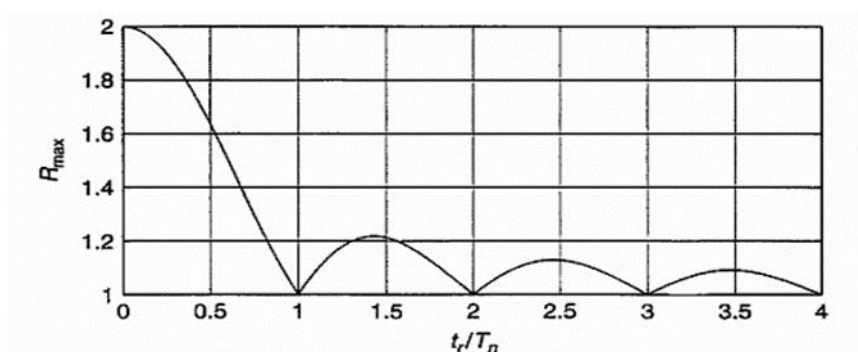


Figure 2: Maximum Response of an undamped SDOF system to ramp inputs [1]

From the figure it is clear in order to have DAF value less than 2, the rise time should be longer than about t_n .

1.2 STUDY ON DYNAMIC AMPLIFICATION FACTOR

Many studies have focused on determination of dynamic amplification factors of bridges by considering the loads acting on i.e., traffic live loads of bridges for road bridges and railway bridges

In 2007, Ruiz et al. studied the Dynamic Amplification Factor in cable stayed bridge under two conditions, one involving the abrupt application of loads to a simply supported beam and the other the accidental breakage of a stay cable in a bridge with under-deck cable-staying. In cable-stayed structures, the DAFs related to internal forces as a result of the abrupt breakage of a stay cable can be larger than 2. Thus, following the guidelines for cable-stayed bridges and carrying out a static calculation in which the forces are amplified by a DAF of 2 can underestimate loads even when the maximum internal forces in critical sections of the structure are assessed. Due to such an accidental action we can predict DAFs related to bending moments larger than 2 in critical sections not only in bridges with under-deck cable-staying, but also unconventional cable-stayed bridges[3].

In 2013, Aoki et al. conducted a numerical study on the potential progressive collapse of cable stayed bridges due to different cable loss scenarios. A comprehensive parametric study is undertaken and effect of location, duration and number of lost cables as well as applied load case and the structural damping ratio on the dynamic amplification factor (DAF) was investigated. With regard to the parametric studies undertaken in this paper, the following conclusions are drawn; The DAF for the bending moment and axial force at different sections along the deck, towers and cables can take values much higher than two and conditions at which it exceeded[4].

In 2010, C.M. Mozos and A.C. Aparicio studied the dynamic response of cable stayed bridges to the sudden loss of a stay. Its objectives are to quantify the relative importance of the accidental ultimate limit state of failure of a stay in the design of the bridge, and to determine the safety level provided by the simplified procedure of using a static analysis with a D.A.F. of 2.0. In the case of load combinations for negative bending moments, the D.A.F. greatly exceeds the value of 2.0. The average of the obtained D.A.F. in the undamped and damped movements is 3.35 and 2.52, respectively, and an extreme value of 7.96 was observed. A fan pattern and a stiffer deck lead to a larger D.A.F which also increases with the distance of the cross section to the pylon[6].

In 2020, G. T. Michaltsos et al. studies analytically the problem of the sudden failure of a number of stays through a suitable mathematical model, based on the analytical method exposed by authors in previous publications and extended in this study through a 3D analysis. The analysis is carried out by the modal superposition method, and the gathered equations of the problem are solved through the Galerkin procedure and the Duhamel's Integrals. Characteristic examples are solved and useful diagrams and plots are drawn, while interesting results are obtained. The maximum forces developing on the left line of cables have values that exceed more than twice the value of the corresponding static one, i.e., larger than the DAF suggested by the related standards. This phenomenon is also be encountered at later stages of the oscillation[7].

In 2009, Ruiz and Aparicio studied the DAFs of an under-deck cable-stayed bridge subjected to accidental breakage of cables. Parametric studies showed that the maximum DAFs of bending moment, shear force and axial force reach 2.79, 8.49 and 30.68, respectively[8]. In 2011 Gerasimidis and Baniotopoulos compared the dynamic responses of a cable-stayed steel roof by two different methods, i.e., the fully dynamic method and the equivalent static method with DAF equaling to 2. Numerical results show the responses from the fully dynamic analysis are usually larger than those obtained based on the equivalent static analysis. In other words, assuming DAF equals to 2 in the equivalent static method may underestimate the structural responses[9].

II. RESULT AND DISCUSSION

The results obtained are as discussed below;

It is well-known that the maximum DAF of an undamped SDOF system subjected to a sudden dynamic load is 2. Many design codes (e.g., PTI and EuroCode 1) therefore use 2 as the upper bound in the design to consider the dynamic effect. From the journals reviewed it is concluded that the use of DAF value as 2 during designing can lead to unsafe structures. Most of the studies reveals that the DAF value obtained during the analysis varies throughout the nodes, one of the main reason is that the non-loaded node (DOFs) on which no loading is applied, the displacement DAF can be larger than 2. The maximum value of displacement DAF is affected by the load spatial distribution and mode shapes of the MDOF system.

III. CONCLUSION

It was observed ;

Many design guidelines specify 2 as an upper bound for the DAF even for the design of multiple-degree-of-freedom (MDOF) systems against dynamic loads, From the analysis it is proven that assumption of 2 as the upper bound of DAF can lead to unsafe structural designs.

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