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# Study of Asymptotic Attenuation of Oscillations Using HydraulicSupports with Inertial Masses

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***Abstract***
*The objective of this work is to investigate the dependencies between the physical parameters of vibration isolating modules for the complete attenuation of harmful oscillations in mobile machinery. To achieve this, algebraic criteria for the stability of dynamic systems developed by V.S. Voronov were utilized. Their good adaptability to dynamic processes in technical devices is demonstrated. A numerical calculation is presented based on the developed methodology for selecting proportional coefficients between physical characteristics of the same dimension. The achieved desirable result is demonstrated. A method for selecting the physical parameters of mechanical vibration damping modules is developed based on asymptotically stable, rapidly decaying oscillations to ensure optimal vibration isolation settings.*
***Keywords:*** *vibration isolation, vibration damping, hydraulic support, inertial mass, damping, stability criteria, elasticity coefficient, damping coefficient, dynamic scheme, mathematical model.*
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## I. INTRODUCTION

The complex technological design of modern vibration isolation elements in machinery is closely linked with the mathematical modeling of corresponding types of dynamic vibration dampers. In laboratory conditions, such actions are accompanied by the installation of elastic multilayer materials, rubber gaskets, pneumatic and hydraulic shock absorbers, spring modules, and other components. Vibration dampers are used to dampen the vibrations of a machine that occur when moving over uneven terrain and roads. These elements can dampen or minimize the transmission of high-amplitude vibrations originating from the source and suppress low frequencies. The most effective is the use of a combination of these elements, forming a single unified block [1]. Recently, the automotive market has widely presented ready-made hydraulic support modules. Their varieties differ in two main types: controlled and passive. Controlled hydraulic supports include those containing rheological, magnetorheological fluids, and electronic devices that provide control over the movement of the vibration-isolated object, with position sensors, speed and acceleration controls. They are called active vibration dampers, capable of changing their stiffness characteristics in accordance with the event that occurred during the operation of the mechanical device. The principle of operation of passive hydraulic dampers is based on the use of physical properties of structural elements during operation and converting the kinetic energy of the body's vibrations into the thermal energy of the fluid flowing through restricted openings. These single-axis damping devices occupy leading positions in the global automotive industry in terms of cost, simplicity of manufacture, economy, replaceability; they do not require additional expenses for creating electronic devices that maintain electrical andmagnetic fields in active hydraulic supports.

The vibration isolation system of a modern technical device includes several elastodamping elements, working sequentially and in parallel, performing different functional purposes. Elastic metal springs soften vibrations and support the structure, being part of the load-bearing support. Damping elements, by absorbing excess kinetic energy, provide quality parameters of smooth running in various road conditions.

For each specific case, individual schemes and methods of vibration isolation are developed, and their corresponding mathematical models are studied.

## II. Derivation of the Motion Laws for the Mathematical Model of a Car Suspension with aHydraulic Support and Inertial Mass

We investigate the effect of an additional inertial mass inside the hydraulic support on the damping of both natural and forced oscillations of a part of the car. This considers a variant of passive vibration isolation of mobile machinery with a conventional metal spring, hydraulic damper, and hydraulic support with inertial mass

[1]. The overall model of the machine part's vibration isolation is represented in Figure 1 as a dynamic scheme with a single-axis hydraulic support and an inertial transformer [3-9]. It includes elements: $m_0$ – the mass of the base, $m_2$ – the mass of the inertial transformer, $M$ – the vibration-isolated mass, $c_2$ – the linear elasticity coefficient of the hydraulic support, $c_1$ – the elasticity coefficient of the cylindrical spring of the car suspension, $b_2$ – the damping coefficient of the hydrodynamic medium of the hydraulic support, $b_1$ – the damping coefficient of the hydraulic damper of the suspension with linear properties relative to the vertical speed of displacement movement.

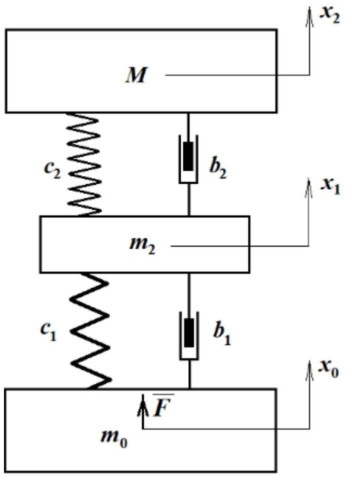


**Figure 1: General Dynamic Scheme of Vibration Isolation for a Body with Mass *M* and InertialMass $m_2$ of the Hydraulic Support**

The dynamic scheme in Figure 1 recreates the movements of a three-mass mechanical system, in which the masses $m_2$ and $M$ have finite values, and the mass $m_0$ is considered infinitely large, as it is identified with the base. The car wheel running over natural road surface irregularities leads to the application of a periodically varying vibrational load $F(t) = A\cdot\sin(\omega t)$, causing its kinematic excitation with vibration displacement $x_0$, having an amplitude a and frequency $\omega$, vibrational velocity $\dot{x}_0$, and vibrational acceleration $\ddot{x}_0$. These can be easily established through experimental measurements, using diagnostic sensors. To simplify the understanding of the influence of physical parameters on the most effective method of vibration isolation in this scheme, only vertical oscillations are considered, applying the equations of static equilibrium of the mechanical system as a whole. Based on these observations, the general mathematical model of the movements of the two-mass system will take the form:

$$\begin{cases} \ddot{x}_1 = b_{10}\dot{x}_0 + c_{10}x_0 - b_{11}\dot{x}_1 - c_{11}x_1 + b_{12}\dot{x}_2 + c_{12}x_2, \\ \ddot{x}_2 = b_{21}\dot{x}_1^2 + c_{21}x_1 - b_{22}\dot{x}_2^2 - c_{22}x_2, \end{cases} \quad (1)$$

where:

$$\begin{array}{lll} b_{10} = b_1/m_2, & b_{11} = (b_1 + b_2)/m_2, & b_{12} = b_2/m_2, \\ & b_{21} = b_2/M, & b_{22} = b_2/M, \\ c_{10} = c_1/m_2, & c_{11} = (c_1 + c_2)/m_2, & c_{12} = c_2/m_2, \\ & c_{21} = c_2/M, & c_{22} = c_2/M, \end{array} \quad (2)$$

$$x_0 = -a\cdot\sin(\omega t),$$

$$\dot{x}_0 = -a\cdot\omega\cdot\cos(\omega t),$$

$$\ddot{x}_0 = a\cdot\omega^2\cdot\sin(\omega t),$$

$$F(t) = m_0\ddot{x}_0(t).$$

Initial conditions at $t = 0$:

$$x_1(0) = 0.02 \text{ m}, \quad \left.\frac{dx_1}{dt}\right|_{t=0} = 0.05 \text{ m/s}, \quad x_2(0) = -0.01 \text{ m}, \quad \left.\frac{dx_2}{dt}\right|_{t=0} = 0.03 \text{ m/s}. \tag{3}$$

In canonical form [4, 6, 8], the system of equations (1) will become:

$$\begin{cases} L_1(x_1) - d_{12}(x_2) = -b_{10} a\omega \cos(\omega t) - c_{10} a \sin(\omega t), \\ -d_{21}(x_1) + L_2(x_2) = 0, \end{cases} \tag{4}$$

where $L_1(\bullet) = \frac{d^2}{dt^2} + b_{11}\frac{d}{dt} + c_{11}$, $L_2(\bullet) = \frac{d^2}{dt^2} + b_{22}\frac{d}{dt} + c_{22}$, $d_{12}(\bullet) = b_{12}\frac{d}{dt} + c_{12}$, $d_{21}(\bullet) = b_{21}\frac{d}{dt} + c_{21}$ – a collection of differential operators. In these notations, it is quite simple to separate the equations by variables $x_1(t)$ and $x_2(t)$

$$L_1(L_2(x_i)) - d_{12}(d_{21}(x_i)) = \frac{d^4 x_i}{dt^4} + (b_{11} + b_{22})\frac{d^3 x_i}{dt^3} + (c_{22} + c_{11} + b_{11}b_{22} - b_{12}b_{21})\frac{d^2 x_i}{dt^2} + \\ +(c_{22}b_{11} + c_{11}b_{22} - c_{21}b_{12} - c_{12}b_{21})\frac{dx_i}{dt} + (c_{22}c_{11} - c_{12}c_{21})x_i, \quad i = 1, 2. \tag{5}$$

then

$$\begin{cases} L_1(L_2(x_1)) - d_{12}(d_{21}(x_1)) = a(\omega^2(b_{11} + c_{10}) - c_{10}c_{11})\sin(\omega t) + a\omega(\omega^2 b_{10} - b_{11}c_{10} - b_{10}c_{11})\cos(\omega t), \\ L_1(L_2(x_2)) - d_{12}(d_{21}(x_2)) = a(b_{10}b_{21} - c_{10}c_{21})\sin(\omega t) - a\omega(b_{10}c_{21} + b_{21}c_{10})\cos(\omega t). \end{cases} \tag{6}$$

Based on the left-hand sides of system (6) and formula (5), the general characteristic equation is formed as follows:

$$\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0. \tag{7}$$

In this case, the stable, rapidly decaying movements of the dynamic scheme shown in Figure 1 and the mathematical model (4) will be solutions of the form:

$$x_i(t) = C_{i1}e^{-\lambda_1 t} + C_{i2}e^{-\lambda_2 t} + C_{i3}e^{-\lambda_3 t}\sin(\lambda_4 t) + C_{i4}e^{-\lambda_3 t}\cos(\lambda_4 t) + C_{i5}\sin(\omega t) + C_{i6}\cos(\omega t), \quad i = 1, 2. \tag{8}$$

The unknown arbitrary constants $C_{ij}$, $i = 1,2$, $j = \overline{1,6}$, must be determined from the solution of the homogeneous system of differential equations, taking into account the initial conditions (3) and the nonhomogeneous (4). The first four terms ensure the rapid convergence of the graphs of natural oscillation laws of motion $x_1(t)$ and $x_2(t)$ to their resting state $x_1(t) \to 0$ and $x_2(t) \to 0$. Their graphs rapidly decrease to zero values, thereby ensuring the rapid asymptotic damping of oscillations using hydraulic supports with inertial masses.

## III. Investigation of the Stability Criteria of the Dynamic System for Ensuring the Fastest Damping of Oscillations

We investigate the stability of the dynamic system based on the known algebraic criteria of V.S. Voronov [2]. For this purpose, from the characteristic equation (7), we write down the coefficients in accordance with the notations adopted in the stability criteria

$$\begin{aligned} a_0 &= c_{22}c_{11} - c_{12}c_{21}, \\ a_1 &= c_{22}b_{11} + c_{11}b_{22} - c_{21}b_{12} - c_{12}b_{21}, \\ a_2 &= c_{22} + c_{11} + b_{11}b_{22} - b_{12}b_{21}, \\ a_3 &= b_{11} + b_{22}, \\ a_4 &= 1. \end{aligned} \tag{9}$$

A necessary condition for the stability of the dynamic system will be the fulfillment of the inequalities

$$\frac{a_0}{a_2} < \frac{a_1}{a_3} < \frac{a_2}{a_4} < \ldots < \frac{a_{n-2}}{a_n}. \tag{10}$$

The sufficient conditions for stability are associated with restrictions of the form:

$$\Omega_k = \frac{a_k^2}{a_{k-1}a_{k+1}} > \sqrt{2.148}, \quad k = \overline{1, n-1}. \tag{11}$$

The conditions for stability with a margin are the requirements

$$W_k = \frac{a_k a_{k+1}}{a_{k-1}a_{k+2}} > 3, \quad k = \overline{1, n-2}. \tag{12}$$

The stability conditions formed according to the quality indicators of the dynamic system must satisfy the inequalities

$$\Omega_k > \sqrt{3}, \quad k = \overline{1, n-1}. \tag{13}$$

To ensure asymptotically stable, rapidly decaying oscillations, we require the fulfillment of the combination of the last two inequalities (12) and (13). Substituting formulas (2) into (9) and (12), (13) will lead to nonlinear multiparametric conditions, which in general cannot be resolved. To simplify these calculations, we will consider the variables in the form of corresponding comparisons of physical parameters with the same dimensions, let's assume

$$\frac{c_1}{c_2} = k, \quad \frac{b_1}{b_2} = l, \quad \frac{M}{m_2} = n \cdot \tag{14}$$

Then, assuming $\alpha = c_2 m_2 / b_2^2$ ($\alpha < 1$), we obtain new expressions for the coefficients (9) in the form:

$$a_0 = \frac{c_2^2 k}{m_2^2 n}, \quad a_1 = \frac{b_2 c_2 (k+l)}{m_2^2 n}, \quad a_2 = \frac{c_2 m_2 (n(k+1)+1) + b_2^2 l}{n m_2^2}, \quad a_3 = \frac{b_2 (n(l+1)+1)}{n m_2}, \quad a_4 = 1. \tag{15}$$

By substituting these values into the dependencies of the quality indicators of the dynamic system and the indicators of stability with a margin, we obtain

By substituting these values into the dependencies of the quality indicators of the dynamic system and the indicators of stability with a margin, we obtain

$$\begin{cases} \Omega_1 = \dfrac{(k+l)^2}{k(l+\alpha(n(k+1)+1))}, \\ \Omega_2 = \dfrac{(l+\alpha(n(k+1)+1))^2}{\alpha(k+l)(n(l+1)+1)}, \\ \Omega_3 = \dfrac{(n(l+1)+1)^2}{n(\alpha(n(k+1)+1)+l)}, \end{cases} \tag{16}$$

$$\begin{cases} W_1 = \dfrac{(k+l)(l+\alpha(n(k+1)+1))}{\alpha k(n(l+1)+1)}, \\ W_2 = \dfrac{(n(l+1)+1)(l+\alpha(n(k+1)+1))}{\alpha n(k+l)}. \end{cases} \tag{17}$$

Let's check what the coefficients $\Omega_i$ and $W_i$ will be for unlimitedly large values of the parameters $k$, $n$, and $l$. To do this, we calculate the corresponding limits

$$\begin{aligned} &1)\ \lim_{k\to\infty}\Omega_1 = \frac{1}{\alpha n} > \sqrt{3}, \\ &2)\ \lim_{n\to\infty}\Omega_1 = 0, \\ &3)\ \lim_{l\to\infty}\Omega_1 = \infty, \\ &4)\ \lim_{k\to\infty}\Omega_2 = \infty, \\ &5)\ \lim_{n\to\infty}\Omega_2 = \infty, \\ &6)\ \lim_{l\to\infty}\Omega_2 = \frac{1}{\alpha n} > \sqrt{3}, \\ &7)\ \lim_{k\to\infty}\Omega_3 = 0, \\ &8)\ \lim_{n\to\infty}\Omega_3 = \frac{(l+1)^2}{\alpha(k+1)} > \sqrt{3}, \\ &9)\ \lim_{l\to\infty}\Omega_3 = \infty. \end{aligned} \tag{18}$$

Based on the calculations made, it can be concluded that the limits in items 3-5 and 9 are always met, conditions 2 and 7 show that the parameters $k$, $n$ should never take excessively large values. Conditions 1 and 6 lead to the same answer

$$n < \frac{1}{\sqrt{3}\alpha}. \tag{19}$$

From item 8, we obtain a condition that is imposed on the parameter $k$

$$k < \frac{(l+1)^2}{\sqrt{3}\alpha} - 1. \tag{20}$$

Following this principle, let's write down the limits for the stability margin indicators $W_1$ and $W_2$

$$\begin{aligned}
&1)\ \lim_{k\to\infty} W_1 = \infty,\\
&2)\ \lim_{n\to\infty} W_1 = \frac{(k+l)(k+1)}{k(l+1)} > 3,\\
&3)\ \lim_{l\to\infty} W_1 = \infty,\\
&4)\ \lim_{k\to\infty} W_2 = 1 + n(l+1) > 3,\\
&5)\ \lim_{n\to\infty} W_2 = \infty,\\
&6)\ \lim_{l\to\infty} W_2 = \infty.
\end{aligned} \tag{21}$$

The four conditions from items 1, 3, 5, and 6 are always met. From the second condition, we derive the relationships between $k$ and $l$

$$k > l + 1 + \sqrt{l^2 + l + 1}. \tag{22}$$

Condition 4 from (21) imposes restrictions between $n$ and $l$

$$n > \frac{2}{l+1}, \tag{23}$$

which is always valid when $l \geq 1$.

Based on the calculations performed for the dynamic system in Figure 1 and to ensure its fastest damping of emerging vibrations, the following recommended methodology for selecting physical parameters and coefficients $k$, $n$, and $l$ should be adhered to:

1) the constant n is selected based on the inequality (19), given the known characteristics of the hydraulic support $\alpha = c_2 m_2 / b_2^2$;

2) the coefficient l is set arbitrarily and must be greater than one. From the double inequality (20) and (22), the magnitude of $k$ is selected

$$l + 1 + \sqrt{l^2 + l + 1} < k < \frac{(l+1)^2}{\sqrt{3}\alpha} - 1. \tag{24}$$

3) at this stage, the validity of all chosen coefficients $n$, $l$, and $k$ is checked from the condition (13) $\Omega_1 > \sqrt{3}$, then

$$n < \frac{k + (2-\sqrt{3})l}{\sqrt{3}(k+1)\alpha} + \frac{l^2}{\sqrt{3}(k+1)k\alpha} - \frac{1}{k+1}. \tag{25}$$

**IV. Verification of the Credibility of the Developed Methodology for Selecting Parameters of the Dynamic Damping of Harmful Oscillations**

Let's assume that for the hydraulic support scheme shown in Figure 1, all physical parameters such as $m_2 = 0.05$ kg, $b_2 = 70$ N·s/m, $c_2 = 50$ N/m are already known. To choose the value of $n$, we use the first point of the methodology $n < 1/\sqrt{3}\alpha$ or $n < 1131.61$. Naturally, the vibration-isolated body with mass $M$ is a thousand or more times greater than the inertial mass of the hydraulic support, so let's set $n = 1000$ and $M = 50$ kg.

Let's assume that the coefficient $l = 50$, then $b_1 = 3500$ N·s/m, and based on the second point of the methodology, we choose the value of $k$ from condition (24) $68.35 < k < 2943307.58$. Let's set $k$ to be 140, then $c_1 = 7000$ N/m.

The final stage will be the coordination of the found parameters that ensure rapid asymptotic damping of harmful oscillations affecting the vibration-isolated object. Substituting all the found parameter values into inequality (25), we get $n < 1374.41$. The chosen $n = 1000$ meets the last verification condition.

Let's verify whether the necessary stability conditions (10) of the dynamic system are satisfied: $a_0/a_2 = 0.586$, $a_1/a_3 = 3.725$, $a_2/a_4 = 239001$. The ratios of these parameters are arranged in ascending order, which precisely corresponds to inequality (10).

The quality stability conditions of the dynamic system, satisfying inequalities (13), have the values: $\Omega_1 = 2.115$, $\Omega_2 = 3.008$, $\Omega_3 = 21331.1238$. As can be seen, they are also fully met.

The stability margin conditions (12) are given by the values $W_1 = 6.36$ and $W_2 = 64154.158$, which also fully comply with these stability criteria.

All newly adopted values of physical quantities of the dynamic scheme in Figure 1 should be substituted into the problem conditions (4) with initial data (3). The new limitations on the parameters that we have adopted should be reflected in the asymptotic decrease in the amplitude of natural oscillations and the minimization of the amplitude of forced oscillations. Precise solutions to the original system of differential equations have been obtained at a frequency $\omega = 5$ rad/s and an amplitude $a = 0.007$ m of the external load regime, in which we distinguish the part with natural oscillations $x_1^o(t)$ and $x_2^o(t)$

$$x_1^o(t) = 0.01994412358 \cdot e^{-1.954579867 \cdot t} - 1.247729132 \cdot 10^{-6} \cdot e^{-71398.05261 \cdot t} - \\ - 0.00004010782238 \cdot e^{-0.6964060968 \cdot t} \cdot \sin(0.7198751436 \cdot t) + \\ + 0.00005706029411 \cdot e^{-0.6964060968 \cdot t} \cdot \cos(0.7198751436 \cdot t), \tag{26}$$

$$x_2^o(t) = - 0.01661789834 \cdot e^{-1.954579867 \cdot t} + 2.446617935 \cdot 10^{-11} \cdot e^{-71398.05261 \cdot t} + \\ + 0.002958048177 \cdot e^{-0.6964060968 \cdot t} \cdot \sin(0.7198751436 \cdot t) + \\ + 0.006615967231 \cdot e^{-0.6964060968 \cdot t} \cdot \cos(0.7198751436 \cdot t) \tag{27}$$

and forced $x_1^*(t)$ and $x_2^*(t)$

$$x_1^*(t) = - 6.887602129 \cdot 10^{-6} \sin(5 \cdot t) + 6.385292456 \cdot 10^{-8} \cos(5 \cdot t), \tag{28}$$

$$x_2^*(t) = - 2.576252663 \cdot 10^{-7} \cdot \sin(5 \cdot t) + 1.931082713 \cdot 10^{-6} \cdot \cos(5 \cdot t). \tag{29}$$

The general solutions satisfy the posed problem (4) with initial conditions (3) and should be combined as $x_1(t) = x_1^o(t) + x_1^*(t)$ and $x_2(t) = x_2^o(t) + x_2^*(t)$. All free coefficients are established in the form of dependencies relative to parameters and are not presented here due to their complexity [10 – 14].

Using the obtained formulas, it is easy to recreate the graphs of the laws of motion of the bodies under consideration, which are the vibration-isolated body and the inertial mass of the hydraulic support. Formulas for the damping natural oscillations are presented in Figures 2 and 3, and the general oscillations in Figures 4 and 5.

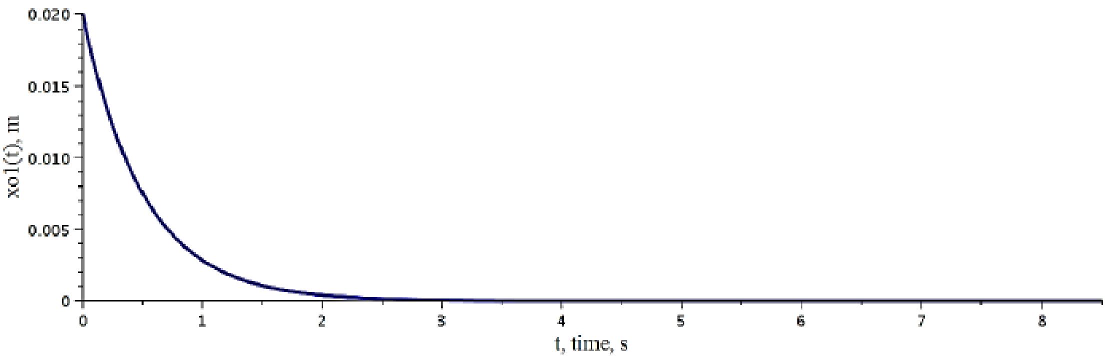


**Figure 2: Natural Oscillations of the Vibration-Isolated Body**

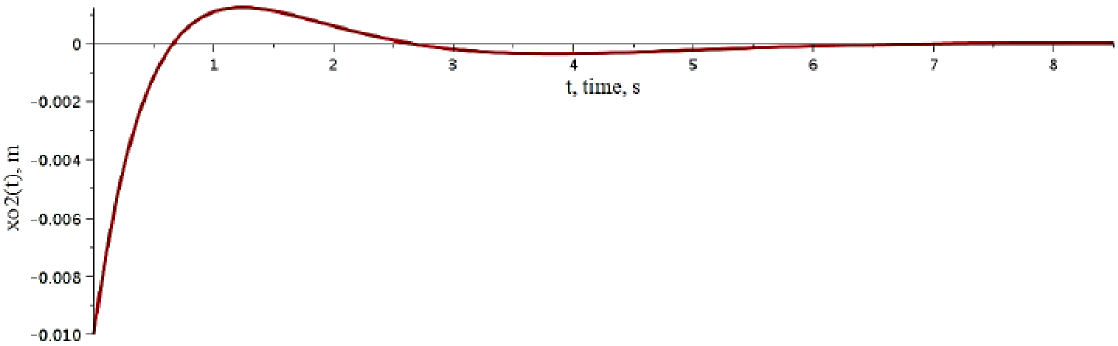


**Figure3: Natural Vibrations of the Inertial Mass of a Hydraulic Support**

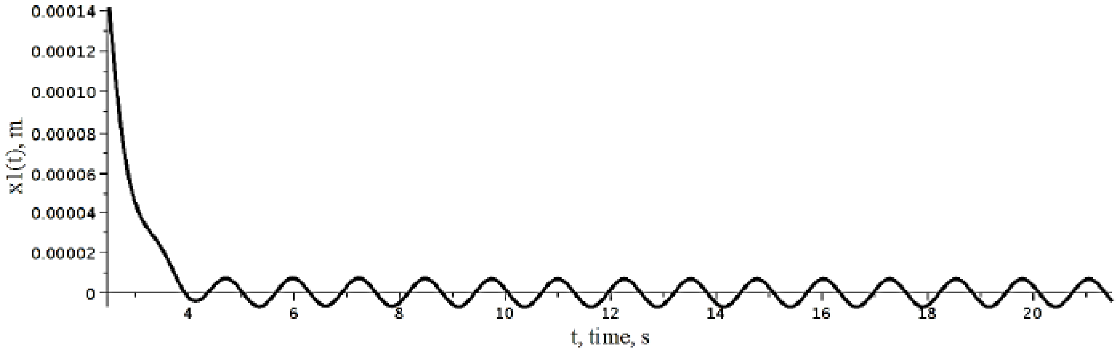


**Figure 4: General Vibrations of the Vibration-Isolated Body from the Time Moment *t* = 2.5 s**

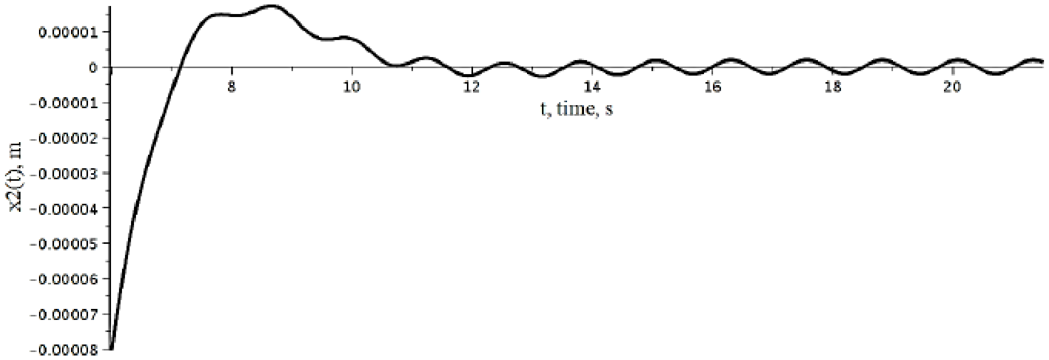


**Figure 5: General Vibrations of the Inertial Mass of a Hydraulic Support from the Time Moment *t* = 6 s**

Let's try to compare the vibration damping model with a vibro-support as shown in Figure 1 with new selected parameters, with the standard model without a hydraulic support, using the same parameters $m_2$, $b_2$, $c_2$. Let the parameters of the new problem remain the same: $M = 50$ kg, $b_1 = 3500$ N·s/m, $c_1 = 7000$ N/m, $\omega = 5$ rad/s, $a = 0.007$ m. We will find out which model of vibration damping, single-stage or two-stage, turns out to be better. Then, the differential equation of motion for a body with mass $M$ is

$$\frac{d^2x_1(t)}{dt^2}+b_{11}\frac{dx_1(t)}{dt}+c_{11}x_1(t)=-b_{10}a\omega\cos(\omega t)-c_{10}a\sin(\omega t)\,. \tag{30}$$

The solution to equation (30) with all known coefficients will be a dependency of the form $x_1(t) = x_1^o(t) + x_1^*(t)$, consisting of natural $x_1^o(t)$ and forced $x_1^*(t)$ vibrations

$$x_1^o(t) = 0.02146170134\cdot e^{-2.060661816\cdot t} - 0.001912981504\cdot e^{-67.93933818\cdot t}, \tag{31}$$
$$x_1^*(t) = -\,0.007148277768\cdot\sin(5\cdot t) + 0.0004512801621\cdot\cos(5\cdot t), \tag{32}$$

whose graph is shown in Figure 6.

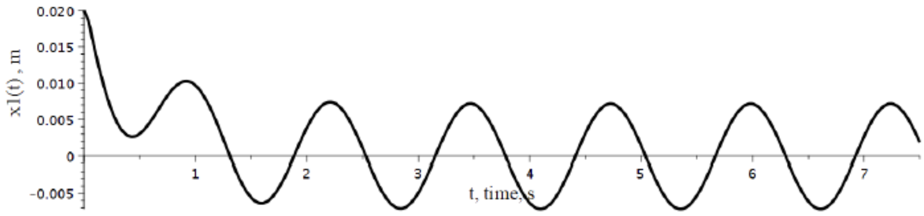


**Figure 6: General Vibrations of the Vibration-Isolated Body without a Hydraulic Support**

## V. CONCLUSION

Comparing the two models of vibration damping, the two-stage model with a hydraulic support as shown in Figure 4 and the single-stage model without a hydraulic support as shown in Figure 6, we conclude that the use of additional inertial masses $m_2$ in their designs, as in Figure 1, significantly increases the resistance of the medium, thereby reducing the vibrational load on the body isolated from harmful vibrations.

Stricter requirements and conditions imposed on physical parameters lead to complete asymptotic damping of natural vibrations in the two-element system, Figures 2 and 3, and a significant reduction in forced vibrations, Figures 4 and 5.

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