

Generation of Stepped and Combined Twill Weaves

Yumnam Kirani Singh

C-DAC Kolkata, Plot E2/1, Block-GP, Saltlake Electronics Complex, Kolkata-91 India

Abstract

Fabrics are formed by interlacement of warps and weft threads. Many different patterns can be produced on the fabric depending on how the interlacement are made during the weaving. There are different types of basic interlacement of the weaves. The most popular one is the twill weave. In this paper, we will study and derive the method or algorithms for generating stepped twill and combined twill weaves so that these weaves can be easily produced in a computer without having to draw manually by clicking individual grids using mouse. Three different methods of generating stepped twill weaves namely warp-way, weft-way, warp-weft stepped twill weaves are given. Four different types of combined twill weaves, warp-way, weft-way, warp-weft way and weft-warp combined twill weaves are also given in which the last two namely warp-weft and weft-warp are new methods introduced here. Using these methods, many different new and interesting stepped and combined twill weaves can be generated by simply changing the parameters.

Keywords: Twill Weave, Twill-weave derivatives, Stepped Twill weave, Warp-way Stepped Twill, Weft-way Stepped Twill, Warp and Weft-way Stepped Twill, Combined Twill weave, Warp-way Combined Twill, Weft-way Combined Twill, Warp-Weft combined Twill, Weft-Warp Combined Twill.

Date of Submission: 14-10-2023

Date of acceptance: 29-10-2023

I. INTRODUCTION

Weaving is the process of making fabrics by interlacement of warps and weave threads. There are mainly three different methods of interlacement known as basic, twill and satin weaves are used which gives three different types of weave patterns on the woven fabrics [1, 2, 3,7]. Usually, these weaves are drawn on a graph paper by the artists or weavers to understand more about the interlacement process before actually translating them on the fabric. This process is quite cumbersome and error prone. So, with the development in computing technology, many software applications have been developed [6] to enable the artists or weavers draw the weave patterns on a computer screen with mouse. Drawing graphs on computer has the flexibility of spotless editing and color filling options but still drawing weave pattern or graph is time consuming. On study of weave patterns, it is found they follow certain patterns which can be represented mathematically. If these weaves are represented mathematically or algorithmically, they can be easily generated instead of drawing by hands using pencil on a graph paper or with mouse on a computer screen. Another advantage is that different weave patterns can be generated by simply changing the parameters. It has been mentioned in [8] that the interlacement in weave patterns can be represented or expressed using circulant matrices. Methods for generating plain and twill weaves patterns using circulant matrices are given in [9]. Also, methods for generating regular satin and sateen weaves from circulant matrices are given in [9]. Moreover, new methods of generating irregular satin weaves are given in [10]. From these methods, it is circulant matrices mainly the circular shifts can represent the weave interlacement process. Circulant matrices have many useful properties which have been used in many mathematical transforms [4,5].

Many different weave patterns can be produced or formed by combining or rearranging the basic twill weaves such as plain, twill and satin weaves. These weaves which are obtained from the basic weaves are known as their derivatives. Of the three basic twill weaves, twill weaves have more derivatives. Some of the twill will derivatives are broken twill weaves, combined twill weaves, elongated twill weaves, re-arranged twill weaves, shaded twill weaves, stepped twill weaves etc. In this paper, we will be discussing more details about the stepped and combined twill weaves which are derivatives of the twill weaves. Stepped wills are generated from a given twill weave while the combined twill weaves are formed by combining two or more twill weaves. But usually only two twill weaves are used to draw combined twill weaves. In this paper also, we will manly study the methods for generating combined twill weaves from two twill weaves only. The paper is divided into five sections. In section II, we mainly analyze the methods of drawing different types of stepped twill weaves. Section III describes the different possible ways of forming combined twill weaves. Usually only two different types of combined twill weaves- namely warp way combined twill weaves and weft-way combined twill weaves. In addition, two new

combined twill weaves warp-weft combined twill weaves and weft-warp combined twill weaves are introduced. In Section IV, different types of stepped and combined weaves are provided for different parameters of twill weaves. Some conclusions are given in Section V.

II. Stepped Twill Weaves

These weaves are generated from a twill weave having step like patterns. These weaves have twill lines with angles more than or less than 45 degrees. Stepped twill weaves having angles less than 45 degrees are known as Warp-way twill weaves. Stepped twill weaves having angles more than 45 degrees are known as weft-way twill weaves. Warp-way twill weaves are elongated horizontally and weft-way twill weaves are elongated vertically. By making a twill weave extended in both horizontal and vertical directions, we get warp-and weft way stepped twill weaves. So, stepped twill weaves are divided into the following three categories.

Stepped Twill Weaves:

Warp -way Stepped Twill Weaves

Weft-way Stepped Twill Weaves

Warp and Weft way stepped Twill weaves

We will give the mathematical ways or methods for deriving these twill weaves in the following three subsections.

2.1 Warp-Way Stepped Twill Weaves:

Suppose X is a twill weave matrix of u ups and d downs. Then, warp-way stepped twill weave is given by

$$Y = [Y_1 \ Y_2 \ Y_3 \ \dots \ Y_{u+d}]$$

Where Y_1 is the twill weave having u ups and d downs, Y_2 is right circular shift of Y_1 by 1, Y_3 is the right circular shift of Y_2 by 1 and so on.

If $C(x,n)$ denotes the right circular shift function of x by n-columns, then we can write the successive circular shifts of twill weaves as

$$Y_k = C(Y_{k-1}, 1)$$

Expressing Y_k in terms of Y_1 , we can write

$$Y_k = C(Y_1, k - 1)$$

So, we can write the warp-way stepped twill weave in terms of Y_1 as

$$Y = [Y_1, C(Y_1, 1), C(Y_1, 2), \dots, (Y_1, u + d - 1)]$$

Example: Warp-way Stepped Twill weave components of twill 2/2

$$Y_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}, \quad Y_3 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \quad Y_4 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Concatenating the circulant shift matrices along columns, as $Y = [Y_1, Y_2, Y_3, Y_4]$, we get the Warp-way Stepped twill Y which will have the step like weave patterns as shown below.

$$Y = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$



Following is the Scilab code for generating warp-way stepped twill weave from a given twill weave. It takes a weave matrix of a twill weave and concatenates horizontally all the successive circular shifts in a loop.

```
function y=warpSteppedTwill(x)
// x is a twill weave matrix
[m,n]=size(x);
y=[x];
for i=1:(n-1)
    y=[y x(:,1+ modulo([1:n]+i-1,n))];
end
endfunction
```

2.2 Weft-Way Stepped Twill Weaves:

Suppose X is a weave matrix of u ups and d downs. Then, weft-wise stepped twill weave can be generated by stacking the left circular shifts of the twill weaves in vertical direction. is given by

$$Y=[Y_{u+d}; \dots; Y_2; Y_1]$$

Where Y1 is the twill weave having u ups and d downs, Y2 is left circular shift of Y1 by 1, Y3 is the left circular shift of Y2 by 1 and so on.

In short, $Y_1=X$;

$$Y_i=Y_{i-1} \% (u+d) + 1, \text{ where } i=2, 3, \dots, u+d.$$

In terms of X or Y1, we can write

$$Y_i=Y_1+i-1 \% (u+d)+1;$$

Example-2: Weft-way Stepped Twill

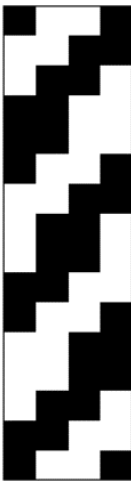
Stacking the left circular shift matrices Y1, Y2, Y3, and Y4 in Example-1, we get the weft-way stepped twill weave Y as

$$Y = \begin{bmatrix} Y_4 \\ Y_3 \\ Y_2 \\ Y_1 \end{bmatrix}$$

If we consider Y1 as weave matrix of twill weave 2/2 as shown below.

$$Y_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Then, by stacking the right circular shifts of generated from the weave matrix of 2/2, we get the following weave matrix as weft-way stepped twill weave matrix.

$$Y = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$


```
function y=weftSteppedTwill(x)
// x is a twill weave matrix
[m,n]=size(x);
y=[x];
for i=1:(m-1)
    y=[x(1+ modulo([1:m]+i-1,m),:); y];
end
endfunction
```

2.3 Warp-Weft Stepped Twill Weaves:

In this stepped twill weave, the size of the generated stepped twill weave will be square. It can be considered as extension of warp-way stepped twill weave weft wise, or extension of weft-way stepped twill warp-wise.

Suppose, Y_1 is a twill weave of size $M \times M$. Let, Y_2, Y_3, \dots, Y_M are respectively the left circular shifts of Y_1 by 1, 2, ..., $M-1$. Then, the Warp and Weft-way Stepped Twill Y is given by the circulant matrix

$$Y = \begin{bmatrix} Y_1, Y_2, Y_3, \dots, Y_M \\ Y_2, Y_3, Y_4, \dots, Y_1 \\ Y_3, Y_4, Y_5, \dots, Y_2 \\ \dots \dots \dots \\ Y_M, Y_1, Y_2, \dots, Y_{M-1} \end{bmatrix}$$

Example-3: Warp and Weft-way Stepped Twill

Here, the stepped twill considered is

$$Y_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

The generated Warp and Weft-way stepped twill generated from Y is given by

$$Y = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$



Following is the Scilab code for generating warp and weft way stepped twill weave. It takes a weave matrix of a twill weave and generates the weave matrix of warp and weft-way twill weaves using two loops. In each iteration of the first loop, a warp-way twill weave is generated using the second loop which is then concatenated vertically to get the warp and weft-way stepped twill weave.

```
function y=WarpWeftSteppedTwill(x)
// x is a twill weave matrix
[m,n]=size(x);
y=[];
for i=1:m
    yt=[];
    for j=1:n
        yt=[yt x(1+ modulo([1:m]+i-1,m),1+ modulo([1:n]+j-1,n))];
    end
    y=[y; yt];
end
endfunction
```

III. COMBINED TWILL WEAVES

Combination twills find extensive use in the worsted industry in the production of garment fabrics, as these weaves are capable of producing compact textures. In these types of weaves, two different types of continuous twills are combined together alternately. The combination may be warp-way or weft-way. Accordingly, warp or weft faced twills may be used suitably. However, two different types of combined twill weaves can be added – i.e., warp-weft combined twill weave and weft-warp combined twill weaves. This is because combining two twill weaves first warp-way then weft-way is not the same combining first weft-way then combine warp-way. All these four different types of combined twill weaves are described in the following subsections.

3.1 Warp-Way Combined Twill Weaves:

In warp-way combined twill weaves, the number of warps is doubled as compared with the number of wefts. This is because the combined weave is formed by alternating warp lines of the two twill weaves. For example, if T is the combined twill weave of two twill weaves T1 and T2 each of which has the same size mxm. Then, the size of T will be mx2m where odd warps of T are successively the warps of T1 and even warps of T are successively the warps of T2. Mathematically, we can write

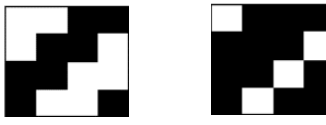
$$T(i, 2*j-1) = T1(1+(i-1\%m), 1+(j-1\%m))$$

$$T(i, 2*j) = T2(1+(i-1\%m), 1+(j-1\%m))$$


Where $i=1, 2, \dots, m$ and $j=1, 2, \dots, m$ and the operator % denotes the modulus operation which gives the remainder.

We can exchange the positions of warps of twill weaves T1 and T2 in the combined twill weave.

Consider the following twill weaves having the same size 4x4. The first twill weave is 2-up 2-down twill, the second twill weave is 3-up 1-down twill whose weaves matrices and weave patterns are shown below.

$$T_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \text{ and } T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$


If the two twill weaves (i.e., 2/2 twill and 3/1 twill) are combined warp-way, the resulting twill weave will have the size 4x8. The resulting weave matrix and the corresponding weave pattern are shown below.

$$T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$


In case, the sizes of the two different twill weaves are not the same, then the resulting combined twill weave will have the size Lx2L for the warp-way combined twill, where L is the LCM of the sizes of the two different twill weaves.

The code for generating the warp-way combined twill weave remains the same except that loop iterates from 1 to L as

$$T(i, 2*j-1) = T1(1+(i-1\%m), 1+(j-1\%n))$$

$$T(i, 2*j) = T2(1+(i-1\%m), 1+(j-1\%n))$$

Where, $i=1, 2, \dots, L$ and $j=1, 2, \dots, L$.

So, the scilab code for generating the weft way combined twill weave is as follows.


```
function y=warp-way(a, b)
    m=size(a,1);
    n=size(b,1);
    k=lcm([m,n]);
    for i=1:k
        for j=1:k
            y(i,2*j-1)=a(1+modulo(i-1,m), 1+modulo(j-1,n));
            y(i,2*j)=b(1+modulo(i-1,m), 1+modulo(j-1,n));
        end
    end
endfunction
```

3.3 Weft-Way Combined Twill Weaves:

In weft-way combined twill weave, the rows of the first twill and the rows of the second twill are combined alternately row-wise. So, in the resulting combined twill weave, the number of wefts is doubled while the number of warps remains the same.

If the two twill weaves having the size 4x4 are combined weft-way, the resulting weave will have the size 8x4.

Consider the two twill weaves of 2/2 and 3/1 used in warp-way combined weave. If these two twill weaves are combined weft-wise, we get the following weave matrix and the weave pattern.

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$


In case, the sizes of the two different twill weaves are not the same, then the resulting combined twill weave will have the size 2LxL for the weft-way combined twill, where L is the LCM of the sizes of the two different twill weaves.

The code for generating the weft-way combined twill weave remains the same except that loop iterates from 1 to L as

```
T(2*i, j)=T1(1+(i-1*m), 1+(j-1*n))
T(2*i, 2*j)= T2(1+(i-1*m), 1+(j-1*n))
Where, i=1, 2, ..., L and i=1,2. ..., L.
```

So, the scilab code for generating the weft way combined twill weave is as follows.

```
function y=weft-way(a, b)
    m=size(a,1);
    n=size(b,1);
    k=lcm([m,n]);
    disp(k);
    for i=1:k
        for j=1:k
            y(2*i-1,j)=a(1+modulo(i-1,m), 1+modulo(j-1,n));
            y(2*i,j)=b(1+modulo(i-1,m), 1+modulo(j-1,n));
        end
    end
endfunction
```

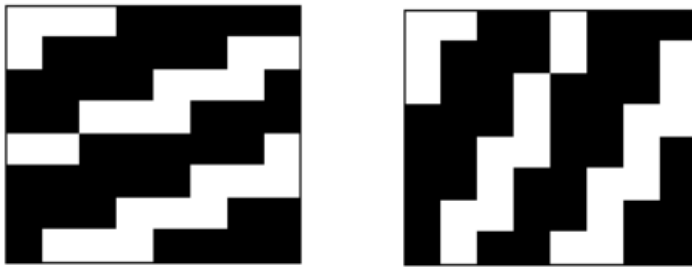
The function takes two twill weave matrices a and b and generate the weft-way combined twill weave matrix y.

3.4 Weft-Way Combined Twill Weaves:

It is seen that the warp-way twill increases the size of twill weave lengthwise and the weft way combined twill weave increases size of the twill weave breadthwise. We are interested to get a combined twill weave which increases the size in both ways -i.e., lengthwise and breadthwise. So, the size of the resulting combined twill weave will have the size 2Lx2L. This can be done in two ways – (a) by filling the L+1 to 2L rows by the left or right circular shift of the first L rows of the warp-way combined twill weave by 1 and (b) by filling the L+1 to 2L columns by left or right circular shift of the left half of the weft way combined twill weave by 1.

$$T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$



The scilab code for generating warp-weft combined twill weave consists of following three statements.

```
Y=warpway(a,b);
Y1=[Y(2:$, : ) ; Y(1,:)];
Y=[Y; Y1];
```

The first line generates the warp-way twill weave from the twill weaves. The second line generates the right circular shift of the warp-way twill weave. The third statement concatenates the two vertically.

In the similar way, we can generate the weft and warp way combined twill weave from the weft-way combined twill weave by concatenating the circular shift in vertical direction. The scilab code for generating warp-weft combined twill weave consists of following three statements.

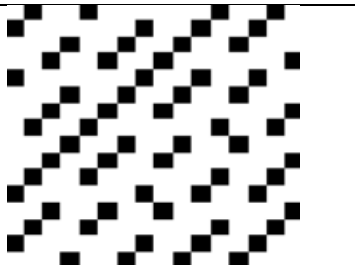
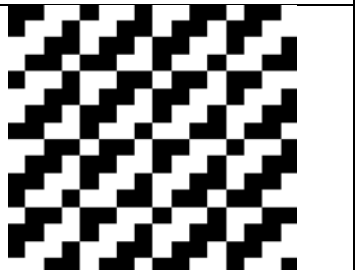
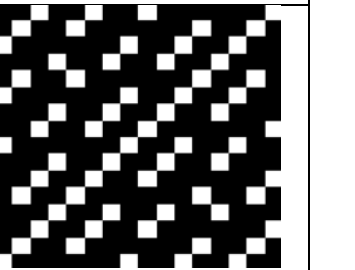

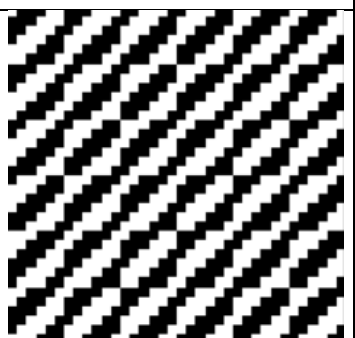
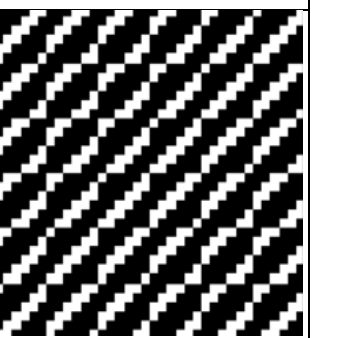
```
Y=weftway(a,b);
Y1=[Y(:, 2:$) Y(:,1)];
Y=[Y Y1];
```

The first line generates the weft-way twill weave from the twill weaves. The second line generates the right circular shift of the weft-way twill weave. The third statement concatenates the two horizontally to generate the warp and weft way combined twill weave.

IV. EXPERIMENTAL RESULTS

We have seen that there are three different types of stepped twill weaves which can be generated from a given twill weave having specific repeat size of M ups and N downs. Warp-way Stepped twill weave will have the size $(M+N)$ -by- $(M+N)^2$, whereas Weft-way Stepped twill weave will have the size $(M+N)^2$ -by- $(M+N)$ and the warp-weft stepped twill of repeat size $(M+N)$ will have the size $(M+N)^2$ -by- $(M+N)^2$. This is because in warp war stepped twill weave the number of warps is increased by power of 2 as compared to the number of wefts. Similarly, the number of wefts is increased by the power of 2 as compared to the number of warps. In the warp-weft stepped twill weave, the numbers of warps and wefts are increased by power of 2. To save space, we are displaying only two different warp-weft stepped twill weaves. Table-1 shows stepped twill weaves generated from the twill weaves having repeat size 4 and 5. The first three warp-weft stepped twill weaves are generated from the twill weaves 1 up 3 down i.e. Twill(1,3) , 2-up 2-down i.e., Twill (2,2) , 3-up and 1-down, each of which has the repeat size 4. The second row of weave patterns are generated from the twill weaves of repeat size 6 having 2-up 4-down, 3-up 3-down and 4-up 2 down twills. It may be seen that these stepped twill weaves generated from the repeat size 4 have size 16x16 and those generated from repeat size 6 have size 36x36.

Table-1: Warp-Weft Stepped twill weaves generated from twill weaves of repeat size 4 and 6.

Repeat Size	Twill(1,3)	Twill (2,2)	Twill(3,1)
4			
Repeat Size	Twill(2,4)	Twill(3,3)	Twill(4,2)
5			

Combined twill weaves are formed from two different twill weaves by combining or inserting warps and wefts in warp-wise and weft-wise directions. Combined have four different types, warp-way, weft-way, warp-weft and weft-warp combined twill weaves. The size of a combined twill weave is dependent on the size of sizes of the two twill weaves to be combined. In combined twill weaves, the warp-weft combined twill weave is not the same as weft-warp combined twill weave unlike in stepped twill weave. However, both have the same size of a square matrix having the number of wefts and warp lines. We will show here only four combined twill weaves – two for warp-weft combined twill weaves and two for weft-warp combined twill weaves.

Table-2: warp-Weft Combined Twill weaves generated from twill weaves of repeat size-4

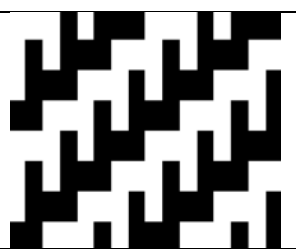
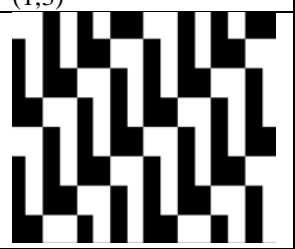
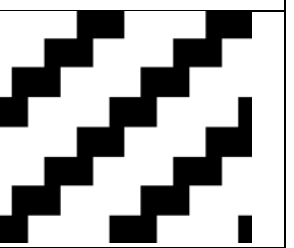
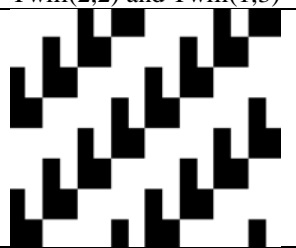
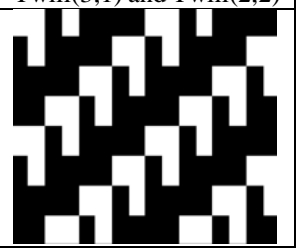
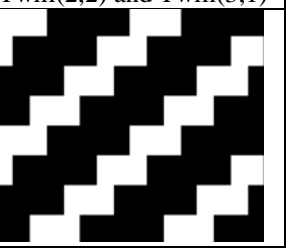
Way of combination	Twill(1,3) and Twill(3,1)	Twill(3,1) and Twill (1,3)	Twill(1,3) and Twill(2,2)
Combined Twill weaves			
Way of Combination	Twill(2,2) and Twill(1,3)	Twill(3,1) and Twill(2,2)	Twill(2,2) and Twill(3,1)
Combined Twill Weaves			

Table-3: Weft-Warp Combined Twill weaves generated from the twill weaves of repeat size-4

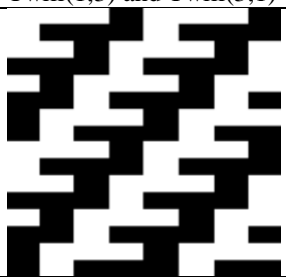
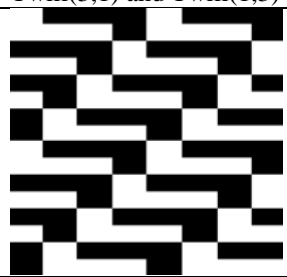
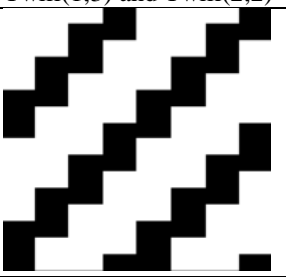
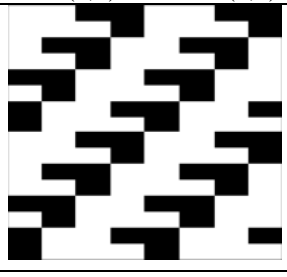
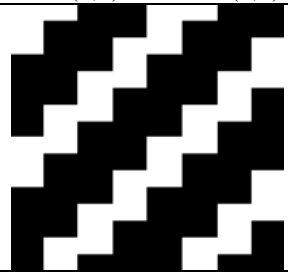
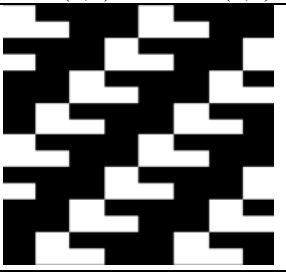
Way of Combination	Twill(1,3) and Twill(3,1)	Twill(3,1) and Twill(1,3)	Twill(1,3) and Twill(2,2)
Combined Twill Weaves			
Way of Combination	Twill(2,2) and Twill(1,3)	Twill(3,1) and Twill(2,2)	Twill(2,2) and Twill(3,1)
Combined Twill Weaves			

Table-2 shows the warp-weft combined twill weaves generated from combination two twill weaves of repeat size 4 in different ways. In the warp-weft combined twill weaves, first the two twill weaves are combined in warp way, then they are combined in weft ways to get square weave pattern, i.e., weave patterns having the same number of rows and same number of columns. The first combined twill weave in Table-2 is formed by combination of Twill(1,3) and Twill(3,1), the second combined twill weave is formed by the combination of the Twill(3,1) and Twill(1,3). It may be seen that two weave patterns are completely different although they are formed the two twill weaves Twill(1,3) and Twill(3,1). This shows that in warp-weft combined twill weaves, we get two different combined twill weaves from the same pair of twills depending on in which order they are combined. So, we get two different combined twill weaves from the combination of twill weaves (1,3) and (2,2) and another two combined twill weaves from the combinations of twill weaves (3,1) and (2,2) in different orders. These are the six possible combined twill weaves which can be generated from the three possible twill weaves having repeat size 4, i.e. (1,3), (2,2) and (3,1). If these three are combined in different ways by taking two of them at a time, we get 6 possible combinations as given in Table-2. All the six weaves are different and have aligned the patterns along the off-diagonal lines as these are derived from Z- twills. If these weaves are generated by combining two S-twills, the combined twill weaves will have patterns aligned along the diagonal line.

Table-3 shows the weft-warp combined twill weaves generated from the combination of two different z-twill weaves of repeat size-4. The first combined twill weave in this table is generated from the combination Twill(1,3) and Twill(3,1). As compared with the combined twill weave of Table-2, this combined twill weave is very significantly different. Similar is the case for the remaining five combined twill weaves in the Table-3, they are different from the remaining five combined twill weaves in Table-2. This shows that warp-weft combined Twill weave is different from weft-warp combined twill weave for the same twill weaves inputs given in the same order. So, altogether we can have 12 different combined twill weaves from the twill weaves of repeat size 4. More weave patterns can also be obtained from these combine twill weave by performing simple operations such as inverse, flip and rotation on them. For example, another 12 different combined weaves can be obtained from these combined twill weaves by simply inverting them, i.e., making white to black and black to white. Also, many more different combined twill weaves can be obtained from the twill weaves of larger repeat sizes or by combination of twill weaves of different repeat sizes.

V. CONCLUSION

In this paper, different types of stepped and combined twill weaves have been described and corresponding algorithms for generating them have also been provided. This will be helpful to students or weavers who are interested to know about the interlacement process of creating stepped and combined twill weaves. All the three types of stepped twill weaves, namely, warp way stepped twill, weft-way stepped twill, warp-weft stepped twill weaves have been covered. In combined twill weaves, in addition to known two types i.e., warp way combined twill, weft-way combined twill weaves, two more new combined twill weaves named warp-weft combined twill weaves and weft-warp combined twill weaves have been introduced. From the given algorithms, it is possible to generate different new patterns of stepped and combined twill weaves can be easily generated by simply changing the parameters without having to draw manually on a graph paper or computer screen when a new stepped or combined twill is desired or required.

REFERENCES

- [1]. Saharon D. Alderman, "Mastering Weave Structures: Transforming Ideas into great fabrics", Interweave Press, 2004.
- [2]. M. K. Bansal, "Basic Weaves", <https://www.slideshare.net/Amitsirohi2/fabric-structure-53197193>
- [3]. Vasant R. Kothari, "Satin and Sateen Weave", http://vasantkothari.com/content/view_presentation/425/16-Satin-and-Sateen-Weave
- [4]. P. J. Davis, "Circulant Matrices", AMS Chelsea Publishing, 1994
- [5]. G. M. Gray, "Toeflitz and Circular Matrices"-A Review, <https://ee.stanford.edu/~gray/toeflitz.pdf>
- [6]. Popular list of software for weave design, <http://www.handweaving.net/weaving-software>
- [7]. Satin and other weaves, https://www2.cs.arizona.edu/patterns/weaving/monographs/ics_507.pdf
- [8]. Y. Kirani Singh, "Generation of Plain and Twill Weaves from Left Circulant Matrices", International Journal of Research in Engineering and Sciences, Volo.10, No. 10, pp. 283-291, 2022.
- [9]. Y. Kirani Singh, "Automatic generation of satin and sateen weaves from circulant matrices", International Journal of Research in Engineering and Sciences, Volo.10, No. 11, pp. 104-110, 2022.
- [10]. Y. Kirani Singh, "Generation of Irregular Satin and Sateen weaves", International Journal of Research in Engineering and Sciences, Volo.11, No. 1, pp. 68-77, 2023.