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Improved Exponential Estimator For Estimating The Product Of Two Population Means Using Auxiliary character

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ABSTARCT
The main aim in estimation of parameters is to getmore précised estimators. It is also observed that with inclusion of more information within the estimation procedure produces better estimators, provided the data must be valid and proper. Auxiliary information has been used in the product method of estimation to attainabetter estimator of the population mean. In product method of estimation,an auxiliary variable is used which is linearly highly negative correlated with the variable under study and help in estimation of population mean.
The problems of estimating the ratio and product of two population means are very common in practice in the field of medical, engineering, socio-economic, forest surveys etc. When on performing sample surveys in agricultural, socio-economic, and forest research, the calculation of the product of two population means may be of significance.

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I. INTRODUCTION

The problems of estimating the ratio and product of two population means are very common in practice like- in medical, engineering, socio-economic, forest surveys etc.

When on performing sample surveys in agricultural, socioeconomic, and forest research, the calculation of the product of two population means may be of importance.

For instance, if we are willing to estimate the total inhabitants of people in a Block then villages can treated as a sampling units. We can estimate the product of (i) average number of occupied houses per village and (ii) average number of people in each house per village, and then the inhabitants of the Block can be calculated by multiplying the estimate of product by the number of villages. The area of the village, the number of cultivators, the number of agricultural laborers, and other auxiliary characteristics may be employed in this context.

Singh (1965, 67) has proposed the estimators for the estimation of $R$and P by using an auxiliary character x_1 as

$$R_1 = \hat{R}\left(\frac{\hat{\bar{X}}_1}{\bar{X}_1}\right), \qquad R_2 = \hat{R}\left(\frac{\bar{X}_1}{\hat{\bar{X}}_1}\right) \qquad R_3 = \hat{R}\left(\frac{\hat{\bar{X}}_1}{\bar{X}_1}\right)^{\alpha}$$

$$P_1 = \hat{P}\left(\frac{\hat{\bar{X}}_1}{\bar{X}_1}\right) \qquad P_2 = \hat{P}\left(\frac{\bar{X}_1}{\hat{\bar{X}}_1}\right) \qquad P_3 = \hat{P}\left(\frac{\hat{\bar{X}}_1}{\bar{X}_1}\right)^{\beta},$$

whereα and β are constants.

Tripathi (1970,80) has considered a generalized estimator R_T for R defined by

$$R_T = \frac{\hat{\bar{Y}}_1 - t_1\left(\hat{\bar{X}}_1 - \bar{X}_1\right)}{\hat{\bar{Z}}_1 - t_2\left(\hat{\bar{X}}_1 - \bar{X}_1\right)}$$

It may be noted that R,$\hat{R}_1$,$\hat{R}_2$ and $\hat{R}_3$ defined above may be derived as a particular member of R_T. The optimum choices of t_1 and t_2 are shown to be the sample regression coefficients.

Singh (1982a, 82b) has proposed the family of estimators for R and P which are given by

$$\tilde{R}_h = \hat{R}h\left(\frac{\hat{\bar{X}}_1}{\bar{X}_1}\right), \qquad \hat{R}_g = g\left(\hat{R}, \frac{\hat{\bar{X}}_1}{\bar{X}_1}\right)$$

$$\tilde{P}_h = \hat{P}h\left(\frac{\hat{\bar{X}}_1}{\bar{X}_1}\right), \qquad \hat{P}_g = g\left(\hat{P}, \frac{\hat{\bar{X}}_1}{\bar{X}_1}\right)$$

Where$h(1)$ = 1, $g(R, 1)$= R, $g(P, 1)$ = P and $h(\cdot)$ and $g(\cdot,\cdot)$ satisfy some regularity conditions. Ray and Singh (1985) have also proposed the estimator for R and P given by

$$\hat{R}_{RS} = (1+W)\hat{R} - W\hat{R}\left(\frac{\hat{\bar{X}}_1}{\bar{X}_1}\right)$$

and $$\hat{P}_{RS} = (1+W)\hat{P} - W\hat{P}\left(\frac{\hat{\bar{X}}_1}{\bar{X}_1}\right)$$

Rao (1957) and Rao and Pereira (1968) have suggested the estimators for R by utilizing the two auxiliary characters x_1 and x_2 given by

$$. R_4 = \hat{R}\left(\frac{\hat{\bar{X}}_1}{\hat{\bar{X}}_2}\right)\left(\frac{\bar{X}_2}{\bar{X}_1}\right)$$

and $$R_5 = \hat{R}\left(\frac{\bar{X}_1}{\bar{X}_2}\right)\left(\frac{\hat{\bar{X}}_2}{\hat{\bar{X}}_1}\right)$$

respectively. Singh (1969) considered the estimators

$$R_6 = \hat{R}\left[\left(\frac{\hat{\bar{X}}_1}{\bar{X}_1}\right)^{\alpha_1}\left(\frac{\hat{\bar{X}}_2}{\bar{X}_2}\right)^{\alpha_2}\right]$$

and $$R_7 = \hat{R}\left[W_1\left(\frac{\hat{\bar{X}}_1}{\bar{X}_1}\right)^{\alpha_1} + W_2\left(\frac{\hat{\bar{X}}_2}{\bar{X}_2}\right)^{\alpha_2}\right]$$

and Shah and Shah (1978) considered

$$R_7 = \hat{R}\left[W_1\left(\frac{\hat{\bar{X}}_1}{\bar{X}_1}\right) + W_2\left(\frac{\hat{\bar{X}}_2}{\bar{X}_2}\right)\right]^{\alpha}$$

where α_1, α_2 and α are constants and $W_1 + W_2 = 1$.

Khare (1987b) modified the estimator of Singh (1986) type for some assumptions regarding the value of derivatives of the function where the estimator of Singh (1986) does not hold. Another estimator by Khare (1988) is

$$\hat{R}_{KH} = h\left(\hat{\bar{Y}}_1, \frac{\hat{\bar{X}}_1}{\bar{X}_1}\right) . g\left(\hat{\bar{Z}}_1, \frac{\hat{\bar{X}}_1}{\bar{X}_1}\right)$$

This class $\hat{R}_{KH}$ attains same minimum value of MSE as $\tilde{R}_h$ and it also includes more members which are neither covered by $\tilde{R}_h$ nor by Singh (1986) estimators.

When the population means $\bar{X}_1, \bar{X}_2, \dots\dots, \bar{X}_p$ of p-auxiliary characters $x_1, x_2 \dots x_p$ are known, Singh (1965) suggested the estimator for R and P given by

$$\hat{R}_p = \hat{R}\prod_{i=1}^{q}\left(\frac{\hat{\bar{X}}_i}{\bar{X}_i}\right)\prod_{i=q+1}^{p}\left(\frac{\bar{X}_i}{\hat{\bar{X}}_i}\right)$$

and $$\hat{P}_p = \hat{P}\prod_{i=1}^{q}\left(\frac{\hat{\bar{X}}_i}{\bar{X}_i}\right)\prod_{i=q+1}^{p}\left(\frac{\bar{X}_i}{\hat{\bar{X}}_i}\right)$$

Tripathi and Chaturvedi (1979) have extended the class of estimators $\tilde{R}$ due to Tripathi (1970,80) which is given as

$$\tilde{R}_T = \frac{\hat{\bar{Y}} - \sum_{i=1}^{p} t_i^{(1)}(\hat{\bar{X}}_i - \bar{X}_i)}{\hat{\bar{Z}} - \sum_{i=1}^{p} t_i^{(2)}(\hat{\bar{X}}_i - \bar{X}_i)}$$

where $t_i^{(1)}$ and $t_i^{(2)}$ are suitably chosen statistics.

Khare (1983, 91a, 92b) suggested the class of estimators for R and P by following Srivastava (1971) which are given by

$$\tilde{R}_k = \hat{R}h(u_1, u_2, \cdots, u_p), \quad R_k^* = g(\hat{R}, u_1, u_2, \cdots, u_p),$$

and $\tilde{P}_k = \tilde{P}h(u_1, u_2, \cdots, u_p)$, $P_k^* = g(\hat{P}, u_1, u_2, \cdots, u_p)$, where $u_i = \frac{\hat{\bar{X}}_i}{\bar{X}_i}$, $h(1,1\cdots1) = 1$, $g(R, 1,1\cdots1) = R$, and $g(P, 1\cdots1) = P$ and $h(\cdot)$ and $g(\cdot,\cdot)$ satisfy some regularity conditions.

Srivastava *et al*. (1988) have given a generalized estimator for R and P by using auxiliary characters defined by $R_{sks}^* = \hat{R}\left(\frac{\bar{x}_{1m}}{\bar{x}_{1n}}\right)^{\alpha_1}\left(\frac{\bar{x}_{2n}}{\bar{X}_2}\right)^{\alpha_2}$ and $P_{sks}^* = \hat{P}\left(\frac{\bar{x}_{1m}}{\bar{x}_{1n}}\right)^{\beta_1}\left(\frac{\bar{x}_{2n}}{\bar{X}_2}\right)^{\beta_2}$ where α_1, α_2 and β_1, β_2 are constants.

Some chain-ratio-type estimators for ratio of the population means using double sampling scheme have been considered by Singh *et al*. (1994), Prasad *et al*. (1996) and Singh & Singh (1997-98). Further Khare and Srivastava (1998) have proposed combined generalized chain (CGC) estimators for ratio and product of the population means using two auxiliary characters and double sampling scheme which becomes unbiased for a proper choice of weights.

In case $\bar{X}_1, \bar{X}_2, \cdots, \bar{X}_p$ are unknown, Khare (1983,92b,93a) extended the classes R_g^* and P_g^* to the case of p-auxiliary characters and the class of estimators are defined by

$$R_g^* = g(\hat{R}_m), \frac{\bar{x}_{1m}}{\bar{x}_{1n}}, \cdots, \frac{\bar{x}_{pm}}{\bar{x}_{pn}} \text{and}$$

$$P_g^* = g(\hat{P}_m), \frac{\bar{x}_{1m}}{\bar{x}_{1n}}, \cdots, \frac{\bar{x}_{pm}}{\bar{x}_{pn}}$$

Khare (1990) proposed a generalized class of estimators for a combination of product and ratio of some population means using multi-auxiliary characters. The parameter considered for estimation is given as

$\theta = \frac{\bar{Y}_1, \bar{Y}_2, \cdots, \bar{Y}_m}{\bar{Y}_{m+1}, \bar{Y}_{m+2}, \cdots, \bar{Y}_k}$

using p-auxiliary characters $x_1, x_2, \ldots x_p$ the class of estimators for θ is given by

$\theta^* = \hat{\theta} h(u_1, u_2, \cdots, u_p)$

Where $h(1,1,\cdots,1) = 1$ and satisfies regularity conditions. Further Khare (1993a) extended the class of estimators θ^* to the case of double sampling and determined the optimum values of first and second phase sample which have minimum mean square error for the given cost.

Further, BahlandTuteja (1991)introduce an exponential ratio-type estimator for population mean as given by

$$\boldsymbol{T_1} = \bar{\boldsymbol{y}} exp\left[\frac{\bar{\boldsymbol{X}} - \bar{\boldsymbol{x}}_{\boldsymbol{c}}}{\bar{\boldsymbol{X}} + \bar{\boldsymbol{x}}_{\boldsymbol{c}}}\right]$$

and exponential product-type estimator as

$$\boldsymbol{T_2} = \bar{\boldsymbol{y}} exp\left[\frac{\bar{\boldsymbol{x}}_{\boldsymbol{c}} - \bar{\boldsymbol{X}}}{\bar{\boldsymbol{x}}_{\boldsymbol{c}} + \bar{\boldsymbol{X}}}\right]$$

Now our main objective is to study and analysis the properties of above existing estimators and suggests a new generalized estimator for product of two population mean using auxiliary information to reduce the error occurred in the estimation.

II. THE SUGGESTED ESTIMATOR

The proposed estimator is

$T = \bar{y}_1 \bar{y}_2 + \alpha\,(\bar{X} - \bar{x}_c) + (1 - \alpha)(\bar{P} - \bar{p}_c)$ … (1)

Here $\bar{x}_c = \frac{(N\bar{X} - n\bar{x})}{(N-n)}$, $\bar{p}_c = \frac{(N\bar{X} - n\bar{x})}{(N-n)}$

$\bar{y}_1 = \bar{Y}_1(1+\varepsilon_0)$, $\bar{y}_2 = \bar{Y}_2(1+\varepsilon_0{}')$

$\bar{x} = \bar{X}(1+\varepsilon_1)$, $\bar{p} = \bar{P}(1+\varepsilon_2)$

S.t $E(\varepsilon_0) = E(\varepsilon_0{}') = E(\varepsilon_0) = E(\varepsilon_0) = 0$, $\left(\frac{1}{n} - \frac{1}{N}\right) = \lambda$

$E(\varepsilon_0{}^2) = \lambda C_{y_1}{}^2$, $E(\varepsilon_0{}'^2) = \lambda C_{y_2}{}^2$

$E(\varepsilon_1{}^2) = \lambda C_x{}^2$, $E(\varepsilon_2{}^2) = \lambda C_p{}^2$

$E(\varepsilon_0 \varepsilon_0{}') = \lambda \rho_{y_1 y_2} C_{y_1} C_{y_2}$, $E(\varepsilon_0 \varepsilon_1) = \lambda \rho_{y_1 x} C_{y_1} C_x$

$E(\varepsilon_0 \varepsilon_2) = \lambda \rho_{y_1 p} C_{y_1} C_p$, $E(\varepsilon_0{}' \varepsilon_1) = \lambda \rho_{y_2 x} C_{y_2} C_x$

$E(\varepsilon_0{}' \varepsilon_2) = \lambda \rho_{y_2 p} C_{y_2} C_p$, $E(\varepsilon_1 \varepsilon_2) = \lambda \rho_{xp} C_x C_p$

The notations used are defined as follows:

N- Population size

n - Sample size

f=n/N (Sampling fraction)

$C_{y_1}, C_{y_2}, C_x, C_p$- Coefficient of Variations

$\bar{X}, \bar{P}$- Population means

$\overline{y_1}, \overline{y_2}, \bar{x}, \bar{p}$ -Sample means

$$\rho_{y_1 x} = Correlation\ coefficient\ between\ Y_1\ and\ X$$
$$\rho_{y_1 p} = Correlation\ coefficient\ between\ Y_1\ and\ P$$
$$\rho_{y_2 x} = Correlation\ coefficient\ between\ Y_2\ and\ X$$
$$\rho_{y_2 p} = Correlation\ coefficient\ between\ Y_2\ and\ P$$
$$\rho_{y_1 y_2} = Correlation\ coefficient\ between\ Y_1\ and\ Y_2$$
$$\rho_{xp} = Correlation\ coefficient\ between\ X\ and\ P$$

Now simplify

$$\bar{X} - \bar{x}_c = n\frac{(\bar{x} - \bar{X})}{N-n} = \frac{n(\bar{X}(1+\varepsilon_1) - \bar{X})}{N-n} = \frac{n\bar{X}\varepsilon_1}{N-n}$$
$$\bar{P} - \bar{p}_c = n\frac{(\bar{p} - \bar{P})}{N-n} = \frac{n(\bar{P}(1+\varepsilon_2) - \bar{P})}{N-n} = \frac{n\bar{P}\varepsilon_2}{N-n}$$

Put the value in given eq.

$$T = \bar{y}_1 \bar{y}_2 (1+\varepsilon_0)(1+\varepsilon_0') + \alpha\left\{\frac{n(\bar{x} - \bar{X})}{N-n}\right\} + (1-\alpha)\left\{\frac{n(\bar{p} - \bar{P})}{N-n}\right\}$$

$T = \bar{Y}_1 \bar{Y}_2 (1+\varepsilon_0)(1+\varepsilon_0') + \alpha\left(\frac{n\bar{X}\varepsilon_1}{N-n}\right) + (1-\alpha)\left(\frac{n\bar{P}\varepsilon_2}{N-n}\right)$ (2)

Since we know that

Given $\left(\frac{1}{n}-\frac{1}{N}\right)=\lambda$, therefor $\frac{(N-n)}{n}=N\lambda$

Then by eq. (2)

$$T = \bar{Y}_1\bar{Y}_2(1+\varepsilon_0)(1+\varepsilon_0^{'}) + \left(\frac{\alpha\bar{X}\varepsilon_1}{N\lambda}\right) + (1-\alpha)\left(\frac{\bar{P}\varepsilon_2}{N\lambda}\right)$$

$$T = \bar{Y}_1\bar{Y}_2(1+\varepsilon_0^{'}+\varepsilon_0^{'}+\varepsilon_0\varepsilon_0^{'}) + \left(\frac{\alpha\bar{X}\varepsilon_1}{N\lambda}\right) + (1-\alpha)\left(\frac{\bar{P}\varepsilon_2}{N\lambda}\right) \qquad (3)$$

It is the expansion of the estimator

III. THE BIAS AND MEAN SQUARE ERROR

Taking expectation on both side and ignoring terms of degree greater than two then we get

$$\mathrm{E}\,(T) = \bar{Y}_1\bar{Y}_2(1+E\,(\varepsilon_0^{'})+E(\varepsilon_0)++E(\varepsilon_0\varepsilon_0^{'}) + \left(\frac{\alpha\bar{X}E(\varepsilon_1)}{N\lambda}\right)$$
$$+(1-\alpha)\left(\frac{\bar{P}E(\varepsilon_2)}{N\lambda}\right)$$

Put the value of these expectation

$$E\,(T) = \bar{Y}_1\bar{Y}_2\left[1+\lambda\rho_{y_1y_2}C_{y_1}C_{y_2}\right]$$

Therefore

$$\text{Bias}\,(T) = \mathrm{E}\,(T) - \bar{Y}_1\bar{Y}_2$$

$$\text{Bias}\,(T) = \bar{Y}_1\bar{Y}_2\left(\lambda\rho_{y_1y_2}C_{y_1}C_{y_2}\right) \qquad (4)$$

Since mean square error is given by following eq.

$$MSE(T_3) = E(T_3 - \bar{Y}_1\bar{Y}_2)^2 \qquad (5)$$

$$[T_2-\bar{Y}_1\bar{Y}_2]^2 = \left[\begin{array}{c} \overline{Y_1^2Y_2^2}\left(\varepsilon_0^{'2}+\varepsilon_0^2+2\varepsilon_0\varepsilon_0^{'}\right)+\alpha^2\bar{X}^2\frac{\varepsilon_1^2}{N^2\lambda^2}+(1-\alpha)^2\bar{P}^2\frac{\varepsilon_2^2}{N^2\lambda^2}+ \\ 2\frac{\bar{Y}_1\bar{Y}_2\alpha\bar{X}}{N\lambda}(\varepsilon_0\varepsilon_1+\varepsilon_0^{'}\varepsilon_1)+ \\ 2\bar{Y}_1\bar{Y}_2(1-\alpha)\frac{\bar{P}}{N\lambda}(\varepsilon_0\varepsilon_2+\varepsilon_0^{'}\varepsilon_2)+ \\ 2\frac{\alpha(1-\alpha)\bar{X}\bar{P}\varepsilon_1\varepsilon_2}{N^2\lambda^2} \end{array}\right]$$

Taking expectation on both side, we have

Therefor by eq. (5)

$$MSE(T) = \bar{Y}_1^{\,2}\bar{Y}_2^{\,2}\left(E(\varepsilon_0^{'\,2})+E(\varepsilon_0^{\,2})+2E(\varepsilon_0\varepsilon_0^{'})\right) + \frac{\alpha^2\bar{X}^2}{N^2\lambda^2}E(\varepsilon_1^{\,2})$$
$$+\frac{(1-\alpha)^2\bar{P}^2}{N^2\lambda^2}E(\varepsilon_2^{\,2}) + 2\bar{Y}_1\bar{Y}_2\frac{\alpha\bar{X}}{N\lambda}\left(E(\varepsilon_0^{'}\varepsilon_1)+E(\varepsilon_0\varepsilon_1)\right)$$
$$+2\bar{Y}_1\bar{Y}_2\frac{\bar{P}(1-\alpha)}{N\lambda}\left(E(\varepsilon_0^{'}\varepsilon_2)+E(\varepsilon_0\varepsilon_1)\right) + \frac{2\alpha(1-\alpha)\bar{X}\bar{P}}{N^2\lambda^2}E(\varepsilon_1\varepsilon_2)$$

Put the value of the given expectation

$$MSE(T) = \bar{Y}_1^{\,2}\bar{Y}_2^{\,2}\left(\lambda C_{y_2}^{\;2}+\lambda C_{y_1}^{\;2}+2\lambda\rho_{y_1y_2}C_{y_1}C_{y_2}\right) + \frac{\alpha^2\bar{X}^2\,\lambda C_x^{\;2}}{N^2\lambda^2} + \frac{(1-\alpha)^2\bar{P}^2\lambda C_p^{\;2}}{N^2\lambda^2}$$
$$+2\bar{Y}_1\bar{Y}_2\frac{\alpha\bar{X}}{N\lambda}\left(\lambda\rho_{y_1x}C_{y_1}C_x+\lambda\rho_{y_2x}C_{y_2}C_x\right) + 2\bar{Y}_1\bar{Y}_2\frac{(1-\alpha)\bar{P}}{N\lambda}\left(\lambda\rho_{y_1p}C_{y_1}C_p+\lambda\rho_{y_2p}C_{y_2}C_p\right)$$
$$+\frac{2\alpha(1-\alpha)\bar{X}\bar{P}}{N^2\lambda^2}\lambda\rho_{xp}C_xC_p$$

$$MSE(T) = \begin{bmatrix} \overline{Y_1^2 Y_2^2}\lambda\left(C_{y_1}^2 + C_{y_2}^2 + 2\rho_{y_1y_2}C_{y_1}C_{y_2}\right) + \frac{\alpha^2\bar{X}^2}{N^2\lambda}C_x^2 + \\ \frac{(1-\propto)^2\bar{P}^2}{N^2\lambda}C_p^2 + 2\frac{\alpha\overline{Y_1Y_2}\bar{X}C_x}{N}\left(\rho_{y_1x}C_{y_1} + \rho_{y_2x}C_{y_2}\right) + \\ 2\bar{Y}_1\bar{Y}_2\frac{\frac{\bar{P}C_p}{x}}{N}(1-\propto) \\ \left(\rho_{y_1p}C_{y_1} + \rho_{y_2p}C_{y_2}\right) + \\ 2\propto(1-\propto)\frac{\bar{X}\bar{P}}{N^2\lambda}\rho_{xp}C_xC_p \end{bmatrix} \quad \textbf{(6)}$$

IV. MINIMIZATION OF MEAN SQUARE ERROR

Minimize the M.S.E of estimator differentiate partially eq.(6) with respect to α and equal to zero then we have

$$\frac{\partial}{\partial\alpha}[MSE(T)] = \begin{bmatrix} \frac{2\alpha\bar{X}^2{C_x}^2}{N^2\lambda} - \frac{2(1-\alpha)\bar{P}^2{C_p}^2}{N^2\lambda} + \\ 2\bar{Y}_1\bar{Y}_2\frac{\bar{X}}{N}\left(\rho_{y_1x}C_{y_1}C_x + \rho_{y_2x}C_{y_2}C_x\right) - 2\bar{Y}_1\bar{Y}_2\frac{\bar{P}}{N}\left(\rho_{y_1p}C_{y_1}C_p - \rho_{y_2p}C_{y_2}C_p\right) \\ + \frac{2\bar{X}\bar{P}}{N^2\lambda}\rho_{xp}C_xC_p - \frac{4\alpha\bar{X}\bar{P}}{N^2\lambda}\rho_{xp}C_xC_p \end{bmatrix}$$

Therefore

$$\begin{bmatrix} \frac{2\alpha\bar{X}^2{C_x}^2}{N^2\lambda} - \frac{2(1-\alpha)\bar{P}^2{C_p}^2}{N^2\lambda} + 2\bar{Y}_1\bar{Y}_2\frac{\bar{X}}{N}\left(\rho_{y_1x}C_{y_1}C_x + \rho_{y_2x}C_{y_2}C_x\right) - \\ 2\bar{Y}_1\bar{Y}_2\frac{\bar{P}}{N}\left(\rho_{y_1p}C_{y_1}C_p - \rho_{y_2p}C_{y_2}C_p\right) + \frac{2\bar{X}\bar{P}}{N^2\lambda}\rho_{xp}C_xC_p - \frac{4\alpha\bar{X}\bar{P}}{N^2\lambda}\rho_{xp}C_xC_p \end{bmatrix} = 0$$

Now

$$\alpha\left(\frac{2\bar{X}^2{C_x}^2}{N^2\lambda} + \frac{2\bar{P}^2{C_p}^2}{N^2\lambda} - \frac{4\bar{X}\bar{P}}{N^2\lambda}\rho_{xp}C_xC_p\right) = \left[2\bar{Y}_1\bar{Y}_2\frac{\bar{X}}{N}C_x\left(\rho_{y_2x}C_{y_2} + \rho_{y_1x}C_{y_1}\right)2\bar{Y}_1\bar{Y}_2\frac{\bar{P}}{N}C_p\left(\rho_{y_1p}C_{y_1} - \rho_{y_2p}C_{y_2}\right) - \frac{2\bar{X}\bar{P}}{N^2\lambda}\rho_{xp}C_xC_p - \frac{2\bar{P}^2{C_p}^2}{N^2\lambda}\right]$$

Above eq. multiply by $\frac{N^2\lambda}{2}$ then we get the value of α

$$[(\alpha)] = \frac{\begin{matrix}\bar{P}^2C_p^2 - Y_1Y_2\bar{X}C_xN\lambda\left(\rho_{y_1x}C_{y_1} + \rho_{y_2x}C_{y_2}\right) + \overline{Y_1Y_2P}C_pN\lambda \\ \left(\rho_{y_1p}C_{y_1} + \rho_{y_2p}C_{y_2}\right) \\ -\overline{XP}\rho_{xp}C_xC_p\end{matrix}}{\bar{X}^2C_x^2 + \bar{P}^2C_p^2 - 2\overline{XP}\rho_{xp}C_xC_p}$$

$$\alpha^2 = \frac{\begin{bmatrix}\bar{P}^2C_p^2 - Y_1Y_2\bar{X}C_xN\lambda\left(\rho_{y_1x}C_{y_1} + \rho_{y_2x}C_{y_2}\right) + \overline{Y_1Y_2P}C_pN\lambda\left(\rho_{y_1p}C_{y_1} + \rho_{y_2p}C_{y_2}\right) \\ -\overline{XP}\rho_{xp}C_xC_p\end{bmatrix}^2}{\left(\bar{X}^2{C_x}^2 + \bar{P}^2{C_p}^2 - 2\bar{X}\bar{P}\rho_{xp}C_xC_p\right)^2}$$

$$[(1-\alpha)] = \frac{\begin{matrix}\bar{x}^2C_x^2 - \bar{Y}_1\bar{Y}_2\bar{X}C_xN\lambda\left(\rho_{y_1x}C_{y_1} + \rho_{y_2x}C_{y_2}\right) - \overline{\bar{Y}_1\bar{Y}_2P}C_pN\lambda \\ \left(\rho_{y_1p}C_{y_1} + \rho_{y_2p}C_{y_2}\right) \\ +\overline{XP}\rho_{xp}C_xC_p\end{matrix}}{\bar{X}^2C_x^2 + \bar{P}^2C_p^2 - 2\overline{XP}\rho_{xp}C_xC_p}$$

$$(1-\alpha)^2 = \frac{\begin{bmatrix} \bar{\mathrm{x}}^2 \mathrm{C}_\mathrm{x}^2 - \bar{Y}_1\bar{Y}_2\bar{\mathrm{X}}\mathrm{C}_\mathrm{x}\mathrm{N}\lambda\left(\rho_{\mathrm{y}_1\mathrm{x}}\,\mathrm{C}_{\mathrm{y}_1} + \rho_{\mathrm{y}_2\mathrm{x}}\,\mathrm{C}_{\mathrm{y}_2}\right) - \overline{\bar{Y}_1\bar{Y}_2}\bar{\mathrm{P}}\mathrm{C}_\mathrm{p}\mathrm{N}\lambda \\ \left(\rho_{\mathrm{y}_1\mathrm{p}}\,\mathrm{C}_{\mathrm{y}_1} + \rho_{\mathrm{y}_2\mathrm{p}}\,\mathrm{C}_{\mathrm{y}_2}\right) \\ +\bar{\mathrm{X}}\bar{\mathrm{P}}\rho_{\mathrm{xp}}\,\mathrm{C}_\mathrm{x}\mathrm{C}_\mathrm{p} \end{bmatrix}^2}{\left(\bar{X}^2{C_x}^2 + \bar{P}^2{C_p}^2 - 2\bar{X}\bar{P}\rho_{xp}C_xC_p\right)^2}$$

Substitute the value of α, α^2 *and* $(1-\alpha)^2$ in eq. (6) than we get the

$$\mathrm{Min[MSE(T)]} = \frac{\begin{matrix} \left(\bar{X}^2{C_x}^2 + \bar{P}^2{C_p}^2 - 2\bar{X}\bar{P}\rho_{xp}C_xC_p\right)\left[\lambda{\bar{Y}_1}^2{\bar{Y}_2}^2\left({C_{y_2}}^2 + {C_{y_1}}^2 + 2\rho_{y_1y_2}C_{y_1}C_{y_2}\right)\right] + \\ \begin{bmatrix} 2\bar{Y}_1\bar{Y}_2\frac{\bar{X}C_x}{N}\left(\rho_{y_1x}C_{y_1} + \rho_{y_2x}C_{y_2}\right) \\ \left[\bar{P}^2{C_p}^2 - \bar{X}\bar{P}\rho_{xp}C_xC_p - \bar{Y}_1\bar{Y}_2N\lambda\bar{X}C_x\left(\rho_{y_2x}C_{y_2} + \rho_{y_1x}C_{y_1}\right) + \bar{Y}_1\bar{Y}_2N\lambda\bar{P}C_p\left(\rho_{y_1p}C_{y_1} + \rho_{y_2p}C_{y_2}\right)\right] \end{bmatrix} + \\ \begin{bmatrix} 2\bar{Y}_1\bar{Y}_2\frac{\bar{P}C_p}{N}\left(\rho_{y_1p}C_{y_1} + \rho_{y_2p}C_{y_2}\right) \\ \left[\bar{X}^2{C_x}^2 - \bar{X}\bar{P}\rho_{xp}C_xC_p + \bar{Y}_1\bar{Y}_2N\lambda\bar{X}C_x\left(\rho_{y_2x}C_{y_2} - \rho_{y_1x}C_{y_1}\right) - \bar{Y}_1\bar{Y}_2N\lambda\bar{P}C_p\left(\rho_{y_1p}C_{y_1} - \rho_{y_2p}C_{y_2}\right)\right] \end{bmatrix} + \\ \frac{1}{N^2\lambda}\begin{bmatrix} {\bar{Y}_1}^2{\bar{Y}_2}^2N^2\lambda^2\left(\bar{X}C_x\left(\rho_{y_2x}C_{y_2} + \rho_{y_1x}C_{y_1}\right) - \bar{P}C_p\left(\rho_{y_1p}C_{y_1} + \rho_{y_2p}C_{y_2}\right)\right)^{\mathbf{2}} \\ + \bar{X}^2\bar{P}^2\left(1 - {\rho_{xp}}^2\right){C_x}^2{C_p}^2 \end{bmatrix} \end{matrix}}{\left(\bar{X}^2{C_x}^2 + \bar{P}^2{C_p}^2 - 2\bar{X}\bar{P}\rho_{xp}C_xC_p\right)}$$

This is the final result of minimum MSE of the suggested estimator.

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