International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 10 Issue 8 ǁ August 2022 ǁ PP. 673-681

Duality for Octonians and Gravity

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Abstract
The different sections calrify the authors view that gravitons need for their different characters different spaces. Spactime is not sufficient, octonians are needed, also in one version the GellMann SU(3) matrices are needed. Both are 8-dimensional and a 4-dimensional extension of R^4 is added as complex Hilbert space C^4. This makes three different vector spaces with different presentations for gravitons: as spin-like 126 field quantums with particle character they need octonians, for graviton waves descriptions R^4 is used with x,y,z,t coordinates. The SU(3) strong interaction contributes to gravitons as projection its first three GellMann matrices $\lambda_{1,2,3}$ presenting the extended three Pauli matrices which map spactime to a color charge carrying complex Riemannian sphere S^2. A note is added for the different presentations and how they can be transformed into one another [6].
Missing is a nonlinear cross ratio spin-like graviton, as well as the seven Fano memo spin-like triples. Octonians are in this article seen as presenting particle or field quantums characters and geometries, projected into spacetime. They can show this in different experiments, compared with their C^4 wave character. SU(3) and SU(2) present nucleon and atomic kernels strong and weak forces . Both SU(3), SU(2) have presentations as fiber bundles with fiber S^1, a circle like U(1). They are not including gravity. As field quantums they use gluons and weak bosons. U(1) is for photons and the electromagnetic interaction. The standard model of physics unifies them in U(1)xSU(2)xSU(3).
Gravity cannot be treated with it. It is necessary to introduce color charges as an octonian force, independent of the QCD, SU(3) use of a property quarks have.
According to the Noether theorem S^2 as geometry with color charges as invariants under Moebius transformations has to be added to it. They are cross ratios, perspective projections which are defined by permuting four elements, a complex variable z and three reference numbers $0,1,\infty$ or
$0,-1,\infty$. Observe that cross ratios are not linear, but fractions z, 1/z, (1-z), 1/(1-z), z/(z-1), (z-1)/z.
For octonians there is a factorization due to the permutation group S_4 of four elements.The group factors through its normal CPT Klein group Z_2xZ_2 to D_3, the quark triangle symmetry. D_3 attributes to the six color charges in their factors a symmetry whose eigenvalues sets unit measures for associated energies. The octonian subspace 123456 adds a coordinate to each factor class, containing four members. Perspective projections and projective geometry are used for gravity. In the projective form, a real or complex linear space is replaced by taking in $R^{(n+1)}$ lines through the center 0 and replace them by their two rays intersection with the unit sphere S^n in the linear space. The standard model uses geometries S^1 for U(1), the Hopf sphere S^3 for SU(2) and for SU(3) the toroidal product S^3 x S^5. For real projective spaces the parity operator P factors out from the S^n antipodal points which are identified. The homogeneous coordinates are $[x_0,x_1,x_2,...,x_n]$ with an equivalence relation where the tuple can be multiplied by a (real or complex) number λ with $|\lambda| = 1$.
These geometries are needed for describing gravity in the Noether theorem sense. Their projective transformations, correlations which interchange dimensions and introduce quadrics for metrics are not available in the standard model. Nonlinearity of Schwarzschild metric is due to cross ratios which are not preserving usual measures. They preserve geometrical incidence and cross ratios or four collinear points. This is not covered by the use of 8-dimensional vector spaces as normed octonians, a Lie algebra symmetry SU(3) or a real or complex Hilbert space R^4 or C^r.
Keywords: *rgb-gravitons, gravity waves, duality, octonian, projective geometry, cross ratio*

Date of Submission: 18-08-2022 Date of acceptance: 02-09-2022

I. Introduction

Duality means for physics wave-particle duality, experimentally found for instance in the double slit experiment for electrons as particles and waves. Recall: if from two slits one is closed, the received patttern for their distribution as spots form a strip, with the two slits open, the spots where electrons hit make the interference pattern from a wave.

It is postulated that the particle character is using the octonians as coordinate base for energies while the wave character uses spacetime xyzt-coordinates of physics. The geometries and symmetries are different and related in experiments by different perspective projections. For the particle character it is essential that color charges are defined as an independent octonian force for physics, not only a QCD proberty experimentally found for quarks. They are complex cross ratios, preserving (as invariants of the Moebius transformations for the 2-dimensional Riemannian sphere) geometrical incidence of a point with its dual hyperplane and the cross ratios of four points as in figure 2. Projective geometry and octonians replace for the particle character of energies a Minkowski metric and Euclidean xyz-space to which time as fourth coordinate is added.

Gravity waves have been experimentally found by LIGO, discussed for octonians in the first section. It is open in 2022 how octonian gravitons as particles are detected. The use of color charge *rgb*-gravitons as complex cross ratios, perspective transformations is for nucleons. In octonians a model for graviton waves a superposition of two helix waves is invented. In octonian coordinates listed as indices numbers 01234567, it is presented as a spin-like orthogonal base triple 126

red-green-blue *rgb*. It is observed as neutral color charge of nucleons. In contrary to spin its third coordinate 6 blue is not the octonian third space z-coordinate 3. It is replaced by a frequency 6 coordinate. The spin as measuring base triple in spacetime has for its coordinates spin length as measure. Length is for the octonian *rgb*-graviton measure replaced by the three color charges attached to three quarks of a nucleon.

A geometry: the 2-dimensional quark triangle replaces projective the center of the spin base vectors.They span with their endpoints a triangle which is dually the barycenter of the nucleon. The three triangle sides map to one another. For the quark triangle they carry the observed gluon exchange between paired quarks (confinement of quarks in the nucleon). Distances between quarks can be stretched and squeezed. This is one property belonging also to the *rgb*-graviton as octonian 3-dimensional 126 base for nucleons. This is not the quaternionic spin $s = (s_x,s_y,s_z)$ base for physics 4-dimensional spacetime. Cross ratios replace the real cross product. They can be applied as functions, not as Pauli matrices with the quaternionic multiplication.

Another example: as geometry for photons (and similar for *rgb*-gravitons) is suggested an octonian 7 Kaluza-Klein circle U(1), S^1. In both cases, the energy is a rotating point on S^1 for angular frequency. Only full contour windings are stored as energy, parts not. This is the quantization for the photons energy. The particle character is for photons that they are only existing in a time interval Δt for one spin revolution on S^1 where the point rotation of its energy occurs on the universal cover of S^1 as one helix winding on a circular cylinder with S^1 as (base) transversal section. In octonians frequency 6 the spin 1 windings in time are counted as f = n, n a natural number. Complex residual contour integration theory can be used. This paritlce character (photons as time generated helix winding) shows up when one of the two slits in the above experiment is closed. In case both slits are open not this discrete spots are observed, but the wave presentation of the electromagnetic interactions EMI energy in Minkowski spacetime. The exponential wave (coming in wave packages) is projected into quaternionic spacetime to its real cosine part which allows interference.

The physics variables x,t for the $\exp(-i\omega(t-x/v)$ wave function are not the octonian variables 12 for $\exp(i\varphi)$ S^1 (as Kaluza-Klein circle), and 6 $f = 1/\Delta t$ of photons. The circle arises as Lissaous figure for two orthogonal hitting frequencies in proportion 1:1. The octonian central projection of the circle is to its real helix line as coordinate 7. One projective point at infinity is deleted from S^1. The cylinder as surface is closed at projective infinity to a pinched torus where one transversal torus circle is retracted to a perspective point. Mostly quoted in physics are polarization experiments for

spin 1 of photons. This is not including a geometrical particle description of photons as generated helix winding on a cylinder in a time interval.

For the Minowski metric, a similar geometry as projective Minkowski cone $r^2 – c^2t^2 = 0$, r = |x| radius, exits. The cone is closed to a pinched torus by identifying at infinity the upper and lower circles. Every line through the center and on the cone is closed at infinity to a circle. The conic geometry applies to *rgb*-gravitons. For the quark triangle a pulsation on three Bohr radii is postulated. In time intervals there are sudden jumps between their potentials for rotating gravitational charges on two helix lines. One exponential Ψ wave is replaced by two interlinked

Ψ_j, j = 1,2, They start on a basic circle at diametrical opposite points. One serves at observer, the other ane as observed system. The roles are changed for stretching and squeezing. The observer

j = 1 males stretching of the Bohr radii, the observer j = 2 squeezing. For the sqeezing, after a gb gluon exchange between the g,b color charged quarks the Bohr radius jumps from scaled 2 to 1, the Ψ_j two points are interchanged which makes for each wave only one half a (cw) circular rotation. This is once more repeated for a br gluon exchange with a (mpo) circular rotation and ome half turns for each wave. There are two quark triangle reflections $\alpha\sigma_1$, σ_1 generated. Composed as

$\alpha\sigma_1\sigma_1 = \alpha$ a (mpo) triangle rotation by 120^0 is obtained as cross ratio. This corresponds to the Pauli matrix multiplication where two matrices mulitplied give the third. For color charges the third one is not a reflection, but a rotation. The rotation lets the radius for the circular point rotation fixed. After that the roles of observer j and observed system $k \neq j$ is changed for stretching the Bohr radii in the opposite direction. In a fixed time interval Δt from large to small or small to large Bohr radii, there are two gluon exchanges with spin1 used.

For the rgb-graviton Ψ_j half turn helix parts has to be spin 2. The above quoted polarization for photons is replaced by the finding of graviton waves [1]. For stretching and squeezing in a Laser measurement two orthogonal lenghts l_k small, large are observed. Graviton wave packages are passing by the detector. The Ψ_j observer is for sqeezing in figure 3 on the lower ray of the Minkowski watch, for stretching on the upper ray. Radius and

frequency for a vibrating triangle side as phonon are l_p squeezed (alternatively stretched), hence time as inverse frequency interval is l_Q stretched. For the $l_{P,Q}$ measures are quoted two Minkowski coordinate systems, turned towards one another by 45^0. They correspond to the two Ψ_k waves. The small, large Bohr radii are measured simultaneously. The change of observer is responsible for reverting squeezing stretching in a time interval. Spin 2 for gravitons is not a polarization as quoted for photons. It is due to the SI rotor and the gravitational stretching squeezing [see the last section].

This setup is repeated later on for clarifying the new concept.

Other octonian particle related models are described below. The Gleason measuring orthogonal base triples in octonians are for spin 123, for electromagnetism 145, for EMI 167, for heat 246, for mass 257, for rotations 347 and for an SI rotor 356. Added is 126 for *rgb*-gravitons as a strong interaction base triple (use as projections the GellMann 3x3-matrices λ_j, j = 1,2,3, as extended Pauli matrices σ_j). Observe that the two *rgb*-graviton presentations belong to different multiplication tables for SU(3) (quaternionic 123) matrices and (126) octonians, the spin 3 and its length measure for 12 is replaced in octonians by composing the two 12 rg cross ratios as functions for getting 6 b as rotational cross ratio, 1/z, z/(z-1) → 1/(1-z). A D_3 symmetry$\alpha\sigma_1$ reflection applied to σ_1 generates a 120^0 rotation α (as angular frequency) of the nucleon triangle. The cross ratio measures for the *rgb*-graviton octonian particle character are for the gravity field quantum Γ in the SU(3) case quaternionic spin-like for a wave presentation.

If coefficient matrices for 2 are compaired, as second (real noted) Pauli matrix the fisrt row is (0 1), the second row (-1 0) as orthogonal base in an xy-space; for $\alpha\sigma_1$ the first row is (1 0), the second row (1 -1) in an rg-space. The Moebius transformation 1/(z-1) multiplied with 1/(z+1) as coefficient matrices gives the quaternionic σ_2 matrix (1). This matrix has the property of the imaginary unit i. Applied to itself gives -id as matrix. Complex numbers z = x +iy are presented in orthogonal bases 2x2-matrix form as quaternionic transformations $x\cdot\sigma_1 + y\cdot\sigma_2$. For the quaternion measure of spin length the matrix multiplication is $\sigma_3 = \sigma_2\sigma_1$. The Moebius transformations for getting σ_2 are not the color charge cross ratios. In the first case 1/(1-z) (of (1)) is multiplied by -id, in the second case

1/(1-z) is applied to -z. The multiplications of the two coefficient matrices for -id, -z gives the Klein CPT group Z_2xZ_2 from physics (conjugation with rows i(1 0), i(0 -1), parity -z, time reflection -id).

Quaternionic complex numbers are then presented as 2x2-matrices with first row (z_1 z_2) and second row (-c(z_2) c(z_1)), c(z) the conjugate of z, z_1 = z +ict, z_2 = x +iy, x,y,z as space coordinates, t time, c speed of light.

In this view the CPT group makes the transition from the color charges complex Riemannian sphere S^2 with coordinate z = x +iy to the quaternions (z_1,z_2), doubling complex coordinates (z_1 can alternatively be replaced by iz_1). The Hopf map h for SU(2) maps first (z_1 z_2) to space coordinates by norming time to a constant. The image $h(S^3) = S^2$ is stereographic mapped from the north pole ∞ to a tangent plane E of S^2 at the south pole. Renorming the E coordinates to (x,y) with the space coordinate z as constant, the E coordinates are z_2/z_1 for $z_1 \neq 0$ and ∞≡ [0,1] for z_1 = 0. Important is the geometry of the S^3 SU(2) geometry as h-map fiber bundle with fiber S^1 and base space S^2.

Heegard decompositions of S^3 generate fermions, entropy (heat) volumes or other genus n brezel decompositions, having surfaces with nontrivial homology groups. Decompositions of S^2 are into two hemispheres on Bohr shells about the S^2. There are six such hemispheres for the six color charges, paired according to the Heisenberg uncertainties (see the hedgehog figure). In a projective version S^2 hasas topological factor P^2 by identifying with the parity P operator diametrical opposite points p,-p. A Moebius strip inside P^2 allows spin s = (s_x,s_y,s_z) to change from an up +s direction as normal to the Moebius strip by a 360^0 rotation to turn to down -s. For fermions spin ½ it means that after a 720^0 rotation the original state +s is obtained again.

Added is that for bosons spin 1 no Moebius strip rotation applies, neither for gravitons spin 2. Spin 1 measn one full rotation about a flat S^1as Lissajous figure, spin2 means that two frequencies in proportion 1:2 hit orthogonal for a lemniscate flat figure generated. A circular rotation as vibrating string with accoustic tone c has a central origin added on the circles diameter for an added overtone c' having a shorter frequency. Spin 2 for gravitons is the for the lengths proportion 1/2:1. In

exponential angular notation it means that a *rgb*-graviton turns spiralic a nucleon triangle by 30^0 (see figure 5), changing the nucleon radius. This is for 60^0 repeated in time and stretching in time makes as dynamics a six cycle for nucleon states changes. As carrier for the vibrating string can be used a gluon exchanged between paired quarks. Physics draws them mostly as a coil about the quarks connecting interval which can be stretched or squeezed. There are two Moebius transformations involved, z/(z-1) for the accoustics (heat phonons 2) and frequency 1/(1-z) (octonian 6 as angular frequency ω = 2πf). If f is substituted by a time interval, the stretching squeezing means for them that time (1-z) gets squeezed when the length (of 2) is stretched and reversely. The Minkowski metrical rescaling of (x,t) coordinates shows this (see figure 3). The noncommuting of z/(z-1) and (1-z) means that the circular spiralic rotation changes its orientation, for instance clockwise cw for contration and mpo counterclockwise for expansion.

Most triples have a 4-dimensional extension like 1234. 2356 is for nucleons and atomic kernels in a particle description. It is projected into spacetime 1234, localized in a solid entropy containing 3-dimensional ball with bounding potential Bohr shells S^2. 145 is extended to 1456 for a circular 6 frequency $\omega = 2\pi f$ in form of a rotating electrical charged point on a latitude circle of a Hopf S^2 sphere. The Hopf fiber bundle is used. 1267 adds *rgb*-gravitons to the EMI space. The heat equation is volume times pressure is proportional to temperature, the extension is 1246, also including *rgb*-gravitons. For mass the frequency extension is 2567. 2347 can be for heat included to rotations. There is a 4-dimensional Hilbert subspace H request: in case three coordinates as points on an interval, containing 4 points as blocks, are incident as vertices in a triangle configuration they are contained in an additional 4-dimensional block. The H closed subspace lattice consists of overlapping blocks, Boolean subalgebras 2^4. The orthogonal projections onto subspaces U for $H = U + U^{\perp}$ are not commuting. The Boolean distributive law for subspaces join and meet has to be replaced by the weaker orthomodular law. Another request is for a 4-cycle replacing a triangles 3-cycle is the existence of a central astroid [1]. Enumerating the points of the astroid requieres that all eight are pairwise different. Octonian coordinates are used for this, extending spacetime coordinates. The pairings are 03, 15, 27, 46. Real numbers for spacetime coordinates R^4 are extended to complex numbers C^4. There are three 8-dimensional vectorial coordinate presentations as octonians, as complex Hilbert space C^4 and as SU(3) GellMann 3x3-matrices generators. The vector or matrix multiplication tables are different. C^4 has the usual Hermitian measure uc(u) for radial length.

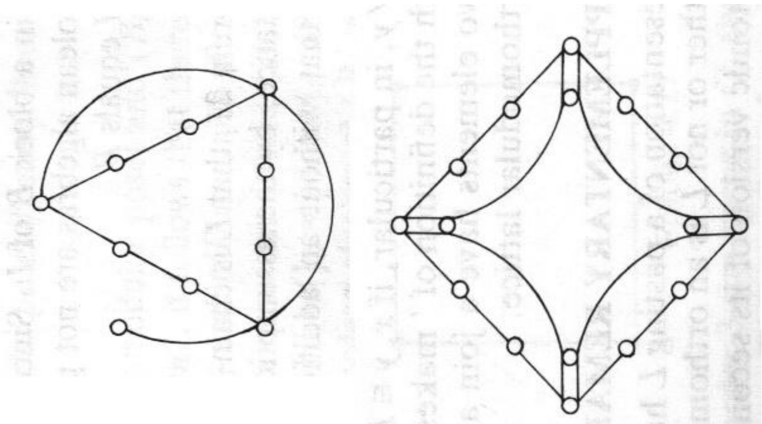


Figure 1 subspaces 3-cycle at left (triangle vertices rgb 126 or 167 extended to 1267 or space 123 to spacetime 1234 with the Hopf fiber bundle projection $h:R^4 \rightarrow R^3$); central astroid 03 at left, 15 at top, 27 at right, 46 at bottom, in a subspace 4-cycle at right (quadrangle), doubling quaternions 1234 q to octonians (q_1,q_2) with a new fiber bundle of the strong interactions SI topological factor S^5, mapped with a fiber S^1 to octonians subspace 2356 as complex projective CP^2 space bounded by a shell S^2

The Hopf geometry is S^3, the SI geometry is a topological product $S^3 \times S^5$ where the S^3 part can be represented by the extended Pauli 2x2-matrices σ_j as GellMann 3x3-matrices λ_j, extending them through a third row and column of elements 0. An *rgb*-graviton can map as projection the λ_j down to the σ_j, j = 1,2,3. In an evolution, the weak interaction WI splits off from SI and gravity GR.

Stretching Squeezing

In detecting gravton waves with LIGO, the quantum theoretical Ψ wave description is not used. The reason is that from general relativity is taken the rescaling of Minkowski metric to Schwarzschild metric. For probagating graviton waves GW in spacetime this is shown in the figure 2 from [1] as two arms of GW.

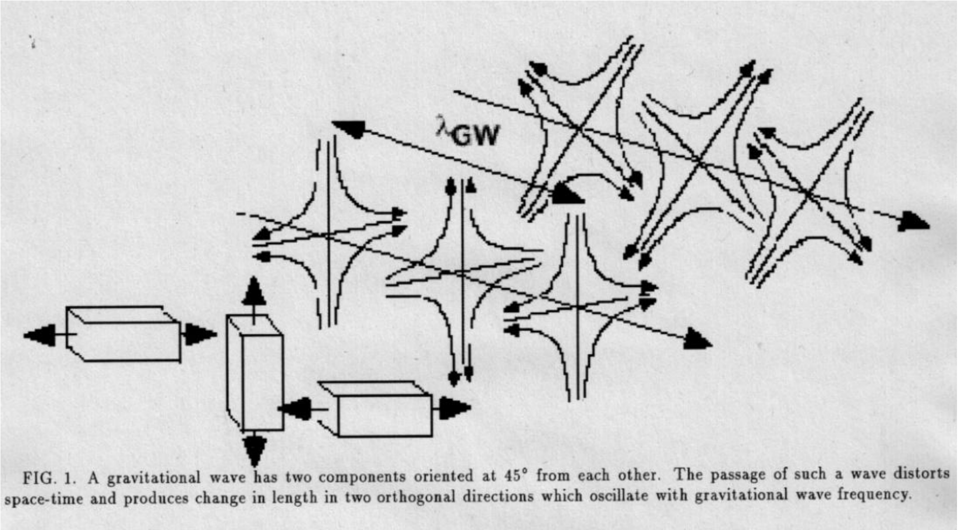


FIG. 1. A gravitational wave has two components oriented at 45° from each other. The passage of such a wave distorts space-time and produces change in length in two orthogonal directions which oscillate with gravitational wave frequency.

Figure 2 from the article [1]

This is shown in figure 3 as a rescaling of spacetime coordinates between two test systems of LIGO the role of the observer P and observed system Q is oscillating in a time interval.
If P measures Q, the radius differential dr of Q is measured as (squeezed to) dr' = dr·cos β, $\sin^2 \beta$ = Rs/r, Rs the Schwarzschild radius of Q, at the same time dr' is measured (stretched from dr) for Q as observer. In the [1] experiment the computation is not using cos β, but the length of the interferometer arms which show two orthogonal mirror lengths l_k, k = P,Q, for the distortion of local metrics. This is an unsymmetrical distance measure for $|PQ| = l_P \neq l_Q = |QP|$ in space.

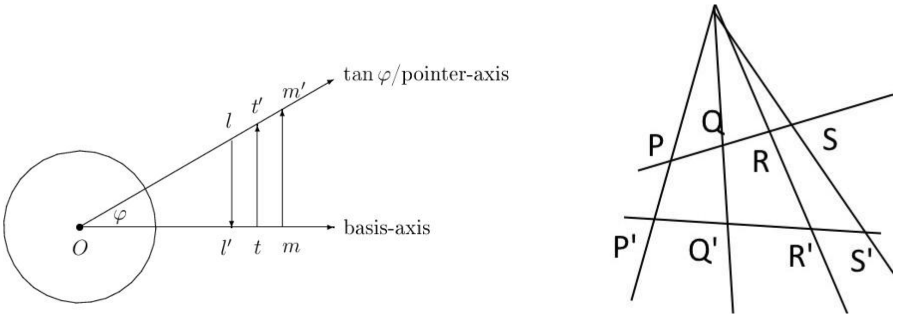


Figure 3 Mwatch for rescalings of length, time, mass between an observers coordinate system and the one of an observed system; perspective projection (P,Q;R,S) = (P',Q';R',S')

If the complex cross ratio for (r-Rs)/r is projective linearized it means that |PQ| = (r-Rs) < r = |QP| before the oscillation, interchanging the roles of P,Q. The linearization uses a central projection and
solves it by a projective homogeneous transformation. From [2] is repeated a 3-dimensional version where for the P,Q case the coordinate y can be deleted and the parameters and variables are substituted for P,Q: project a point P in R^3 down from a point P_0 = (0,0,a), 0≤z<a, along the ray from P_0 through P to the xy-plane z = 0. The nonlinear parameter is t = -a/(z-a); in homogeneous extended coordinates it can be multiplied by (z-a) in form of [-ax,-ay,0,(z-a)]. The transformation matrix A is a projection similar to GellMann matrix projections with first row (-a 0 0 0), second row (0 -a 0 0)
third row (0 0 0 0), fourth row (0 0 -a 1). It is applied to [x,y,1,z].
The action of a *rgb*-graviton Γ as complex cross ratio (z-1)/z with coefficient matrix of order 6 is used. Γ is a perpective projection. These maps preserve incidence (used for getting the quadrics for metrics) and the cross ratio of four collinear points (figure 2). In octonian coordinates, Γ is a spin-like base triple named 126 where 12 are xy-coordinates and 6 is a frequency f = 1/Δt, Δt a time interval, energy coordinate (measured in Hz).
In a geometry for the electromagnetic EMI waves Ψ functions, the complex polar angle φ is substituted by the exponential function φ → exp(iφ). This function describes the EMI symmetry U(1) as circle for two hitting

freqeuencies in proportion 1:1. On its universal cover the angular speed for rotation traces out in time a helix line for photons which are one helix winding, generated in a time interval for the waves frequency. Its wave length satisfies $\lambda f = c$. Geometrical spin frequency is counted by photons spin winding numbers f = n, n = 1,2,...
If for Γ as spin-like triple, not having space coordinates of spin as $s = (s_x,s_y,s_z)$, a geometry is invented, the third s_z coordinate needs another geometry. A cylinder as for the EMI Ψ waves has to be replaced.
The use in figure 2 of an observer and an observed system is substituted by setting for Γ two Ψ helix lines and for the cylinder surface a pulsation such as for Bohr radii. The pulsation is in six steps a dynamics which in nucleon occurs in a fixed time interval Δt, changing together with gluon exchanges as SI rotor the nucleon state. In figure 3 the six steps are drawn in one figure earthworm where the second part as time expansion has to be vertical reflected, contraction from large to small put on top. The frequency for the earthworm as wave is $f = 1/\Delta t$, taken from the nucleons SI rotor time interval Δt. The six cycle of the SI rotor is presented as the pulsation. The two helix lines are diametrical opposite located on the earthworm surface. Their color charge interaction is in pairs

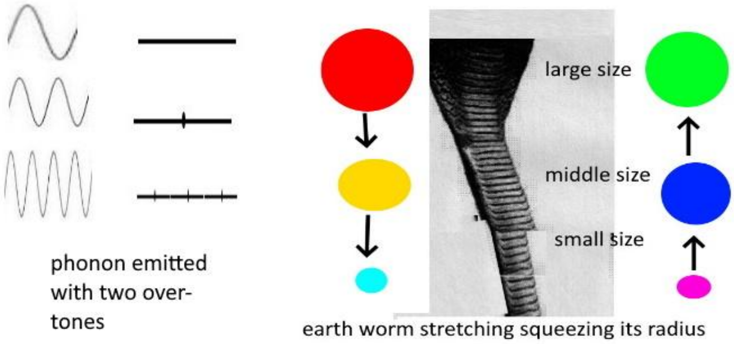
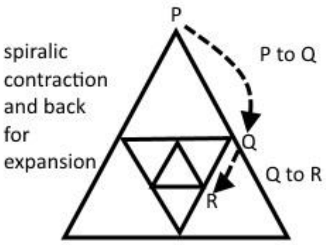


Figure 4 earthworm for Γ with an accoustic action, pulsation of the cylinders Bohr radii and two helix lines, replacing the role of an observer and observed system

uc(u), u = r red or g green or b blue, c(u) the conjugate color charge. The pairing is as in mesons,
not as gluons. In proportions for lengths contraction expansion the small radius is ½ as spin of quarks, the next number is for gluons spin 1, the third for Γs spin 2, rescaling is by ½:1:2.The numbers are a degenerate numerical orbit of the quark triangles symmetry D_3. It has a presentation as the six complex cross ratios. They are invariants under the Moebius transformations of a
Riemannian sphere S^2. In comparison with the EMI case photons spin 1 as one geometrical helix winding, for Γ one helix winding is from a point P as 1 on the base circle small over point
$\exp(i\pi) = -1$ on the middle circle back to $\exp(i2\pi) = 1$ on the large circle. The spin length as geometrical frequency is 2. At the fourth and sixth steps of the cycle, no radius change occurs, only when passing the middle the radii proportions in 2 are changed. The cycle is repeated when a graviton wave expands in spacetime. Since the Γ come like EMI waves in wave packages the earthworm model can serve for detecting them as spin 2 duality particles.

The SI rotor and models

This model for nucleons inner dynamics is constructed in octonian coordinates. The quaternionic spacetime coordinates are enumerated as 1234 xyzt and doubled for energies, forces to 03,15,46,27.
Subspaces of 01234567 are listed by indices as a number. Octonians have seven cross products as
3-dimensional measuring Gleason frames, orthogonal triples like spin. Added is 126 for Γ, also for the strong interaction GellMann matrices λ_j , j = 1,2,3. in another SI strong interaction presentation.
The Bohr radii pulsation of a nucleons quark triangle QT uses Γ as tip of a tetrahedron configuration with base QT. The QT symmetry D_3 is presented as permuation of three color charges r,g,b of Γ. Either r is kept fixed and by a gluon exchange gb are reflected by a conic cw (clockwise) rotation of the triangle side opposite of r. The first barycentrical coordinate of the QT is generated as rotation axis. At its new place g is kept fixed and by a gluon exchange rb are reflected by a conic mpo (counterclockwise) rotation of the triangle side opposite of r. The second barycentrical coordinate of the QT is generated. This is repeated in a six cycle for the new locations of r,g until the first rgb permutation is obtained. A barycenter for the nucleon mass is set.



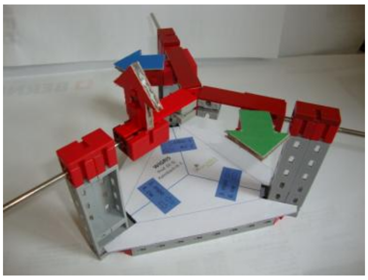


Figure 5 quark triangle pulsation spiralic contraction (or expansion); SI rotor model at right

Their are associated integrations of forces as six energies. In an enumeration of figure 4 1 electrical potential/force EM is red, 4 is yellow magnetic force, turquoise is 5 mass, magenta is 3 rotational force, blue is 6 frequency and green is 2 heat. The two potentials 1,5 are radial integrations
$e/r^2 \rightarrow -e/r$, the two kinetic, rotational energies 6, or 3 are time integrated $\int f(t)dt$ as linear momentum p = mv or rotational angular momentums L = ωJ speeds v or ω. Magnetic energy 4 is area integrated from induction and heat 2 is volume integrated for entropy inside a spacial volume.
A tetrahedron configuration shows as model the action of Γ combined with the gluon exchanges of the strong interaction. As permutation group S_4 of the tetrahedron, the D_3 symmetry is obtained as factor group from S_4 by its normal Klein sugroup Z_2 x Z_2. The factor classes contain each four members: a color charge, an energy, an octonian coordinate 1,2,3,4,5,6 and a D_3 symmetry whose eigenvector sets a unit for the energy measure 1 Ampere or length, 2 Kelvin or length, 3 Joule or length, 4 TESLA or second (for time), 5 kg, 6 Hz. An old listing from the 1980th of the author is added, where the coordinate numbers in line have to be replaced in 2022 as sequence to 123465.

In the first line of the table are the spherical SI coordinates, possibly 7- (not 8-)dimensional extended with exponential/polar coordinates. In the second line are the linear Pauli/Euclidean coordinates, in the third line a distribution of color charges to the SI coordinates. The fourth (fifth) line contains the D_3 (SU(2)/Pauli) MTs as cross ratios. Their matrix names are in the sixth line, together with the Einstein matrices. The following line is a numbering for a strong 6-fold integration series (not the Fano figures numbers which are for octonians). The next line contains the Planck numbers. Energy vectors are in the second to last line and the last line contains natural constants and three more operators, C (conjugation for quantum numbers), T (time reversal) and P (space parity) of physics.

r or $re^{i\varphi_1}$	φ	θ	ict	iu	iw
$x \in \mathbb{R}$	$iy \in i\mathbb{R}$	$z \in \mathbb{R}$			
r	g	$\overline{g}$	$\overline{b}$	b	$\overline{r}$
z	$\frac{z}{-z-1}$	$\frac{-z-1}{z}$	$(-z-1)$	$\frac{1}{-z-1}$	$\frac{1}{z}$
$\frac{1}{z}$	$-\frac{1}{z}$	$-z$	z		
$id; \sigma_1$	$\alpha\sigma_1; \sigma_2$	$\alpha^2; \sigma_3$	$\alpha^2\sigma_1; id$	$\alpha; \delta$	$\sigma_1; id; \beta$
1	6	4	2	5	3
length λ_P	temp. T_P	dens. ρ_P	time t_P	ener. E_P	mass m_P
EM_{pot}	E_{heat}	E_{rot}	E_{magn}	E_{kin}	E_{pot}
c, e_0, ϵ_0	k,C	N_A,T	μ_0	h	γ_G, R_S,P

Table 1

The octonian Gleason measuring triple is 356. It is extended 4-dimensional to 2356 as octonian space for a nucleon. It arises as base space CP^2 of a strong interactions fiber bundle with fiber S^1 as image of the 5-dimensional sphere S^5. The CP^2 has the 2356 space and is closed by a Riemannian sphere for Bohr shells. Color charges are presented as hemispheres in the Heisenberg uncertainty relation on x-,y- of z-axes. There is an energy vector attached on the axis which can change its

orientation like a spin from a normal in or out direction where energy is absorbed or emitted as exhcange with the environment (figure 5). The valve for sudden changes are due to a cusp or fold catastrophe where the measuring potential jumps from one to another level. The use of Γ is a projection of 2356 into spaetime 1234 where a nucleon is observed inside an entropy volume.
The energy space 56 mass 5, frequency 6, is substituted by 14 radius or x-coordinate 1, time 4.
The polar color charge caps are a substitute of electrons in an atoms shells.

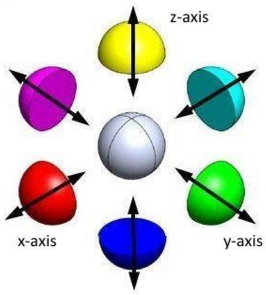


Figure 6 hedgehog, nucleon or atomic kernel solid 3-dimensional ball in the middle, polar color charged caps for the balls energy exchange with its environment

Note

In [6] a computation is given how the different coordinate systems can be related. This is not the view of the author, but may be useful for physics. Octonians are not associated with SU(3) matrices as quoted in [6]. The authors first compare spin wiht isospin coordinates. Then octonians and GellMann λ matrices are compared, disregarding the different multiplication tables. This list is not correct. Octonians are color charge defined, not by λ matrices. In [6] the hyper charge Y with λ_8 and an extension from 8 to 9 (each three) I,V,U dimensions is presented for spin I-, V-. U-multiplets. I has associated $\lambda_{1,2,3}$. λ_8 is replaced by a noremd e_3 and the eight GellMann matrices appear for $I_{1,2,3}$ with $\lambda_{1,2,3}$, for $V_{1,2}$ with $\lambda_{4,5}$ and V_3 with scaled e_3, for $U_{1,2}$ with $\lambda_{6,7}$ and U_3 with scaled e_3. For the replacement of λ_8 is added by the author that in the σ_3 extension to the λ matrices two of the three linear dependent extensions $\lambda_{9,10}$ are added. An e_3 replacement is not needed for λ_8 . Isospin triples are extended by V,U spin-like triples. The Fano memo for octonians have seven of them where the spin-like *rgb*-graviton 126 is added. Spin is 123 (extended to 1234 and for 23 by the cross product), I can correspond to angular momentum L = rxp, known for 347; in [6] for the $\lambda_{1,2,3}$ projections into SU(2) generators are used the quaternions i,j,k, added for weak bosons $W^{+,-}$ for i,j and I_3 for k.
145 (extended to 1456) for electromagnetism and the weak interaction 14 (cross product extended for induction as angular momentum), V can correspond to 167 (extended to 1267 and for
rgb-gravitons 126) for the electromagnetic interaction and gravity 07, U can correspond to 356 (extended to 2356) for the strong interaction 56 (as energy plane for mc^2 = hf). The Fano memo 246, 257 are missing under the above spin-like triples. A Heegard decomposition of the Hopf S^3 into two solid volumes containing an entropy is for 246. Barycentrical coordinates for 257 where Higgs can set a mass scalar for a mass system are constructed by the SI rotor of the author for nucleons. Three quark triangle reflections are associated with this. The D_3 symmetry is not used in physics.
The Riemannian nonlinear cross ratio 0 coordinate of octonians is not spin-like. It is not in the [6] listing transferring SU(3) to octonians. For 0 the Riemannian sphere S^2 with nonlinear Moebius transformations has to be used. Spin needs three dimensions. The Hopf geometry h for SU(2) adds the missing space z-coordinate as a north-south pole axis of S^2 with the stereographic projection st of S^2 to its tangent plane at the south pole. Pauli spin matrices define the S^2 spacial coordinates and $st \circ h$ maps spacetime coordinates to scaled complex z = x +iy coordinates where the third space coordinate (no spin for the octonian 0-coordinate S^2) and time is factored out. The projective norming of S^2 to the projective plane P^2 is essential for treating gravity, a geometrical tool for orbits
and metrical quadrics not used in physics.

II. Conclusion

Since rgb-gravitons are like color charges as new force on the octonian coordinate 0, it is argued that a search for them as field quantums for gravity and mass relates to their definition as rotational cross ratio. The color charge geometry is a 2-dimensional Riemannian sphere S^2. Cross ratios are

the six invariants under the S^2 Moebius transformations. They preserve neither length nor time or other energies. Perspective, central or projective projections are allowed. This geometry S^2 has to be

added to the standard models symmetries and geometries U(1)xSU(2)xSU(3). It is in the sense of the Noether theorem. The computations need an extension of quaternionic spacetime coordinates to octonians.

For photons spin 1 has been experimentally observed. This was taken above as a frequency, counting winding numbers about a U(1) circle. The value of the photon spin is considered constant for all wave lengths. For gravitons the "earthworm" circle is stretched squeezed, the winding of the helix is in two parts from a radius size ½ small to a middle size 1 as a winding by π; in a second large size part the winding by 2π covers the full circle (spin frequency 2 for two spin 1 exchanged gluons). If the LIGO experiment can detect spin 2 of its GW (constant for all wave lengths), this can

be seen as an observed field quantum Γ. The measured two squeezing stretching length are an otion for this. In the second section the *rgb*-gravitons 126 arise in octonians 2356 nucleon version for a Γ tetraedron configuration with the permution symmetry S_4 of four elements z,r,g,b as rotational cross ratio (z-1)/z. For the cross ratio presentation r,g,b are replaced by $(z,0;\infty,1)$. Γ has in this interpretation as cross ratio several uses: for the tetrahedron as color charged *rgb*-graviton spin-like particles (field quantums), for graviton waves [1] as two P,Q systems Minkowski coordinate systems of an observer and observed system for stretching squeezing, for an unsymmetrical distance measure arising in the rescaling of Minkowski to Schwarzschild metric. It is observed that the tetrahedron configuration is in the octonian subspace 2356, projected into spacetime. The cross ratio presentation of the *rgb*-graviton allows the perspective projection for the Schwarzschild rescaling factor as unsymmetrical distance measure. The graviton wave packages [1] are observed in the Minkowski 1234 spacetime. In another listing for a graviton, the first three SU(3) GellMann matrices as projections are presenting the topological S^3 factor of the SU(3) geometry S^3xS^5. As projection, this $\lambda_{1,2,3}$ presented graviton maps the SU(3) sphere by the Hopf map h to $h(S^3) = S^2$. The three Pauli matrices define the three space coordinates for R^3 with time normed to a constant, but used for defining the Hopf coordinates as $(c(z_1),c(z_2))\sigma_j(z_1,z_2)^{tr}$. As mentioned earlier, also the real z-coordinate is normed. The graviton maps spacetime for a black hole location to a 2-dimensional (color charges carrying) Riemannian sphere with complex or real xy-coordinates.

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