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A Unified Approach for Reliability Evaluation of Probabilistic FlowNetworks in Terms of Minimal Cutsets

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Abstract
Many real-world systems such as manufacturing systems, telecommunication systems, transportation systemsand logistics/distribution systems that play important roles in our modern society can be regarded as flow networks whose arcs have independent, finite and multi-valued random capacities. Such a flow network is indeed a multistate system with multistate components and its source-to-terminal reliability can be calculated in terms of minimal cut vectors to level d (denoted d-MCs here). The main purpose of this article is to extend this result to develop a simple and efficient algorithm to evaluate the source-to-K-terminal reliability of a probabilistic flow network in terms of minimal cutsets. Two examples are given to illustrate how all d-MCs are generated by our algorithm and the reliabilities are then computed by further applying the state space decomposition method

Keywords: *Probabilistic flow network, Minimal Cutsets, Demand level vector, System reliability, d-MC.*

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I. INTRODUCTION

Reliability analysis often assumes that the system under study is represented by a probabilistic graph or network in a two-state model so that one of the following network indices is taken into consideration:

$R_1(G)$: source-to-terminal reliability: $\Pr\{$ communication from a specified source node to a specified sink node is possible in G}

$R_2(G)$: source-to-all-terminal reliability: $\Pr\{$ communication from a specified source node to all sink nodes is possible in G}

$R_3(G)$: source-to-K-terminal reliability: $\Pr\{$ communication from a specified source node to a specified set K of sink nodes is possible in G}

In such a modeling, the system operates successfully if and only if there exists at least one operative path (respectively spanning tree; K-tree) from the specified node to another one (respectively all sink nodes; all sink nodes in K) and so its reliability is in fact to be considered as a matter of connectivity only. However, system represented in two-state models do not always reflect the real-world situations reasonably. For example, many physical systems such as manufacturing systems, telecommunication systems, transportation systems, and logistics/distribution systems can be regarded as multistate probabilistic flow networks in which arcs have independent, limited, and integer-valued random capacities. For such a probabilistic flow network, the reliability indices should be re-defined as follows:

$R_1(G)$: $\Pr\{$ at least a specified desired amount of flow can be transmitted from one specified source node s to one specified sink node t}

$R_2(G)$: $\Pr\{$ flow can be transmitted from one specified source node s to all sink nodes whose demands are all satisfied}

$R_3(G)$: $\Pr\{$ flow can be transmitted from one specified source node s to a specified set K of sink nodes whose demands are all satisfied}

Several different approaches to evaluate $R_1(G)$ for such a flow network G has been presented [6,12, 14, 20]. However, so far, it seems that no method to calculate $R_2(G)$ or $R_3(G)$ in terms of minimal cutsets is available. In fact, both $R_1(G)$ and $R_2(G)$ may be regarded as special cases of $R_3(G)$ with $|K| = 1$ and

$|K| = n - 1$, respectively if G is consisted of n nodes. Hence, all three can be calculated by a common formula for $R_3(G)$. The main objective of this article is to develop a intuitive and efficient algorithm to evaluate $R_3(G)$ by extending the results obtained in evaluating $R_1(G)$. Two examples are given to illustrate how all d-MCs are generated and the reliabilitiesarethen calculated by further applying the state-space decomposition method [3].

II. BASIC ASSUMPTION

Notation

G $G = (N, A, C, K)$;a directed probabilistic flow network where N is the set of nodes, $A = \{a_i \mid 1 \le i \le n\}$, theset of directed arcs, and $C = (M^1, M^2, ..., M^n)$ where M^i (an integer) denotes themaximum capacityof each arc a_i for $i = 1,2,...,n.$ $K = \{t_1, t_2, ..., t_k\}$, the specified set of sink nodes.

$\boldsymbol{d}$ $\boldsymbol{d} = (d_1, d_2, ..., d_k)$; the system demand vector of G where d_j is the demand level of t_j for $j = 1, 2, ..., k.$

$\boldsymbol{d}_s$ $\boldsymbol{d}_s = d_1 + d_2 + ... + d_k$; the total demand of the system.

$\boldsymbol{X}$ $\boldsymbol{X} = (x_1, x_2, ..., x_n)$; a system-state vector of G where $x_i = 0,1,2,...,M^i$ denotes the capacity level of arc a_i for $i = 1, 2, ..., n.$

$V(\boldsymbol{X})$ $V(\boldsymbol{X}) = (V(\boldsymbol{X})_1, V(\boldsymbol{X})_2, ..., V(\boldsymbol{X})_k)$; the system maximal flow vector of G under the system-state vector $\boldsymbol{X}$ where $V(\boldsymbol{X})_j$ notes the maximal flow from s to t_j under $\boldsymbol{X}$. Whenever $k \ge 2$, there may be more than one maximal flow vector for each system-state vector $\boldsymbol{X}$. Hence, a priority list which order such sinks may be introduced in order to search for a particular maximal flow vector to satisfy the demand of the system under such an order.

$B_{\boldsymbol{X}}$ $B_{\boldsymbol{X}} = \{a_i \mid x_i = M^i\}$; the set of saturated arcs under $\boldsymbol{X}$.

$U_{\boldsymbol{X}}$ $U_{\boldsymbol{X}} = \{a_i \mid x_i < M^i\}$; the set of unsaturated arcs under $\boldsymbol{X}$.

$S_{\boldsymbol{X}}$ $S_{\boldsymbol{X}} = \{a_i \mid a_i \in U_{\boldsymbol{X}}, V(\boldsymbol{X} + e_i) > V(\boldsymbol{X})\}$; the set of sensitive arcs under $\boldsymbol{X}$. For any particular system-state vector $\boldsymbol{X}$, $A = S_{\boldsymbol{X}} \cup (U_{\boldsymbol{X}} \setminus S_{\boldsymbol{X}}) \cup B_{\boldsymbol{X}}$ is a disjoint union of A.

K_j jth MC (minimal cutset) for $j = 1, 2, ..., m,$ where m is the number of MCs in the system.

$C_{K_j}(X)$ the capacity of K_j under $\boldsymbol{X}$. That is $C_{K_j}(\boldsymbol{X}) = \sum \{x_i \mid a_i \in K_j\}$.

Nomenclature

Demand (or load) level vector $\boldsymbol{d}$ of G is denoted by a k-tuple vector $\boldsymbol{d} = (d_1, d_2, ..., d_k)$ where d_j is a non-negative and integer-valued load stress to t_j. Usually, it may be a random vector whose joint probability distribution $\{p_{\boldsymbol{d}}\}_{\boldsymbol{d}}$ can be determined through continuous observation or forecasting of the load or customer's demand to the network.

System reliability: The system reliability is defined as

$$R_3(G) = \Pr\{\boldsymbol{X} \mid V(\boldsymbol{X}) \ge \boldsymbol{d}\} = \Pr\{\boldsymbol{X} \mid V(\boldsymbol{X})_1 \ge d_1, V(\boldsymbol{X})_2 \ge d_2, ..., V(\boldsymbol{X})_k \ge d_k\}.$$

d-MC: A system-state vector $\boldsymbol{X}$ in G which has a unique source and a unique sink is said to have the capacity level d if $V(X) = d$. A system-state vector $\boldsymbol{X}$ is a d-MC if and only if $V(X) = d$ and $U_X = S_X$.

Assumptions

The probabilistic flow network under study satisfies the following assumppptions:

1. It has only one source node s and k sink nodes $t_1, t_2, ..., t_k$.
2. Each node is perfectly reliable. Otherwise, the network will be enlarged by treatingeach of such nodes as an arc [1].
3. The capacity of each arc a_i is an integer-valued random variable that takes integervalues from 0 to u_i according to a given distribution.
4. The capacities of different arcs are statistically independent.
5. Flow in the network must be integer-valued and satisfy the so-called flow-conservation law [10]. This

means that no flow will disappear or be created during the transmission.

6. Each sink node attempts to meet its own demand. If there is excess flow, it may be transmitted to another sink node whose load demand has not been satisfied yet according to the order specified by priority lists.

III. MODEL CONSTRUCTION

For $G = (N, A, C, K)$ with the unique source node s and a specified set $K = \{t_1, t_2, ..., t_k\}$ of sink nodes, the evaluation of $R_3(G) = \Pr\{\boldsymbol{X} \mid V(\boldsymbol{X}) \geq \boldsymbol{d}\}$ essentially relies on the much more difficult problem on how to obtain the maximal flow vector $V(\boldsymbol{X}) = (V(\boldsymbol{X})_1, V(\boldsymbol{X})_2, ..., V(\boldsymbol{X})_k)$ for each system-state $\boldsymbol{X}$ in G. To avoid such a problem, enlarging G into $\hat{G}$ that has the unique source s and the unique sink $\hat{t}$ and then exact k artificial arcs $a_{n+1}, a_{n+2}, ..., a_{n+k}$ directed respectively from $t_1, t_2, ..., t_k$ to $\hat{t}$ are all added to G such that each arc a_{n+j} is given a deterministic capacity d_j (i.e. $M^{n+j} = d_j$ and $\Pr\{X_{n+j} = d_j\} = 1$ for $j = 1, 2, ..., k$). In other words, $\hat{G} = (N \cup \{\hat{t}\}, A \cup \{a_{n+1}, a_{n+2}, ..., a_{n+k}\}, \tilde{M}, \{\hat{t}\})$ where $\tilde{M} = (M^1, M^2, ..., M^n, M^{n+1}, ..., M^{n+k})$. In fact, $R_3(G) = R_1(\hat{G})$. That is, the evaluation of $R_3(G)$ is essentially reduced to a much more easier problem on how to obtain the maximal flow $V_e(\hat{\boldsymbol{X}})$ in $\hat{G}$. The maximal flow from s to $\hat{t}$ in $\hat{G}$ is less than or equal to $d_s = \sum_{j=1}^{k} d_j$. Thus, under any system-state vector $\hat{\boldsymbol{X}}$ of $\hat{G}$, the demands of all sink nodes in K are all satisfied if and only if its maximal $s-\hat{t}$ flow is equal to $d_s = \sum_{j=1}^{k} d_j$. The relationship between $\hat{\boldsymbol{X}}$ and $\boldsymbol{X}$ is shown in the following lemma.

Lemma 1. Foreach system-state vector $\boldsymbol{X} = (x_1, x_2, ..., x_n)$ in G, let $\hat{\boldsymbol{X}} = (x_1, x_2, ..., x_n, M^{n+1}, ..., M^{n+k})$ be the corresponding system-state vector in $\hat{G}$. Then $V(\boldsymbol{X}) \geq \boldsymbol{d}$ if and only if $V_e(\hat{\boldsymbol{X}}) = d_s = \sum_{j=1}^{k} d_j$.

From lemma 1, we conclude that $R_3(G) = \Pr\{\boldsymbol{X} \mid V(\boldsymbol{X}) \geq (d_1, d_2, ..., d_k)\} = \Pr\{\hat{\boldsymbol{X}} \mid V_e(\hat{\boldsymbol{X}}) = d_s\}$ $= \Pr\{\hat{\boldsymbol{X}} \mid V_e(\hat{\boldsymbol{X}}) \geq d_s\} = R_1(G)$. However, $R_1(\hat{G}) = \Pr\{\hat{\boldsymbol{X}} \mid V_e(\hat{\boldsymbol{X}}) \geq d_s\}$ can be calculated if all $(d_s - 1)$-*MCs* of $\hat{G}$ are known in advance. Thus, we will concentrate on how to search for all $(d_s - 1)$-*MCs* of $\hat{G}$ first.

Lemma 2. If $\boldsymbol{X}$ is a $(d_s - 1)$-*MC* of $\hat{G}$, then

$$S_{\hat{\boldsymbol{X}}} \subseteq \bigcap_j \{C_{K_j}(\hat{\boldsymbol{X}}) = d_s - 1\}.$$

Lemma 3. If $\boldsymbol{X}$ is a $(d_s\text{-}1)$-*MC* of $\hat{G}$, then there exists at least one MC $K_r = \{a_{r_1}, a_{r_2}, ..., a_{rn_r}\}$ such that

$$x_{r_1} + x_{r_2} + ... + x_{rn_r} = d_s - 1 \quad (1)$$

$$(x_{r_1}, x_{r_2}, ..., x_{rn_r}) \leq (M^{r_1}, M^{r_2}, ..., M^{rn_r}) \quad (2)$$

$$x^i = M^i \text{ for } a_i \notin K_r. \quad (3)$$

Any $\hat{\boldsymbol{X}} = (x_1, x_2, ..., x_n, M^{n+1}, ..., M^{n+k})$ that satisfies constraints (1), (2), and (3) simultaneously for each MC K_r will be a $(d_s - 1)$-*MC* candidate of $\hat{G}$. And so each $(d_s - 1)$-*MC* of $\hat{G}$ is obviously a $(d_s - 1)$-*MC* candidate of $\hat{G}$ by the theorem. According to the definition, a candidate $\hat{\boldsymbol{X}}$ of $\hat{G}$ will be a $(d_s - 1)$-*MC* if (1) $V_e(\hat{\boldsymbol{X}}) = d_s - 1$ and (2) $U_{\hat{\boldsymbol{X}}} = S_{\hat{\boldsymbol{X}}}$.

Suppose that $\{\hat{Y}^1, \hat{Y}^2, ..., \hat{Y}^{m_d}\}$ is the family of all $(d_s - 1)$-*MCs* in $\hat{G}$, then $R_3(G) = \Pr\{\boldsymbol{X} \mid V(\boldsymbol{X}) \geq \boldsymbol{d}\} = \Pr\{\hat{\boldsymbol{X}} \mid V_e(\hat{\boldsymbol{X}}) = d_s\} = 1 - \Pr\{\hat{\boldsymbol{X}} \mid V_e(\hat{\boldsymbol{X}}) \leq d_s - 1\} = 1 - \Pr\left\{\bigcup_{i=1}^{m_d} \{\hat{\boldsymbol{X}} \mid \hat{\boldsymbol{X}} \leq \hat{\boldsymbol{Y}}^i\}\right\}$. To compute it, several methods such as inclusion-exclusion [7, 12], disjoint subset [13], and state space decomposition [3] are available.

IV. ALGORITHM

Step 1.Input G and its demand level vector $\boldsymbol{d} = (d_1, d_2, ..., d_k)$:

(a) Input graph G and specify the source node s and the set K of sink nodes.

(b) Input the demand level of each sink node in K.

Step 2. Supergraph creation:

(a) Enlarge G into $\hat{G}$ by adding one node $\hat{t}$ and then k additional arcs $\{a_{n+1}, a_{n+2}, ..., a_{n+k}\}$ directed from $t_1, t_2, ..., t_k$ respectively to $\hat{t}$ are added.

(b) Let $x_{n+1} = M^{n+1} = d_1$ for $1 = 1, 2, ..., k.$

Step 3. Find all MCs, $K_1, K_2, ..., K_m$ from s to $\hat{t}$ in $\hat{G}$ by assuming that each arc has two states only [13].

Step 4. With respect to each $K_r = \{a_{r_1}, a_{r_2}, ..., a_{rn_r}\}$

4.1. Find all non-negative integer-valued solutions of $x_{r_1} + x_{r_2} + ... + x_{rn_r} = d_s - 1$ and $(x_{r_1}, x_{r_2}, ..., x_{rn_r}) \le (M^{r_1}, M^{r_2}, ..., M^{rn_r})$

4.2. Set $x^i = M^i$ for $a_i \in K_r$.

4.3. Obtain the family of all $(d_s - 1)$-MC candidates

$$\hat{\boldsymbol{X}} = (x_1, x_2, ..., x_n, M^{n+1}, ..., M^{n+k}).$$

4.4. Check each $(d_s - 1)$-MC candidate $\hat{\boldsymbol{X}} = (x_1, x_2, ..., x_n, M^{n+1}, ..., M^{n+k})$ whether it is a $(d_s - 1)$-MC.

Step 5.Suppose that $\{\hat{Y}^1, \hat{Y}^2, ..., \hat{Y}^{m_d}\}$ is the family of all $(d_s - 1)$-MCs in $\hat{G}$. Then, $R_3(G) = R_1(\hat{G}) = \Pr\{\hat{\boldsymbol{X}} \mid V_e(\hat{\boldsymbol{X}}) \ge d_s\} = 1 - \Pr\left\{\bigcup_{i=1}^{m_d} \{\hat{\boldsymbol{X}} \mid \hat{\boldsymbol{X}} \le \hat{\boldsymbol{Y}}^i\}\right\}$ can be obtained by applying the state decomposition method.

V. NUMERICAL EXAMPLES

The following two examples are used to illustrate the proposed algorithm:

Example 1.

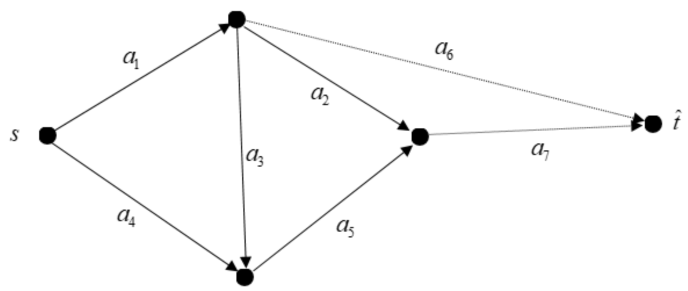

Figure 1: Network G and its supergraph in Example 1

Table 1: Probability distributions of arc capacities in Example 1

Arc	Capacity	Probability	Arc	Capacity	Probability
a_1	3	0.60	a_3	1	0.90
	2	0.25		0	0.10
	1	0.10	a_4	1	0.90
	0	0.05		0	0.10
a_2	2	0.60	a_5	2	0.70
	1	0.30		1	0.25
	0	0.10		0	0.05

The system reliability for demand level $\boldsymbol{d} = (d_1, d_2) = (2,1)$ can be evaluated by the following steps:

Step 1. $C = (M^1, M^2, M^3, M^4, M^5) = (3,2,1,1,2)$, $K = \{t_1, t_2\}$, $d_1 = 2$, $d_2 = 1$, and $d_s - 1 = d_1 + d_2 - 1 = 2$.

Step 2. (a) Enlarge G into $\hat{G}$ by adding $\hat{t}$ and two artificial arcs a_6 and a_7 directed respectively from t_1 and t_2 to $\hat{t}$.

(b) Set $x_6 = M^6 = d_1 = 2$, $x_7 = M^7 = d_2 = 1$, and $\Pr\{x_6 = 2\} = \Pr\{x_7 = 1\} = 1$.

Step 3. Total MCs from s to $\hat{t}$ in $\hat{G}$ are $K_1 = \{a_1, a_4\}$, $K_2 = \{a_1, a_5\}$, $K_3 = \{a_1, a_7\}$, $K_4 = \{a_6, a_7\}$, $K_5 = \{a_2, a_5, a_6\}$, and $K_6 = \{a_2, a_3, a_4, a_6\}$.

Step 4. The result is listed in Table 2.

Table 2: Final results of Example 1.

K_i	**2-MC candidate**	**2-MC?**	K_i	**2-MC candidate**	**2-MC?**
K_1	$(2,2,1,0,2,2,1)$	Yes	K_3	$(1,2,1,1,2,2,1)$	Yes
	$(1,2,1,1,2,2,1)$	Yes	K_4	**None**	
	$(2,2,1,1,0,2,1)$	Yes	K_5	$(3,0,1,1,0,2,1)$	Yes
K_2	$(1,2,1,1,1,2,1)$	No	K_6	$(3,0,0,0,2,2,1)$	Yes
	$(0,2,1,1,2,2,1)$	No			

Step 5. By applying the state space decomposition, we may obtain that $\Pr\left\{\bigcup_{i=1}^{5}\{\hat{X} \mid \hat{X} \le \hat{Y}_i\}\right\} = 0.18982$.

Hence, the system reliability for demand level $\boldsymbol{d} = (d_1, d_2) = (2,1)$ is $R_3(G) = 1 - 0.18982 = 0.81018$.

Example 2.

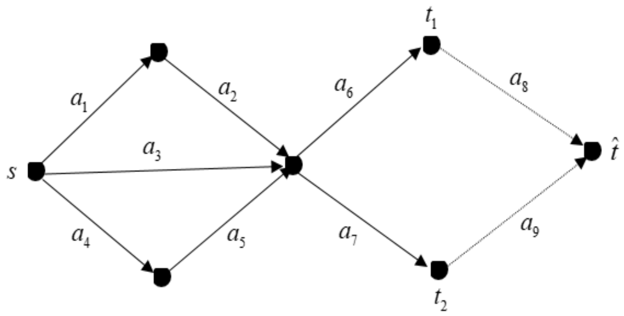

Figure 2: Network G and its supergraph in Example 2

Table 3: Probability distributions of arc capacities in Example 2

Arc	Capacity	Probability	Arc	Capacity	Probability
a_1, a_2	3	0.60	a_4, a_5	2	0.70
	2	0.25		1	0.20
	1	0.10		0	0.10
	0	0.05	a_6, a_7	2	0.60
a_2	1	0.80		1	0.30
	0	0.20		0	0.1

The system reliability for demand level $\boldsymbol{d} = (d_1, d_2) = (1,1)$ can be evaluated by the following steps:

Step 1. $C = (M^1, M^2, M^3, M^4, M^5, M^6, M^7) = (3,3,1,2,2,2,2)$, $K = \{t_1, t_2\}$, $d_1 = 1$, $d_2 = 1$, and $d_s - 1 = d_1 + d_2 - 1 = 1$.

Step 2. (a) Enlarge G into $\hat{G}$ by adding $\hat{t}$ and two artificial arcs a_8 and a_9 directed respectively from t_1 and t_2 to $\hat{t}$.

(b) Set $x_8 = M^8 = d_1 = 1$, $x_9 = M^9 = d_2 = 1$, and $\Pr\{x_8 = 1\} = \Pr\{x_9 = 1\} = 1$.

Step 3. Total MCs from s to $\hat{t}$ in $\hat{G}$ are $K_1 = \{a_1, a_3, a_4\}$, $K_2 = \{a_2, a_3, a_4\}$, $K_3 = \{a_1, a_3, a_5\}$, $K_4 = \{a_2, a_3, a_5\}$, $K_5 = \{a_6, a_7\}$, $K_6 = \{a_6, a_9\}$, $K_7 = \{a_7, a_8\}$, and $K_8 = \{a_8, a_9\}$.

Step 4. The result is listed in Table 4.

Step 5. By applying the state space decomposition, we may obtain that $\Pr\left\{\bigcup_{i=1}^{16}\{\hat{X} \mid \hat{X} \le \hat{Y}_i\}\right\} = 0.12688$.

Hence, the system reliability for demand level $\boldsymbol{d} = (d_1, d_2) = (1,1)$ is $R_3(G) = 1 - 0.12688 = 0.87312$.

Table 4: Final results of Example 2.

K_i	**1-MC candidate**	**1-MC?**	K_i	**1-MC candidate**	**1-MC?**
	$(1,3,0,0,2,2,2,1,1)$	Yes		$(3,1,0,2,0,2,2,1,1)$	Yes
K_1	$(0,3,1,0,2,2,2,1,1)$	Yes	K_4	$(3,0,1,2,0,2,2,1,1)$	Yes
	$(0,3,0,1,2,2,2,1,1)$	Yes		$(3,0,0,2,1,2,2,1,1)$	
	$(3,1,0,0,2,2,2,1,1)$	Yes	K_5	$(3,3,1,2,2,1,0,1,1)$	Yes
K_2	$(3,0,1,0,2,2,2,1,1)$	Yes		$(3,3,1,2,2,0,1,1,1)$	
	$(3,0,0,1,2,2,2,1,1)$	Yes	K_6	$(3,3,1,2,2,0,2,1,1)$	Yes
	$(1,3,0,2,0,2,2,1,1)$	Yes	K_7	$(3,3,1,2,2,2,0,1,1)$	
K_3	$(0,3,1,2,0,2,2,1,1)$	Yes	K_8	$(3,3,1,2,2,0,2,1,1)$	No
	$(0,3,0,2,1,2,2,1,1)$	Yes			

VI. SUMMARY AND CONCLUSIONS

A simple and efficient algorithm to calculate $R_3(G)$ of a probabilistic flow network is proposed in this article. This algorithm is so general that it can also apply to compute $R_1(G)$ and $R_2(G)$. In fact, $R_1(G)$ and $R_2(G)$ are special cases of $R_3(G)$. Hence, easier algorithm [6, 12, 14] for the source-to-terminal is shown to be a special case of this new one.

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