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Formulaic teaching design of pricing interest rate swaps

Dianli ZHAO
College of Science, University of Shanghai for Science and Technology, Shanghai, 200093, China

Abstract: *Interest rate swaps in traditional finance are shown by the sum of the values of a series of forward interest rate agreements, in which the known floating interest rate has only one period. How to extend the corresponding part to n periods for calculation is difficult to understand, especially for science students major in mathematical finance. This paper attempts to explain the origin and theoretical basis of this pricing method from the perspective of formula evolution, to provide intuitive analysis ideas for relevant personnel to clarify this pricing problem.*

Key words*: interest rate swap, forward, formulaic method, mathematical finance*

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In recent years, with the increasing prosperity of the financial industry, the number of workers involved in this industry is almost growing exponentially. The emergence of financial mathematics courses strongly confirms the attention of the educational community to the financial field. Because most of the learners of this course are from science and have relatively little contact with the relevant knowledge of finance, they have some difficulties in understanding when learning the conversion between certain products or mutual pricing. This paper will take the pricing of interest rate swaps as an example to illustrate its formulaic derivation method, to optimize the relevant teaching process.

I. Forward interest rate agreement pricing of interest rate swap (see [1], page 153)

Literature [1] mainly explains the valuation of interest rate swaps using forward rate agreements in the form of examples. The specific process is summarized as the following three steps:

(1) Using the LIBOR swap zero interest rate curve, calculate the forward interest rate for each LIBOR interest rate that determines the swap cash flow.

(2) Assuming that LIBOR interest rate is equal to forward interest rate, calculate swap cash flow.

(3) The swap value is obtained by discounting these swap cash flows.

In this calculation process, it is difficult for science students to understand the whole process of asset price calculation visually from the above words because they do not have much basic knowledge of finance, do not have a thorough understanding of finance, and do not have much understanding of financial logic.From this point of view, this paper proposes to deduce the relevant asset prices based on formula derivation, in order to help you understand this process.

II. Formulaic teaching design

It is not difficult to see from the contract content of the interest rate swap that its value is $V = V_{fl} - V_{fix}$, among which V_{fl} and V_{fix} represent the value of the floating rate part of the bonds received by one party to the contract and the value of the fixed rate part of the bonds paid out. The details are as follows:

$$V_{fl} = \tilde{k}e^{-r_1 t_1} + Qe^{-r_1 t_1}$$

and

$$V_{fix} = \sum_{i=1}^{n} ke^{-r_i t_i} + Qe^{-r_n t_n} .$$

First, we extend the calculation formula of the value of floating rate bonds backward

$$\begin{aligned} V_{fl} &= \tilde{k}e^{-r_1t_1} + Qe^{-r_1t_1} \\ &= \tilde{k}e^{-r_1t_1} + Qe^{-r_1t_1} - Qe^{-r_2t_2} + Qe^{-r_2t_2} \\ &= \tilde{k}e^{-r_1t_1} + Q\left[e^{r_2t_2-r_1t_1} - 1\right]e^{-r_2t_2} + Qe^{-r_2t_2} \\ &= \tilde{k}e^{-r_1t_1} + Q\left[e^{R_1(t_2-t_1)} - 1\right]e^{-r_2t_2} + Qe^{-r_2t_2} \end{aligned}$$

Among them $R_1 = \frac{r_2t_2 - r_1t_1}{t_2 - t_1}$, it is using LIBOR swap zero interest rate curve to calculate forward interest rate" mentioned in document [1].

For convenience, denote $\tilde{k}_1 = \tilde{k}$ and $\tilde{k}_2 = Q\left[e^{R_1(t_2-t_1)} - 1\right]$, then we get

$$V_{fl} = \tilde{k}_1e^{-r_1t_1} + \tilde{k}_2e^{-r_2t_2} + Qe^{-r_2t_2}.$$

Similarly, for each $i \geq 1$, denote $R_i = \frac{r_{i+1}t_{i+1} - r_it_i}{t_{i+1} - t_i}$ and $\tilde{k}_{i+1} = Q\left[e^{R_i(t_{i+1}-t_i)} - 1\right]$, we have

$$V_{fl} = \sum_{i=1}^{n} \tilde{k}_ie^{-r_it_i} + Qe^{-r_nt_n}.$$

The pricing formula of swap can be obtained as follows

$$V = \sum_{i=1}^{n} \left(\tilde{k}_i - k\right)e^{-r_it_i}.$$

That is:

$$\text{Total values}=\sum_{i=1}^{n}\left(Value \ \ of \ swap \ \ at \ \ t_i\right)\times\left(Discount \ \ \ factors\right).$$

Remark 1. Compared with the previous text-based-process pricing method, we use a formulaic derivation method that is more acceptable to science students, so as to strengthen the understanding of the content. At the same time, the pricing is carried out in the form of multi-term forward contract swap. This formula shows that the value of assets is equal to the discount of all future cash flows, which is very easy to understand.

III. Examples And Descriptions

Example 1. Suppose that in a swap contract, a financial institution pays a six-month LIBOR and charges an annual fixed interest rate of 8% (compounded once every six months), with a nominal principal of $100million. The swap has a 1.25 year term. LIBOR (continuous compound interest rate) for 3 months, 9 months and 15 months are 10%, 10.5% and 11% respectively. LIBOR for the six months on the last interest payment date is 10.2% (compound interest once every six months).

Analysis: In this example, $k = 400$ ten thousand and $\tilde{k} = 150$ ten thousand. The cash flow to be exchanged after 3 months is known, and the financial institutions use the annual interest rate of 10.2% to exchange for the annual interest rate of 8%. Therefore, the value of this exchange to financial institutions can be calculated directly: $\left(\tilde{k} - k\right)e^{-r_1t_1} = -1.07$ million.Next, to calculate the value of the cash flow exchange after 9 months, we must first calculate the forward interest rate from 3 months to 9 months from now. According to the calculation formula of forward interest rate, the forward interest rate from 3 months to 9 months is: $\mathrm{R}_1 = 10.75\%$. The exchange value is $\left(\tilde{k}_2 - k\right)e^{-r_2t_2} = -1.41$ million. Similarly, you can get $\mathrm{R}_2 = 11.75\%$, and further get $\left(\tilde{k}_3 - k\right)e^{-r_3t_3} = -1.79$ million. Thus, the exchange value is $V = -1.07 - 1.41 - 1.79 = -4.27$ million.□ ■

Remark 2. From the above pricing process, it is not difficult to find that, compared with the three-step descriptive pricing method, the formula pricing process is not only intuitive, but also explains the sources of various data and specific calculation methods in the calculation process, which is more convenient for understanding and application. This derivation process is very useful for those science students who have weak financial knowledge.

Reference

[1]. John Hull (written), Zhang Taowei (Translated), Options, Futures and Other derivatives, Beijing: People's Posts and Telecommunications Press, 2009.7, 6th Edition.