International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 10 Issue 7 ǁ July 2022 ǁ PP. 779-783

Construction of new exactly solved quantum potentials generated from the Quartic powerlaw potential

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Abstract: *One of the important tasks of quantum mechanics is to solve the Schrödinger equation with a physical potential. We have reported a list of exact bound state solutions of the Schrödinger equation generated from the quartic power law potentials using the extended transformation formalism. The bound state energy eigenvalues of the generated potential systems are obtained. The constraint equations relating the parameters of the potential and angular momentum quantum numbers are also obtained.*
Keywords*:Exactly Solvable Potentials,Schrödinger equation,Transformation Method*

Date of Submission: 10-07-2022 Date of acceptance: 26-07-2022

I. INTRODUCTION

One of the important tasks of quantum mechanicsis to solve the Schrödinger equation with a physical. In recent years, there has been considerable interest in the study of exactly solvable quantum mechanical potentials of physical interest as it provides maximum information of the quantum system. Various methods are used in the calculation of exact analytic solutions (EAS) of the Schrödinger equation for quantum mechanical potentials.

In this paper we present an efficient approach to generate/construct exactly solvable potentials in non-relativistic quantum mechanics. The approach is based on a transformation/mapping procedure and is known as Extended Transformation (ET) method. The method has been found to be a neat and simple method to determine the energy eigenfunctions and associated energy eigenvalues spectrum of the generated quantum mechanical potentials. The extended transformation (ET) includes a coordinate transformation (CT) followed by a functional transformation (FT). A very useful property of the transformation method one should note is that the wavefunction of the generated quantum systems (QSs) are almost always normalizable. We have reported exact bound sate solutions of the Schrödinger equation for quartic and two term fractional power law potential in any chosen dimensional Euclidean spaces.

II. FORMALISM

The most general dimensionless Schrödinger equation for the already known termed as A-QS quartic potential $V_A(r) = ar + br^2 + cr^3 + br^4$ with $(\hbar = 2m = 1)$ is:

$$\frac{d^2\Psi}{dr^2} + \left[E_n^A - \left(ar + br^2 + cr^3 + dr^4\right)\right]\Psi_A(r) = 0$$

The exact analytic solution of A-QS is given by [3, 4]:

$$\Psi_A(r) = \exp\left[-\frac{c}{4\sqrt{d}}r^2 - \frac{\sqrt{d}}{3}r^3\right]$$

The energy eigenvalues for the potential system is given by [3, 4]

$$E_A = \frac{c}{2\sqrt{d}}$$

Under extended transformation (ET), which consist of coordinate transformation $r \to g_B(r)$ and followed by functional transformation:

$$\Psi_B(r) = f^{-1}(r)\Psi_A(g_B(r))$$

Let us choose D_B as the dimension of the B-QS.

Then the co-efficient of the term $\Psi_B'(r)$ will take the form

$$\frac{d}{dr}\ln\frac{f_B^2 g_B^{D_A-1}}{g_B^{'}}=\frac{D_B-1}{r}=\frac{d}{dr}\ln r^{D_B-1}$$

This leads to:

$$\Psi_B(r)=g_B^{'-\frac{1}{2}}(r)g_B^{\frac{D_A-1}{2}}r^{-\frac{D_B-1}{r}}\Psi_A(g_B(r))$$

To obtain the Standard Schrödinger equation form we make the following predefined ansatze [6-9] by taking the first term of A-QS potential as working potential.

To implement Extended Transformation [ET],let the working potential be selected as:

$$V_A^W(r)=ar$$

The analytic form of the transformation function is found as:

$$g_B(r)=\left(\frac{3}{2}\right)^{\frac{2}{3}}\left(-\frac{E_{B1}}{a}\right)^{\frac{1}{3}}r^{\frac{2}{3}}$$

Which satisfies the local property $g_B(0)=0$ and asymptotic property $g(\infty)=\infty$, by putting the integration constant equal to zero.

$$g_B^{'2}(\alpha g_B(r))=-E_{B1}$$

And

$$g_B^{'2}(r)E_n^A=-V_B^1$$

We obtain:

$$V_B^1(r)=C_B^2 r^{-\frac{2}{3}}$$

Where C_B^2 is the Characteristic Constant of B-Qs and is given by:

$$C_B^2=\left(\frac{3}{2}\right)^{-\frac{2}{3}}\left(-\frac{E_{B1}}{a}\right)^{\frac{2}{3}}(-E_A)$$

The another ansatze is:

$$-g_B^{'2}(r)\left[bg_B{}^2(r)+cg_B{}^3(r)+dg_B^4(r)\right]=-V_B^2$$

This leads to:

$$V_B^2(r)=\beta_2 r^2+\beta_3 r^{\frac{4}{3}}+\beta_4 r^{\frac{2}{3}}$$

Therefore the potential of the newly constructed B-QS becomes:

$$V_B(r)=\beta_1 r^{-\frac{2}{3}}+\beta_2 r^2+\beta_3 r^{\frac{4}{3}}+\beta_4 r^{\frac{2}{3}}$$

The parameters of the potential are defined as:

$$C_B^2 = \beta_1, \beta_2 = \left(\frac{3}{2}\right)^2 \left(-\frac{E_{B1}}{a}\right)^2 d$$

$$\beta_3 == \left(\frac{3}{2}\right)^{\frac{4}{3}} \left(-\frac{E_{B1}}{a}\right)^{\frac{5}{3}} c$$

$$\beta_4 == \left(\frac{3}{2}\right)^{\frac{2}{3}} \left(-\frac{E_{B1}}{a}\right)^{\frac{4}{3}} b$$

The constraint equation relating the parameters of the potential are:

$$\beta_1 == -\frac{1}{3}\frac{\beta_3}{\sqrt{\beta_2}}; \beta_4 = \frac{1}{4}\frac{{\beta_3}^2}{\beta_2}$$

The energy eigenvalues of B-QS comes out to be:

$$E_{B1} = \left(D_B + \frac{1}{3}\right)\sqrt{\beta_2}$$

The corresponding energy eigenfunctions of B-QS in desired -dimensional spaces becomes:

$$\Psi_{B1}(r) = N_1 r^{\frac{4-3D_B}{6}} \exp\left[-\frac{3}{8}\frac{\beta_3}{\sqrt{\beta_2}} r^{\frac{4}{3}} - \frac{\sqrt{\beta_2}}{2} r^2\right]$$

(i) Exactly Solved Potentials (ESPs) generated from $V_A(r) = ar + br^2 + cr^3 + dr^4$ and energy eigenvalues taking the other terms as working potential:

QS	$V_A^W(r)$	$g(r)$	ESP	Energy Eigenvalues
B_2	br^2	$\sqrt{2}\left(-\frac{E_{B2}}{b}\right)^{\frac{1}{4}} r^{\frac{1}{2}}$	$\gamma_1 r^{-1} + \gamma_2 r^{-\frac{1}{2}} + \gamma_3 r + \gamma_4 r^{\frac{1}{2}}$	$E_{B2} = -\frac{1}{4}\frac{{\gamma_4}^2}{\gamma_3}$
			$\gamma_1 = \frac{1}{2}\sqrt{\left(-\frac{E_{B2}}{b}\right)}(-E_A)$	
			$\gamma_2 = \frac{1}{\sqrt{2}}\left(-\frac{E_{B2}}{b}\right)^{\frac{3}{4}} a$	
			$\gamma_3 = 2\left(-\frac{E_{B2}}{b}\right)^{\frac{3}{2}} d$	
			$\gamma_4 = \sqrt{2}\left(-\frac{E_{B2}}{b}\right) c$	
		2		

B_3	cr^3		$\sigma_1 r^{-\frac{6}{5}}+\sigma_2 r^{-\frac{2}{5}}+\sigma_3 r^{-\frac{4}{5}}+\sigma_4 r^{\frac{2}{5}}$	$E_{B3}=-2\sqrt{\sigma_2\sigma_4}$
			$\sigma_1=\left(\frac{2}{5}\right)^{\frac{6}{5}}\left(-\frac{E_{B3}}{c}\right)^{\frac{2}{5}}\left(-E_A\right)$	
			$\sigma_2=\left(\frac{2}{5}\right)^{\frac{2}{5}}\left(-\frac{E_{B3}}{c}\right)^{\frac{4}{5}}b$	
			$\sigma_3=\left(\frac{2}{5}\right)^{\frac{4}{5}}\left(-\frac{E_{B3}}{c}\right)^{\frac{3}{5}}a$	
			$\sigma_4=\left(\frac{2}{5}\right)^{-\frac{2}{5}}\left(-\frac{E_{B3}}{c}\right)^{\frac{6}{5}}d$	
B_4	dr^4	$\left[3\left(-\frac{E_{B4}}{d}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}r^{\frac{1}{3}}$	$\rho_1 r^{-\frac{4}{3}}+\rho_2 r^{-1}+\rho_3 r^{-\frac{2}{3}}+\rho_4 r^{-\frac{1}{3}}$	$E_{B4}=-\frac{9}{4}\rho_2^2$
			$\rho_1=\frac{1}{9}\left[3\sqrt{\left(-\frac{E_{B4}}{d}\right)}\right]^{\frac{2}{3}}\left(E_A\right)$	
			$\rho_2=\frac{1}{3}\left[\sqrt{\left(-\frac{E_{B4}}{d}\right)}\right]a$	
			$\rho_3=\frac{1}{9}\left[3\sqrt{\left(-\frac{E_{B4}}{d}\right)}\right]^{\frac{4}{3}}b$	
			$\rho_4=\frac{1}{9}\left[3\sqrt{\left(-\frac{E_{B4}}{d}\right)}\right]^{\frac{5}{3}}c$	

(ii) Constraint equations relating the parameters of the potentials and energy eigenfunctions:

QS	Constraint equations	Energy eigenfunctions
B_2	$\gamma_1=-\frac{1}{4}\frac{\gamma_4}{\sqrt{\gamma_3}};\gamma_2=-\frac{1}{2}\sqrt{\gamma_3}$	$\Psi_{B2}(r)=N_2 r^{\frac{1}{4}}\exp\left[-\frac{1}{2}\frac{\gamma_4}{\sqrt{\gamma_3}}r-\frac{2}{3}\sqrt{\gamma_3}r^{\frac{3}{2}}\right]$
B_3	$\sigma_1=-\frac{2}{5}\sqrt{\sigma_2};\sigma_3=-\frac{4}{5}\sqrt{\sigma_4}$	$\Psi_{B3}(r)=N_3 r^{\frac{3}{10}}\exp\left[-\frac{5}{4}\sqrt{\sigma_2}r^{\frac{4}{5}}-\frac{5}{6}\sqrt{\sigma_4}r^{\frac{6}{5}}\right]$
B_4	$\rho_1=-\frac{1}{3}\sqrt{\rho_3};\rho_4=-3\rho_2\sqrt{\rho_3}$	$\Psi_{B4}(r)=N_4 r^{\frac{1}{3}}\exp\left[-\frac{3}{2}\sqrt{\rho_3}r^{\frac{2}{3}}+\frac{3}{2}\rho_2 r\right]$

III. CONCLUSION

In extended transformation method it is possible to generate a finite number of different exactly solved quantum systems by selecting the working potentials. We have found here a class of exactly solved potentials which includes Coulomb+fractional, fractional, Coulomb+linear+harmonic, coulomb potential perturbed by terms involving powers of , linear +fractional etc. This technique offers a general and efficient scheme for calculating these and other similar potentials of physical and mathematical interest in quantum mechanics.

REFERENCES

[1]. G.P. Flosses, 1979, "Exact Solution for a doubly anharmonic Oscillator", **Phys.Lett.A72, pp 289**

[2]. P.Roy and R. Roychoudhury, 1987,"On exact solutions of the doubly anharmonic oscillator", **J.Phys.A:Math.Gen.20PP6597-6601**

[3]. G.P.Flessas and A. Watt 1981 "An exact solution of the Schrödinger equation for a Multiterm potential", **J.Phys.A.:Math.Gen.14 ppL315-L318**

[4]. G.P.Flessas and K.P.Das 1981 "Exact solution for anharmonic Oscillators", **J.Phys.A.:Math.Gen.14 ppL209-L211**

[5]. R.Adhikari, R.Dutt and Y.P. Varshni, 1989,"Exact Solution for the Polynomial Potentials using Supersymmetric Factorization Method**", Phys. Lett.A 141,pp 1-8**

[6]. SAS Ahmed, 1997 "A transformation method of Generating exact analytic Solutions of the Schrödinger Equation", **International Journal of Theoretical phys., 36,1893(1997)**

[7]. SAS Ahmed & L. Buragohain. (2009). "Generation of new Classes of Exactly solved Potentials**" Physica Scripta, Phys.Scr.80(2009)025004(6pp)**

[8]. SAS Ahmed & L. Buragohain. (2010), "Exactly solvable quantum mechanical systems generated from the Manning Rosen Potential using Extended transformation Method", **Electronic Journal of Theoretical Physics, EJTP,7,No23(2010)145-154**

[9]. SAS Ahmed & L. Buragohain. (2010). "Generation of new Classes of Exactly Solved potentials from the Trigonometric Rosen-Morse Potential" **Indian Journal of Physics, 84(6),741-744(2010))**

[10]. S.K.Bose, N.Gupta,1998, "Exact solution of non-relativistic Schrödinger equation for certain Central Physical Potentials"**, IINuovoCimento 113B,3, PP.299-328**