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Bianchi Type-I Cosmological Models with Bulk Viscosity and Decaying Vacuum Energy Density

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Abstract: *Bianchi Type I cosmological model with bulk viscosity and vacuum energy density is studied.The Hubble parameter H is assumed to be function of scale factor R. It is found that constant deceleration parameter leads to decelerating universe throughout the evaluation. The vacuum energy density decreases with time. Physical and geometrical properties of the model have also been discussed.*
Key words: *Deceleration Parameter, Bulk Viscosity, Hubble parameter*

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I. Introduction:

In general relativity, it is interesting to investigation the cosmological problem to find a model of physical universe that correctly predicts the results of astronomical observations. Spatially homogeneous and isotropic cosmological models i.e. Friedmann-Robertson-Walker cosmological models are the simplest models of the expanding universe. But FRW models are unstable near the singularities as pointed out by Partridge and Wilkinson [1]. Therefore, spatially homogeneous and anisotropic Bianchi Type I models are undertaken to study the universe in its early stages of evolution.

The distribution of matter can be satisfactorily explained by perfect fluid due to the large scale distribution of galaxies in our universe. However, a realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. The different picture of the universe appeared at the initial stage of evolution due to dissipative process caused by viscosity as viscosity counteracts the cosmological collapse. The effect of viscosity on the evolution of cosmological models has been studied by Misner [2]. Cosmological models with viscous fluid source in isotropic as well as anisotropic space-times have been widely considered [3-8].

In this paper, we have studied the main components of bulk viscous fluid Bianchi type-I cosmological model with cosmological term Λ, and introduce the assumptions. These assumptions bring us to study to origin and evolution of universe.

Metric and Field Equations :

We consider the Bianchi type-I space-time represented by the line-element

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \qquad \dots (1)$$

where A, B and C are functions of t only.

We assume the cosmic matter consisting of viscous fluid represented by the energy-momentum tensor

$$T_{ij} = (\rho + \bar{p})v_i\, v_j + \bar{p}\;\, g_{ij} \qquad \dots (2)$$

where $\bar{p}$ is the effective pressure given by

$$\bar{p} = p - \zeta\;\, v^i_{;i} \qquad \dots (3)$$

where ρ and p are energy density and isotropic pressure respectively and vi the four velocity vector of the fluid satisfying $v_i\, v^i = -1$. The Einstein's field equations (in gravitational units 8πG = c = 1) with time-dependent cosmological term Λ (t) are

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j + \Lambda\, g_i^j \qquad \dots (4)$$

For the line-element (1), the field equations (4) in comoving system of coordinates lead to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=\Lambda-\bar{p} \qquad \dots (5)$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{C}\dot{A}}{CA}=\Lambda-\bar{p} \qquad \dots (6)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=\Lambda-\bar{p} \qquad \dots (7)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=\Lambda+\rho \qquad \dots (8)$$

The vanishing divergence of Einstein tensor gives rise to

$$\dot{\rho}+(\rho+\bar{p})\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\dot{\Lambda}=0 \qquad \dots (9)$$

We define average scale factor R for Bianchi I universe as

$$R^3=ABC \qquad \dots (10)$$

In analogy with FRW universe, we define generalized Hubble parameter H and deceleration parameter q as

$$H=\frac{\dot{R}}{R}=\frac{1}{3}(H_1+H_2+H_3) \qquad \dots (11)$$

$$q=-\frac{\ddot{R}}{RH^2} \qquad \dots (12)$$

where $H_1=\dot{A}/A, H_2=\dot{B}/B, H_3=\dot{C}/C$ are directional Hubble factors along x, y and z directions respectively.

We introduce volume expansion θ and shear scalar σ for the Bianchi I metric as

$$\theta=v^i_{;i} \qquad \dots (13)$$

$$\sigma^2=\frac{1}{2}\sigma_{ij}\,\sigma^{ij} \qquad \dots (14)$$

For the metric (1), we have

$$\theta=3\dot{R}/R \qquad \dots (15)$$

$$\sigma=k/R^3 \qquad \dots (16)$$

where $3k^2=k_1^2+k_2^2+k_1k_2$ and k_1,k_2 are integration constant.

Equations (6) – (9) can be expressed in terms of H, σ and q as

$$p-\zeta\theta-\Lambda=(2q-1)H^2-\sigma^2 \qquad \dots (17)$$

$$\rho+\Lambda=3H^2-\sigma^2 \qquad \dots (18)$$

Solution and Discussion :

The equations (5) – (8) are four equations involving seven unknown parameter A, B, C Λ, ρ, p and ξ. We require three more conditions to close the system.
We assume functional relation on Hubble parameter

$$H = aR^{-m},\ a > 0,\ m \geq 0 \quad \ldots (19)$$

From equation (11) and (19), we et

$$R = (maT)^{1/-m} \quad \ldots (20)$$

where $T = t + t_1$ (t_1 is constant of integration).
From equation (12) and (20), we obtain

$$q = m - 1 \quad \ldots (21)$$

we now consider

$$\Lambda = 3\beta H^2 \quad \ldots (22)$$

where β is constant.
and bulk viscosity is taken as

$$\xi = \xi_0 \rho \quad \ldots (23)$$

ξ_0 being constant.

Spatial volume V, Expansion scalar θ, Matter density ρ, cosmological parameter Λ, Isotropic pressure p, Bulk viscosity ξ shear σ for the models are given by

$$V = R^3 = (m\,a\,T)^{3/m}$$

$$\theta = \frac{3}{mT}$$

$$\rho = \frac{3(1-\beta)}{m^2T^2} - \frac{k^2}{(maT)^{6/m}}$$

$$\Lambda = \frac{3\beta}{m^2T^2}$$

$$p = \frac{(2m+3\beta-3)}{m^2T^2} - \frac{k^2}{(maT)^{6/m}} + \frac{3\xi_0}{mT}\left[\frac{3(1-\beta)}{m^2T^2}\frac{k^2}{(maT)^{6/T}}\right]$$

$$\xi = \xi_0\left[\frac{3(1-\beta)}{m^2T^2} - \frac{k^2}{(maT)^{6/m}}\right]$$

$$\theta = \frac{k}{(maT)^{3/m}}$$

We notice that the spatial volume increases with time T and it become infinite for large value of T. As T = 0, the spatial volume is zero and other parameter θ, ρ, Λ, p, σ, ξ all are infinite but zero for large values of T. Thus, the model has a big-bang singularity at the T = 0.
For the model

$$\frac{\sigma}{\theta} = \frac{k}{3(maT)^{\frac{3}{m}-2}}$$

we observe that $\frac{\sigma}{\theta} \to 0$ as $T \to \infty$.Therefore model approach isotropy at late times.

II. Conclusion :

In this paper, we have studied Bianchi type-I universe with bulk viscosity and functional relation on Hubble parameter H in the context of general relativity. Since $q = m - 1 > 0$, the model represents a decelerating universe throughout the evaluation. If $m = 1$, we get $H = 1 / T$ and $q = 0$. Therefore, galaxies move with constant speed. The cosmological term is infinite initially and approaches to zero at recent time. These are supported by recent results from the observations of the type-I a supernova explosion (SNIa).

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