International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 10 Issue 7 ǁ July 2022 ǁ PP. 158-162

Applications of Alenezi Transform for handling Exponential growth and Decay Problems

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Abstract
In this paper we are going to use Alenezi transform for the exact solution of first order ordinary differential equations of exponential growth and decay problems. To prove the efficiency, accuracy and capability of the Alenezi transform, we solve problems on growth and decay

Date of Submission: 25-06-2022 Date of acceptance: 06-07-2022

I. Introduction:

Many quantities in the universe grow or decay at a rate proportional to their size. For example a colony of bacteria may double every hour. If the size of the colony after t hours is given by $y(t)$, then we can express this information in the mathematical language in the form of a first order differential equation,

$$\frac{dy}{dt} = 2y$$

The quantity y that grows or decays at a rate proportional to its size is governed by a first order differential equation,

$$\frac{dy}{dt} = ky$$

If $k < 0$ then the above equation is called the law of natural decay and if If $k > 0$ then the above equation is called the law of natural growth. This equation is solved by separation of variable method. Integral transforms plays an important role in solving differential equations.

Recently, S.R Kushare and D. P. Patil [1] introduce Kushare transform in September 2021.In October 2021, S.S.Khakale and D. P. Patil [2] introduce Soham transform. As researchers are going to introduce new integral transforms at the same time many researchers are interested to apply these transforms to various types of problems. In January 2022,R. S. Sanap and D. P. Patil [3] used Kushare transform to solve the problems based on Newton's law of cooling. In April 2022 D. P. Patil etc. [4] use Kushare transform to solve the problems on growth and decay. In October 2021 D. P. Patil [5] used Sawi transform in Bessel function. D. P. Patil [6] used Sawi transform of error function for evaluating improper integral further, Lpalce and Shenu transforms are used in chemical science by D. P. Patil [7]. Dr. Patil [8] solved the wave equation by Sawi transform and its convolution theorem. Further Patil [9] also used Mahgoub transform for solving parabolic boundary value problems.

Dr. Dinkar Patil [10] obtains solution of the wave equation by using double Laplace and double Sumudu transform. Dualities between double integral transforms are derived by D. P. Patil [11]. Laplace, Elzaki, and Mahgoub transforms are used for solving system of first order and first degree differential equations by Kushare and Patil [12].Boundary value problems of the system of ordinary differentiable equations are by using Aboodh and Mahgoub transform by D. P. Patil [13]. D. P. Patil [14] study Laplace, Sumudu, Elzaki and Mahgoub transforms comparatively and apply them in Boundary value problems. Parabolic Boundary value problems are also solved by Dinkar Patil [15]. For that he used double Mahgoub transform.

Soham transform is used to obtain the solution of system of differential equations by D. P. Patil et al [16]. D. P. Patil et al also used Soham transform for solving Volterra integral equations of first kind [17]. D. P. Patil et al [20] used Anuj transform to solve Volterra integral equations of first kind. Soham transform is used to solve same equations by D. P. Patil et al [21] . Rathi sisters and D. P. Patil used Soham transform for system of differential equations [22]. Recently Zankar, Kandekar and D. P. Patil used general integral transform of error function for evaluating improper integrals[23]. Recently, Dinkar Patil, Prerana Thakare and Prajakta Patil [24] used double general integral transform for obtaining the solution of parabolic boundary value problems. D. P. Patil et al [25] used emad-Sara transform to obtain the solution of telegraph equation. . Shirsath, Gangurde and Patil [26] Applied Soham transform for solving the problems based on Newton's law of cooling. D. P. Patil et al [27] used the HY integral transform for handling growth and Decay problems. Komal Patil, Snehal Patil and

Dinkar Patil [28] solved Newton's law of cooling by using " Emad- Falih Transform". Bachhav, Patil et al [29] used HY transform in Newton's law of cooling. Double Kushare transform is introduced by D. P. Patil et al [30]. Pardeshi, Shaikh, Deshmukh, and Patil [31] used Emad-Sara transform in handling population growth and decay problems

II. Preliminaries

In this we state some preliminary concepts which are required for solving the problems on growth and decay.

2.1. Alenezi Transform [18]

Let h(t) becomes an integrable function realized for t≥0, m(s) and n(s)≠0 are favourable real function, we explain Alenezi integral transform J(s) of h(t) by the formula ,

J(s)= m(s)$\int_0^\infty h(t)e^{-tn(s)}$ dt

2.2. Alenezi integral transform of some functions:[18]

Function	Alenezi integral Transform
1	$\frac{m(s)}{n(s)}$
T	$\frac{m(s)}{n(s)^2}$
t^α	$\frac{\Gamma(\alpha+1)m(s)}{n(s)^{\alpha+1}}$
Cost	$\frac{n(s)m(s)}{n(s)^2+1}$
Sint	$\frac{m(s)}{1+n(s)^2}$
Sin(at)	$\frac{am(s)}{n(s)^2+a^2}$
e^t	$\frac{m(s)}{n(s)-1}$
e^{at}	$\frac{m(s)}{n(s)-a}$
h'(t)	n(s)J(s)-m(s)h(0)

Theorem: Iet h(t) is differentiable and m(s) and n(s) are positive real functions , then

J(h'(t),s)= n(s)J(s)-m(s)h(0)

Convolution Theorem: [18] let $h_1\,(t)$and h_2(t) have new integral transform F(s). Then the new integral transform of the convolution of F(s). Then the new integral transform of the convolution of h_1 *and* h_2 is

$h_1 * h_2 = \int_0^\infty h_1\ (t) * h_2(\text{t-z})\,\text{dz} \;= \frac{1}{m(s)} f_1(s) * f_2(s)$

III. Alenezi Integral Transform for Solving Growth and Decay Problems:

In this section we apply Alenezi integral transform for exponential growth and decay problems

A. Alenezi integral transform For handling growth problems:

For handling population growth problem the population growth problem can be express mathematically, In the term of first order ordinary linear differential equation,

$\frac{dh}{dt} = \text{k h(t)}$ (1)

Where k is positive real constant

Applying Alenezi integral transform equation (1) we get,

J($\frac{dh}{dt}$) = kJ(h(t))

Applying the property Alenezi transform of derivative, we get,

n(s)J{h(t);s} – m(s)h(0) =kJ{h(t);s}

n(s)J{h(t) –kJ{h(t)} = m(s)h(0)

[n(s) – k] J{h(t)} = m(s)h(0)

J{h(t)} = $\frac{m(s)h(0)}{n(s)-k}$

Applying the Alenezi inverse transform, we get,

h(t) = $J^{-1}(\frac{m(s)h(0)}{n(s)-k})$
h(t) = h(0) $J^{-1}\{\frac{m(s)}{n(s)-k}\}$
h(t) = h(0)e^{kt}
here, h(0) denote the population at time t=0, where h(t) is the population at a time t= t.

Alenezi integral transform For handling decay problems:
Decay problem can be express mathematically in the first order ordinary linear differential equation:
$\frac{dh}{dt}$ = -kh(t)
Now applying the Alenezi transform,
J{$\frac{dh}{dt}$} = k J{h(t)}
Applying Alenezi transform for derivative,
N(s)J{h(t)} – m(s)h(0) = -kJ{h(t)}
n(s) J{h(t)} + kJ{h(t)} = m(s)h(0)
(n(s)+k)J{h(t)} = m(s)J(0)
J{h(t)} = $\frac{m(s)h(0)}{n(s)+k}$
Applying Alenezi inverse transform,
h(t) = $J^{-1}(\frac{m(s)h(0)}{n(s)+k})$
=h(0)$J^{-1}\{\frac{m(s)}{n(s)+k}\}$
h(t) = h(0)e^{-kt} which is the required amount of substance time t.

IV. Applications

In this section we solve some problems on growth and decay by using Alenezi transform.
(1) The population of city growth at a rate proportional to the number of people presently living in city. If after 4 years the population has tripled and after 5 years the population is 50000, estimate the number of people initially living in the city.
Solution: The problem can be written mathematically,
$J\{\frac{dh}{dt}\}$ = kJ{h(t)}
Applying the Alenezi transform for derivative,
n(s)J{h(t)} – m(s)h(0) = KJ{h(t)}
n(s)J{h(t)}-kJ{h(t)} = m(s)h(0)
J{h(t)} = $\frac{m(s)h(0)}{n(s)-k}$
Applying the Alenezi inverse transform,
h(t) = $J^{-1}\{\frac{m(s)h(0)}{n(s)-k}\}$
h(t) = h(0)$J^{-1}\{\frac{m(s)}{n(s)-k}\}$
h(t) = h(0)e^{kt} ....(2)
at, t = 4 ,h = 3 h(0), so using (2) we get,
3 h(0) = h(0)e^{4k}
3 = e^{4k}
K = $\frac{\ln(3)}{4}$ =$\frac{1.0986}{4}$ ≈ 0.2746
Approximately, K = 0.275
Now at t = 5, h = 50000 then above equation becomes
h(t) = h(0) $e^{5*0.275}$
50000 = h(0) *3.955
h(0) ≈ $\frac{50000}{3.955}$
h(0) ≈ 12642
h(0) = 12642 which is the required number of people living in city initially.
(2) The radioactive substance is known to decay at rate proportional to the amount of present. if initially there is 100 miligram of radioactive substance present and after 6 hours it is observed that 30 miligram substance is lost. Find half life of radioactive substance.
Solution: The problem mathematically written in the format
$\frac{dh}{dt}$ = -kh(t)

Applying the Alenezi transformation,

$J\{\frac{dh}{dt}\} = J\{-kh(t)\}$

Applying the Alenezi transform for derivative

$n(s)J\{h(t)\} - m(s)h(0) = J\{-kh(t)\}$

$n(s)J\{h(t)\} + kJ\{h(t)\} = m(s)h(0)$

$[n(s) + k]J\{h(t)\} = m(s)h(0)$

$J\{h(t)\} = \frac{m(s)h(0)}{n(s)+k}$

Applying Alenezi inverse transformation

$h(t) = J^{-1}\{\frac{m(s)h(0)}{n(s)+k}\}$

$h(t) = h(0)J^{-1}\{\frac{m(s)}{n(s)+k}\}$

$h(t) = h(0)e^{-kt}$ ...(3)

$t = 0$, $h(t) = 100$

$h(t) = 100e^{-kt}$ ...(4)

at $t=6$,$h(t) = 100-30 = 70$

From equation (4) we get, $70 = 100e^{-6k}$

$e^{-6k} = \frac{70}{100} = 0.70$

$k = \frac{-1}{6}\log_e 0.70$

$k = 0.059$ ...(5)

when $h = \frac{h(0)}{2} = \frac{100}{2} = 50$, we have,

$50 = 100e^{-0.059t}$

$t = -\frac{1}{0.059}\log_e 0.50 = 11.75$ hours

The required half time for radioactive substance is 11.75 hours.

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