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On Homogeneous Quadratic Diophantine Equation with Five Unknowns

$$x^2 + y^2 + z^2 + w^2 = 15t^2$$

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Abstract:
The homogeneous quadratic diophantine equation with five unknowns given by $x^2 + y^2 + z^2 + w^2 = 15t^2$ is analyzed for determining its non-zero distinct integer solutions through employing linear transformations.
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I. Introduction:

The theory of diophantine equations offers a rich variety of fascinating problems. In particular, homogeneous or non-homogeneous quadratic diophantine equations with two or more variables have been an interest to mathematicians since antiquity [1-4]. In this context, one may refer [5-12] for different choices of quadratic diophantine equations with four unknowns. In [13-15], the quadratic diophantine equations with five unknowns are analysed for obtaining their non-zero distinct integer solutions.

This motivated me for finding integer solutions to other choices of quadratic equations with five unknowns. This paper deals with the problem of determining non-zero distinct integer solutions to the quadratic Diophantine equation with five unknowns given by $x^2 + y^2 + z^2 + w^2 = 15t^2$.

Method of analysis:

The second degree diophantine equation with five unknowns to be solved is

$$x^2 + y^2 + z^2 + w^2 = 15t^2 \tag{1}$$

The process of obtaining different sets of non-zero distinct integer solutions to (1) is illustrated below:

Illustration1:

The substitution of the linear transformations

$$x = 2z,\ y = 3(X + 3T),\ t = X + 2T,\ w = X + 3T \tag{2}$$

in (1) leads to the equation

$$X^2 = 6T^2 + z^2 \tag{3}$$

which is satisfied by

$$X = 6r^2 + s^2,\ T = 2rs,$$

and

$$z = 6r^2 - s^2 \tag{4}$$

In view of (2), one has

$$x = 2(6r^2 - s^2),\ y = 3(6r^2 + s^2 + 6rs)$$
$$t = 6r^2 + s^2 + 4rs,\ w = 6r^2 + s^2 + 6rs \tag{5}$$

Thus,(4) and (5) represent the integer solutions to (1).

Illustration 2:

Write (3) as

$$z^2 + 6T^2 = X^2 = X^2 * 1 \tag{6}$$

Assume

$$X = a^2 + 6b^2 \tag{7}$$

The integer 1 on the R.H.S. of (6) may be taken as

$$1 = \frac{(1 + i2\sqrt{6})(1 - i2\sqrt{6})}{25} \tag{8}$$

Substituting (7) & (8) in (6) and employing the method of factorization,consider

$$z + i\sqrt{6}T = \frac{(1 + i2\sqrt{6})(a + i\sqrt{6}b)^2}{5} \tag{9}$$

Equating the real and imaginary parts in (9) ,we have

$$\left.\begin{aligned} z &= \frac{(a^2 - 6b^2 - 24ab)}{5}, \\ T &= \frac{(2a^2 - 12b^2 + 2ab)}{5} \end{aligned}\right\} \tag{10}$$

As our interest is on finding integer solutions,replacing a by 5A and b by 5B in (7) and (10),we get

$$X = 25(A^2 + 6B^2),\ T = 5(2A^2 - 12B^2 + 2AB),$$

$$z = 5(A^2 - 6B^2 - 24AB) \tag{11}$$

In view of (2),one has

$$\left.\begin{aligned} x &= 10(A^2 - 6B^2 - 24AB),\ y = 3(55A^2 - 30B^2 + 30AB) \\ t &= (45A^2 + 30B^2 + 20AB),\ w = (55A^2 - 30B^2 + 30AB) \end{aligned}\right\} \tag{12}$$

Thus,(11) and (12) represent the integer solutions to (1).

Note 1:

The integer 1 on the R.H.S. of (8) is also represented by

$$1 = \frac{(6r^2 - s^2 + i2\sqrt{6}rs)(6r^2 - s^2 - i2\sqrt{6}rs)}{(6r^2 + s^2)^2}$$

Repeating the above process ,an another set of integer solutions to (1) is obtained.

Illustration 3:

Write (3) as

$$X^2 - 6T^2 = z^2 = z^2 * 1 \tag{13}$$

Assume

$$z = a^2 - 6b^2 \tag{14}$$

The integer 1 on the R.H.S. of (13) may be taken as

$$1 = (5 + 2\sqrt{6})(5 - 2\sqrt{6}) \tag{15}$$

Substituting (14) and (15) in (13) and employing the method of factorization, consider

$$X + \sqrt{6}T = (5 + 2\sqrt{6})(a + \sqrt{6}b)^2 \tag{16}$$

Equating the rational and irrational parts in (16) ,we have

$$\left.\begin{aligned} X &= 5(a^2 + 6b^2) + 24ab, \\ T &= 2(a^2 + 6b^2) + 10ab \end{aligned}\right\} \tag{17}$$

In view of (2),one has

$$\left.\begin{aligned} x &= 2(a^2 - 6b^2),\ y = 3(11a^2 + 66b^2 + 54ab) \\ t &= 9a^2 + 54b^2 + 44ab,\ w = (11a^2 + 66b^2 + 54ab) \end{aligned}\right\} \quad (18)$$

Thus,(14) and (18) represent the integer solutions to (1).
Note 2:

The integer 1 on the R.H.S. of (15) is also represented by

$$1 = \frac{(6r^2 + s^2 + 2\sqrt{6}rs)(6r^2 + s^2 - 2\sqrt{6}rs)}{(6r^2 - s^2)^2}$$

Repeating the above process ,an another set of integer solutions to (1) is obtained.

II. Conclusion

In this paper, an attempt has been made to obtain some non-zero distinct integer solutions to the quadratic diophantine equation with five unknowns given by $x^2 + y^2 + z^2 + w^2 = 15t^2$
The readers of this paper may search for finding other sets of integer solutions to the considered quadratic diophantine equation with five unknowns as well as integer solutions to other choices of quadratic equations with five or more unknowns.

References:

[1]. L.E. Dickson, History of theory of Numbers, Vol.II, Chelsea Publishing co., New York 1971.
[2]. L.J. Mordell, Diophantine Equations, Academic Press, New York, 1969.
[3]. Andre Weil, Number Theory: An approach through History, From Hammurapi to Legendre, Birkhauser, Boston, 1984.
[4]. Bibhotibhusan Batta and Avadhesh Narayanan Singh, History of Hindu Mathematics, Asia Publishing House, Bombay, 1938.
[5]. M.A. Gopalan and B. Sivakami, Integral solutions of quadratic with four unknowns $(x+y)(z+w) = xy + 4zw$, Global Journal of Pure and Applied Mathematics, 8(5), 573-578, 2012.
[6]. M.A. Gopalan and G. Srividhya, On the diophantine equation $X^2 + Y^2 = U^2 + V^2$, Impact J. Sci., 6(1), 111-116, 2012.
[7]. M.A. Gopalan, V. Sangeetha and Manju Somanath, Integral point on the quadratic equation with four unknowns $2(x^2 + y^2) + 3xy + x - y + 1 = z^2 + 7w^2$, Diophantus J. Math., 2(1), 47-54, 2013.
[8]. M.A. Gopalan, S. Vidhyalakshmi and K. Lakshmi, On the non homogeneous quadratic equation $x^2 + y^2 + z^2 = t^2 + 1$, American Journal of Mathematical Sciences and Applications, 1(1), 77-85, January-June 2013.
[9]. M.A. Gopalan, S. Vidhyalakshmi and K. Lakshmi, On the non homogeneous quadratic equation $x^2 + y^2 + z^2 = t^2 - 1$, International Journal of Applied Mathematical Sciences, 6(1), 1-6, 2013.
[10]. A. Vijayasankar, M.A. Gopalan, V. Krithika, Observations on $x^2 + y^2 + z^2 = w^2$, IJRTER, 3(5), 378-381, May 2017.
[11]. S. Vidhyalakshmi, M.A. Gopalan, S. Aarthy Thangam, Real and Gaussian integer solutions to $x^2 + y^2 = 2(z^2 - w^2)$, GJESR, 5(9), 46-53, September 2018.
[12]. Shreemathi Adiga,On bi-quadratic equation with four unknowns $7xy + 3z^2 = 3w^4$,AIP conference Proceedings 2261,030087 (2020), https://aip.scitation.org/doi/abs/10.1063/5.0016866
[13]. R. Anbuselvi, S. Jamuna Rani, Integral solutions of quadratic diophantine equation with five unknowns $10w^2 - x^2 - y^2 + z^2 = t^2$, IJERD , 13(9), 51-56, September 2017.
[14]. R. Anbuselvi, S. Jamuna Rani, Integral solutions of quadratic diophantine equation with five unknowns $4w^2 - x^2 - y^2 + z^2 = 16t^2$, IJRAT , 6(11), 3327-3329, November 2018.
[15]. M.A. Gopalan, S. Vidhyalakshmi and EPremalatha, On Equal Sums Of Like Powers $x^2 + y^2 + z^2 = w^2 + U^2$, International Journal of Engineering Research-OnlineA, 6(1), vol 1,issue 3,401-406,2013, http://ijoer.in/Vol%201.3.2013/401-406.pdf.