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Thermal Convection of Maxwell Ferromagnetic Fluid Through Porous Medium

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Abstract
This paper deals with the theoretical investigation of the thermal convection in a Maxwell ferromagnetic fluid layer saturating in a porous medium using linear stability theory. The linear stability analysis is based on the classical normal mode technique. For a flat fluid layer contained between two free boundaries, an exact solution is obtained using Galerkin method. For the stationary convection, the thermal Rayleigh number and the magnetic thermal Rayleigh number has been obtained analytically. It is also observed that the stress relaxation parameter of Maxwell ferromagnetic fluid does not play any role in stationary convection. It is found that all the three parameters buoyancy magnetization parameter, non-buoyancy magnetization parameter, and medium permeability parameter always have destabilizing effect on the system. The principle of exchange of stabilities is valid under certain conditions. The results are depicted both analytically and graphically.
Keywords: *Thermal convection; Maxwell ferromagnetic fluid; Porous medium.*

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I. Introduction

Magnetic fluids or ferrofluids are colloidal suspensions of fine ferromagnetic mono domain nanoparticles in non-conducting fluids. The method for making ferrofluids was developed in the 1960s. Several researchers from time to time have contributed to the development of ferrofluids. Finlayson [1] studied the convective instability of the ferrofluid layer heated from below and obtained a precise solution for the case of free boundaries and approximate solutions (for stationary convection) for rigid boundaries. In the recent past, studies on ferrofluids have attracted the attention of many researchers because of their wide applications in several areas. A detailed account of thermal convection problems in Newtonian fluids was given by Chandrasekhar [2]. Further, a detailed introduction to this subject was provided by Rosensweig [3, 4] in his monograph and review article.

Several convection problems in ferrofluids were investigated by Lalas and Carmi [5], Shliomis [6], Stiles and Kagan [7], Venkatasubramanian and Kaloni [8], Abraham [9], Sunil and their co-workers [10-12]. Jasmine [13] studied thermoconvective stability of a ferrofluids in presence of magnetic field. . Prakash [14] examined the effect of magnetic field dependent viscosity on the thermal convection in a ferromagnetic fluid layer. Siddheshwar *et al.* [15] discussed the finite-amplitude ferro-convection and electro-convection in a rotatory fluid. Nadian *et al.* [16, 17] presented the study on couple stress ferromagnetic fluids under varying gravity field. Meghana and Pranesh [18] investigated four types of effects of rotation modulation on Rayleigh–Bénard convection in a ferromagnetic fluid with couple stress.

Thermal convection in a non-Newtonian fluid layer in porous media has attracted considerable attention of the researchers, due to its wide applications in different fields as bio-rheology, geophysics, and petroleum industries. The first visco-elastic rate type model, which is still used widely, is due to Maxwell. Several studies have been carried on thermal convection in Maxwell fluid. Narayana *et al.* [19] carried out a linear and nonlinear stability analysis of binary Maxwell fluid convection in a porous medium. Chand and Kumar [20] examined the effects of rotation on thermal instability of Maxwell visco-elastic fluid with variable gravity in porous medium. Gaikwad and Kamble [21] investigated the effects of cross diffusion on convective instability of Maxwell fluid in porous medium. Mahajan *et al.* [22] performed a linear stability analysis of penetrative convection via internal heating in a ferrofluid saturated porous layer. Prakash *et al.* [23] studied the ferromagnetic convection in a densely packed porous medium with magnetic-field-dependent viscosity. Awasthi *et al.* [24] considered a problem of triply diffusive convection in a Maxwell fluid saturated porous layer with internal heat source. Singh *et al.* [25] considered a problem on onset of soret driven instability in a Darcy–Maxwell nanofuid.

To the best of our knowledge, the thermal convection in Maxwell ferromagnetic fluid layer through porous medium has not been investigated yet. Therefore, an attempt has been made to study the thermal convection of the ferromagnetic fluid saturated porous layer using Darcy-Maxwell model. In this paper, we consider an infinite, incompressible Maxwell ferromagnetic fluid saturated porous layer contained between two free boundaries. Our aim is to study the effect of stress relaxation parameter as well as other magnetization parameters for stationary convection and to check the validity of principle of exchange of stabilities. Following the linear stability analysis, the analytical thermal Rayleigh number and magnetic thermal Rayleigh number has been obtained using normal mode technique.

II. Mathematical Formulation

Consider an infinite, horizontal layer of thickness d of an electrically non-conducting incompressible Maxwell ferromagnetic fluid layer in a Darcy porous medium. The Maxwell ferromagnetic fluid layer is heated from below and the temperature at the bottom and the top surface is T_0 and T_1 respectively and a uniform temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ is maintained. The gravity field $\vec{g} = (0, 0, -g)$ pervades the system.

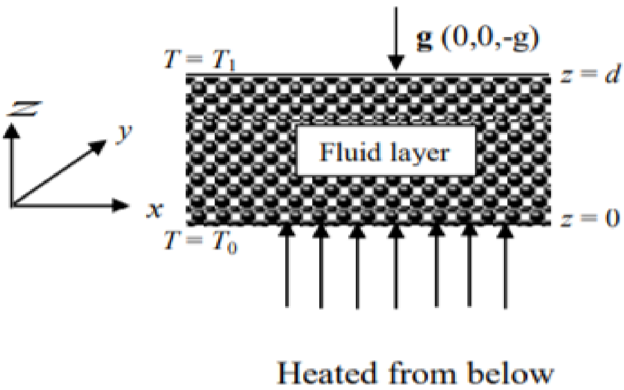

Figure1: Geometrical Configuration

The equations of continuity, motion and heat for a Maxwell ferromagnetic fluid layer through a Darcy porous medium for the above model are as follows:

$$\nabla \cdot \vec{q} = 0 \qquad (1)$$

$$\frac{\rho_0}{\varepsilon}\left(1 + \lambda \frac{\partial}{\partial t}\right)\frac{\partial \vec{q}}{\partial t} = \left(1 + \lambda \frac{\partial}{\partial t}\right)\left(-\nabla p + \rho_0\left(1 - \alpha\left(T_1 - T_0\right)\right)\vec{g} + \nabla \cdot \left(\vec{H} \cdot \vec{B}\right)\right) - \frac{\mu}{k_1}\vec{q} \qquad (2)$$

$$\varepsilon\left[\rho_0 C_{V,H} - \mu_0 \vec{H} \cdot \left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}\right]\frac{dT}{dt} + (1-\varepsilon)\rho_s c_s \frac{\partial T}{\partial t} + \mu_0 T\left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H} \cdot \frac{d\vec{H}}{dT} = K_1 \nabla^2 T \qquad (3)$$

where $\vec{q}$ is the Darcian (filter) velocity, ε denotes medium porosity, k_1 represents the medium permeability, p denotes the pressure, λ denotes stress relaxation time , μ is viscosity, $\vec{H}$ is magnetic field, $\vec{g} = (0, 0, -g)$ is acceleration due to gravity, $C_{V,H}$ is heat capacity at constant volume and magnetic field, μ_0 is magnetic permeability, T is temperature, $\vec{M}$ is magnetization, K_1 is thermal conductivity, α is the coefficient of volume expansion, ρ is the density of the fluid and ρ_0 is the density of the fluid at some reference temperature T_0 . .

For a non conducting fluid with no displacement current, Maxwell's equations are given by

$$\nabla \cdot \vec{B} = 0, \qquad (4a)$$

$$\nabla \times \vec{H} = 0, \qquad (4b)$$

where the magnetic induction is given by

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right), \tag{5}$$

We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field as well as the temperature as:

$$\vec{M} = \left(\frac{\vec{H}}{H} \right) M \left(H, T \right), \tag{6}$$

The linearized magnetic equation of state is given by

$$M = M_0 + \chi \left(H - H_0 \right) - K_2 \left(T - T_0 \right), \tag{7}$$

where M_0 is the magnetization when magnetic field is H_0 and temperature T_0, $\chi = \left(\frac{\partial \vec{M}}{\partial H} \right)_{H_0, T_0}$ is the magnetic susceptibility and $K_2 = -\left(\frac{\partial \vec{M}}{\partial T} \right)_{H_0, T_0}$ is the pyromagnetic coefficient.

3. Basic State

The basic state is assumed to be stationary. Thus the solution of equations (1) to (7) in the basic state is given by

$$\vec{q} = \vec{q_b} = 0, \; \rho = \rho_b \left(z \right), \; \mathrm{p} = \mathrm{p}_b \left(z \right), \; T = T_b \left(z \right) = T_0 - \beta z, \; \beta = \frac{T_0 - T_1}{d},$$

$$\vec{H_b} = \left(H_0 - \frac{K_2 \beta z}{1 + \chi} \right) \vec{k}, \; \vec{M_b} = \left(M_0 + \frac{K_2 \beta z}{1 + \chi} \right) \vec{k}, \; H_0 + M_0 = H_0^{ext} \tag{8}$$

4. Perturbed State

Following Finlayson [1], the perturbations in the basic state are given by;

$$\vec{q} = \vec{q_b} + \vec{q'}, \; \rho = \rho_b \left(z \right) + \rho', \; p = p_b \left(z \right) + p', \; T = T_b \left(z \right) + \theta',$$

$$\vec{H} = \vec{H_b} \left(z \right) + \vec{H'}, \; \vec{M} = \vec{M_b} \left(z \right) + \vec{M'} \tag{9}$$

where $\vec{q'} = \left(u', v', w' \right)$, ρ', p', θ', $\vec{H'}$, $\vec{M'}$ are infinitesimal perturbations in velocity, density, pressure, temperature, magnetic field intensity, and magnetization. Substituting equation (9) into the equations (1) – (7) and using the basic state solutions (8), the following linearized perturbation equations are obtained.

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0, \tag{10}$$

$$\frac{\rho_0}{\varepsilon} \left(1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial u'}{\partial t} = \left(1 + \lambda \frac{\partial}{\partial t} \right) \left(-\frac{\partial p'}{\partial x} + \mu_0 \left(H_0 + M_0 \right) \frac{\partial H_1{}'}{\partial z} \right) - \frac{\mu}{k_1} u' \tag{11}$$

$$\frac{\rho_0}{\varepsilon} \left(1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial v'}{\partial t} = \left(1 + \lambda \frac{\partial}{\partial t} \right) \left(-\frac{\partial p'}{\partial y} + \mu_0 \left(H_0 + M_0 \right) \frac{\partial H_2{}'}{\partial z} \right) - \frac{\mu}{k_1} v' \tag{12}$$

$$\frac{\rho_0}{\varepsilon} \left(1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial w'}{\partial t} = \left(1 + \lambda \frac{\partial}{\partial t} \right) \left(\begin{array}{c} -\frac{\partial p'}{\partial z} + \rho_0 \alpha \theta' g + \mu_0 \left(H_0 + M_0 \right) \frac{\partial H_3{}'}{\partial z} \\ -\mu_0 K_2 \beta H_3{}' + \frac{\mu_0 K_2^2 \beta \theta'}{1 + \chi} \end{array} \right) - \frac{\mu}{k_1} w' \tag{13}$$

$$\rho C_1 \frac{\partial \theta'}{\partial t} - \mu_0 T_0 K_2 \frac{\partial}{\partial t} \left(\frac{\partial \phi'}{\partial z} \right) = K_1 \nabla^2 \theta' + \left(\rho C_2 \beta - \frac{\mu_0 T_0 K_2^2 \beta}{1 + \chi} \right) w' \tag{14}$$

where

$$\rho C_1 = \varepsilon \rho_0 C_{V,H} + \varepsilon \mu_0 K_2 H_0 + \left(1 - \varepsilon \right) \rho_s C_s, \tag{15}$$

and

$$\rho C_2 = \varepsilon \rho_0 C_{V,H} + \varepsilon \mu_0 K_2 H_0, \tag{16}$$

$$\frac{\partial}{\partial x}\left(H_1' + M_1'\right) + \frac{\partial}{\partial y}\left(H_2' + M_2'\right) + \frac{\partial}{\partial z}\left(H_3' + M_3'\right) = 0,\ \overrightarrow{H'} = \nabla \phi', \tag{17}$$

where ϕ' is the perturbed magnetic potential, and

$$H_3' + M_3' = (1+\chi)H_3' - K_2\theta',\ H_1' + M_1' = \left(1 + \frac{M_0}{H_0}\right)H_i',\ (i = 1, 2). \tag{18}$$

where we have assumed $K_2 \beta d << (1+\chi) H_{0.}$

Eliminating $u',\ v',\ p'$ between (11), (12) and (13) and using (10), we obtain

$$\frac{\rho_0}{\varepsilon}\left(1 + \lambda\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\nabla^2 w' = \left(1 + \lambda\frac{\partial}{\partial t}\right)\left(\rho_0 \alpha g \nabla_1^2\theta' - \mu_0 K_2 \beta \frac{\partial}{\partial z}\nabla_1^2\phi' + \frac{\mu_0 K_2^2 \beta \nabla_1^2 \theta'}{1+\chi}\right) - \frac{\mu}{k_1}\nabla^2 w' \tag{19}$$

Now, combining (17) and (18) we obtain

$$(1+\chi)\frac{\partial^2\phi'}{\partial z^2} + \left(1 + \frac{M_0}{H_0}\right)\nabla_1^2\phi' - K_2\frac{\partial\theta'}{\partial z} = 0 \tag{20}$$

5. Normal Mode Technique

Now analyze the disturbances into the normal mode and let the perturbed quantities are of the form

$$(w',\ \theta',\ \varphi')(x, y, z, t) = \left[w''(z),\ \theta''(z),\ \phi''(z)\right]\exp\left[i\left(k_x x + k_y y\right) + nt\right] \tag{21}$$

Using (21), in (19), (14) and (20) and non-dimensionalizing the variables by setting

$$z = \frac{z}{d},\ w' = \frac{d}{\nu}w'',\ a = kd,\ \mathrm{t} = \frac{\nu t}{d^2},\ D' = Dd,\ \theta = \frac{K_1 a R^{1/2}}{(\rho C_2)\beta \nu d}\theta'',$$

$$\phi = \frac{(1+\chi)K_1 a R^{1/2}}{K_2(\rho C_2)\beta \nu d^2}\phi'',\ k_1 = \frac{k_1}{d^2},\ \nu = \frac{\mu}{\rho_0},\ \mathrm{P}_r' = \frac{\nu(\rho C_2)}{K_1},\ \mathrm{P}_r = \frac{\nu(\rho C_1)}{K_1},$$

$$R = \frac{g\alpha\beta d^4(\rho C_2)}{\nu K_1},\ M_1 = \frac{\mu_0 K_2^2 \beta}{(1+\chi)\alpha\rho_0 g},\ \mathrm{M}_2 = \frac{\mu_0 T_0 K_2^2}{(1+\chi)\rho C_2},$$

$$M_3 = \frac{1 + \frac{M_0}{H_0}}{(1+\chi)},\ \omega = \frac{nd^2}{\nu},\ G = \frac{\lambda\nu}{d^2} \tag{22}$$

where $M_1,\ M_2,\ M_3$ are magnetization parameters and G is the non-dimensional stress relaxation parameter. On dropping the asterisks for simplicity we obtain

$$\left[\frac{\omega}{\varepsilon} + \frac{1}{k_1}(1 + G\omega)^{-1}\right]\left(D^2 - a^2\right)w = aR^{1/2}\left[M_1 D\phi - (1 + M_1)\theta\right] \tag{23}$$

$$\left(D^2 - a^2 - P_r\omega\right)\theta + P_r' M_2 \omega D\phi = -(1 - M_2)aR^{1/2}w \tag{24}$$

$$\left(D^2 - a^2 M_3\right)\phi = D\theta \tag{25}$$

6. Exact solution for free boundaries

Using Galerkin method, the exact solution of the system of equations (23) – (25) subject to the boundary conditions

$$w = 0 = \theta = D^2 w = D\phi \quad at\ z = 0\ and\ z = 1 \tag{26}$$

is written in the form

$$w = w_0 \sin\pi z,\ \theta = \theta_0 \sin\pi z \tag{27}$$

where $w_0,\ \theta_0$ are constants. Substituting (27) into the equations (23) – (25), we obtain

$$\left[\frac{\omega}{\varepsilon}+\frac{1}{k_1}(1+G\omega)^{-1}\right]\left(\pi^2+a^2\right)w_0+aR^{1/2}\left[\frac{M_1\pi^2}{\pi^2+a^2M_3}-\left(1+M_1\right)\right]\theta_0=0 \tag{28}$$

$$\left(1-M_2\right)aR^{1/2}w_0-\left[\left(\pi^2+a^2+\mathrm{P}_r\,\omega\right)-\mathrm{P}_r{}'M_2\omega\left(\frac{\pi^2}{\pi^2+a^2M_3}\right)\right]\theta_0=0 \tag{29}$$

For the existence of non trivial solutions of the above equations, the determinant of the coefficients of $w_0,\ \theta_0$ in equations (28) and (29) must vanish. After simplifying this determinant we obtain the equation of the form

$$T_3\,\omega^3+T_2\,\omega^2+T_1\,\omega+T_0=0 \tag{30}$$

where

$$T_3=\frac{1}{\varepsilon}\left(\pi^2+a^2\right)\left[\left(\pi^2+a^2M_3\right)GP_r-\pi^2P_r{}'M_2G\right] \tag{31}$$

$$T_2=\frac{1}{\varepsilon}\left(\pi^2+a^2\right)\left[\left(\pi^2+a^2\right)\left(\pi^2+a^2M_3\right)G+\left(\pi^2+a^2M_3\right)P_r-\pi^2P_r{}'M_2\right] \tag{32}$$

$$T_1=\left[\begin{array}{l}\frac{1}{\varepsilon}\left(\pi^2+a^2\right)^2\left(\pi^2+a^2M_3\right)+\frac{1}{k_1}\left(\pi^2+a^2\right)\left(\pi^2+a^2M_3\right)P_r\\ -\frac{1}{k_1}\pi^2\left(\pi^2+a^2\right)P_r{}'M_2-a^2R\left(1-M_2\right)\left(\pi^2+a^2\left(1+M_1\right)M_3\right)G\end{array}\right] \tag{33}$$

$$T_0=\left[\frac{1}{k_1}\left(\pi^2+a^2\right)^2\left(\pi^2+a^2M_3\right)-a^2R\left(1-M_2\right)\left(\pi^2+a^2\left(1+M_1\right)M_3\right)\right] \tag{34}$$

7. Stationary Convection

When the instability sets in as stationary convection and $M_2\cong 0$, the marginal state will be characterized by $\omega=0$, then the Rayleigh number will become

$$R_1=\frac{(1+x)^2(1+xM_3)}{xP_l\left[1+x\left(1+M_1\right)M_3\right]} \tag{35}$$

where

$$R_1=\frac{R}{\pi^4},\ x=\frac{a^2}{\pi^2},\ P_l=\pi^2k_1$$

which expresses the modified Rayleigh number as a function of the dimensionless wavenumber x, the buoyancy magnetization parameter M_1, the non-buoyancy magnetization parameter M_3 and the medium permeability parameter P_l (Darcy number).

To investigate the effects of non-buoyancy magnetization, buoyancy magnetization and medium permeability, we examine the behavior of $\frac{dR_1}{dM_3},\frac{dR_1}{dM_1}$ and $\frac{dR_1}{dP_l}$ analytically.

$$\frac{dR_1}{dM_3}=-\frac{(1+x)^2M_1}{P_l\left[1+x\left(1+M_1\right)M_3\right]^2} \tag{36}$$

$$\frac{dR_1}{dM_1}=-\frac{(1+x)^2M_3\left(1+xM_3\right)}{P_l\left[1+x\left(1+M_1\right)M_3\right]^2} \tag{37}$$

$$\frac{dR_1}{dP_l}=-\frac{(1+x)^2\left(1+xM_3\right)\left[1+x\left(1+M_1\right)M_3\right]}{xP_l^2\left[1+x\left(1+M_1\right)M_3\right]^2} \tag{38}$$

It is found that, for stationary convection, all of the three, the non-buoyancy magnetization, buoyancy magnetization and medium permeability always have destabilizing effect on the system.

For sufficiently large M_1, we obtain the magnetic thermal Rayleigh number

$$N_1 = R_1 M_1 = \frac{(1+x)^2(1+xM_3)}{x^2 P_l M_3} \tag{39}$$

The role of the medium permeability and the magnetic parameters discussed above can also be illustrated with the help of Figures 2 - 6.

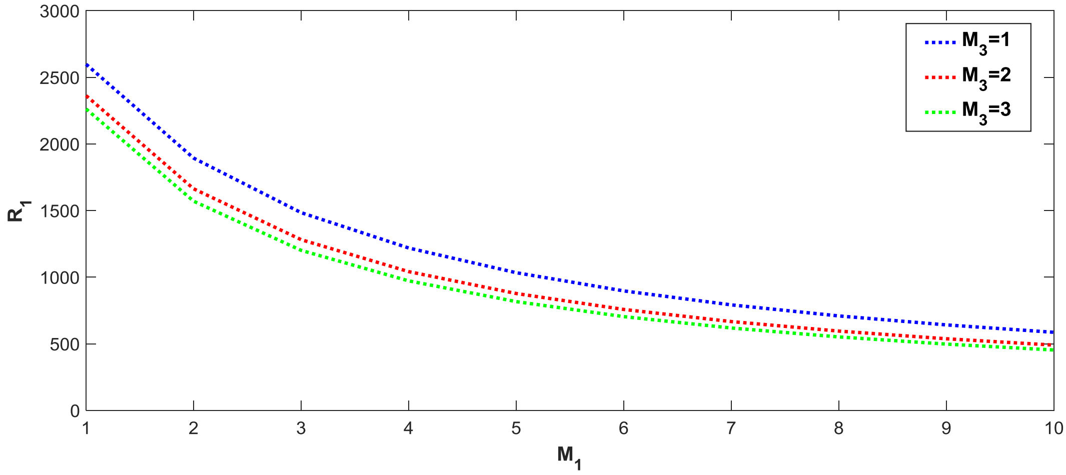

Figure 2: Variation of R_1 with M_1 for $P_l = 0.001$ and $M_3 = 1$, $M_3 = 2$, $M_3 = 3$

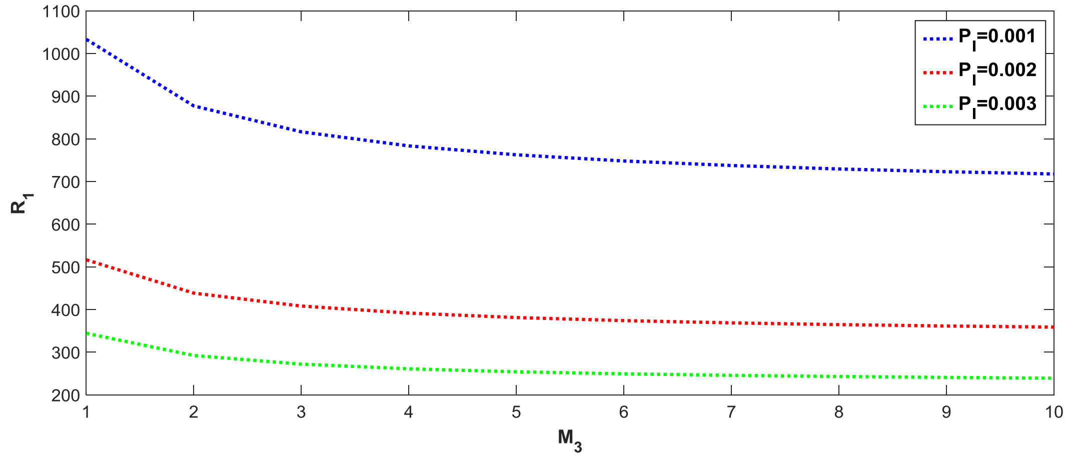

Figure 3: Variation of R_1 with M_3 for $M_1 = 5$ and $P_l = 0.001$, $P_l = 0.002$, $P_l = 0.003$

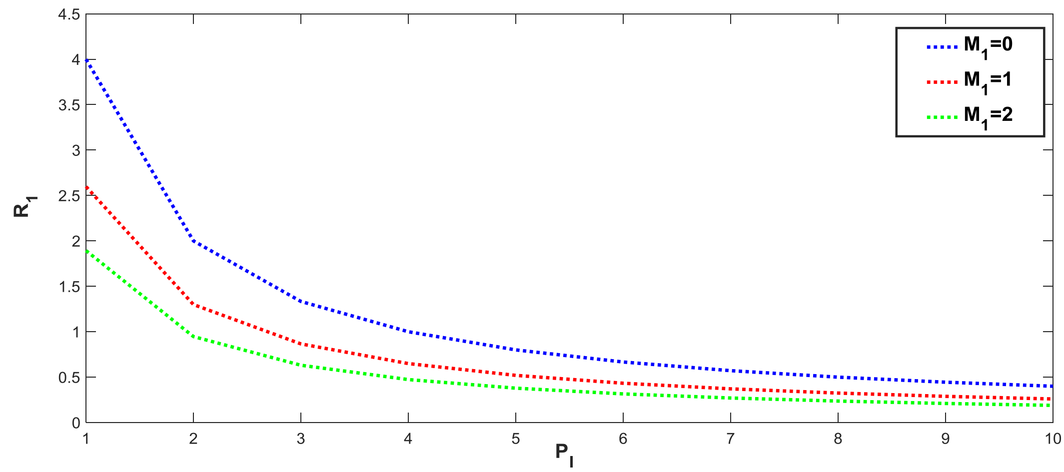

Figure 4: Variation of R_1 with P_l for $M_3 = 1$ and $M_1 = 0$, $M_1 = 1$, $M_1 = 2$

In Figure 2, R_1 is plotted against M_1 for $P_l = 0.001$ and $M_3 = 1$, $M_3 = 2$, $M_3 = 3$. It is clear that with the increase in buoyancy magnetization parameter M_1, the thermal Rayleigh number R_1 decreases, therefore buoyancy magnetization parameter M_1 has destabilizing effect on the system. In Figure 3, R_1 is plotted against M_3 for $M_1 = 5$ and $P_l = 0.001$, $P_l = 0.002$, $P_l = 0.003$. It is clear that with the increase in non-buoyancy magnetization parameter M_3, the thermal Rayleigh number R_1 decreases, thereby non-buoyancy magnetization parameter M_3 showing the destabilizing effect on the system. In Figure 4, R_1 is plotted against P_l for $M_3 = 1$ and $M_1 = 0$, $M_1 = 1$, $M_1 = 2$. It is clear that with the increase in medium permeability P_l, the thermal Rayleigh number R_1 decreases, thus the medium permeability P_l has the destabilizing effect on the system.

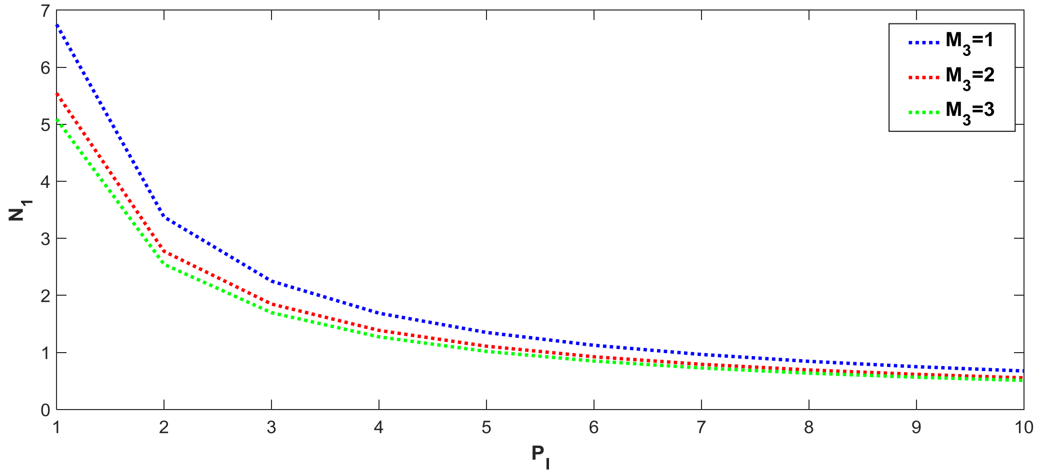

Figure 5: Variation of N_1 with P_l for $M_3 = 1$, $M_3 = 2$, $M_3 = 3$

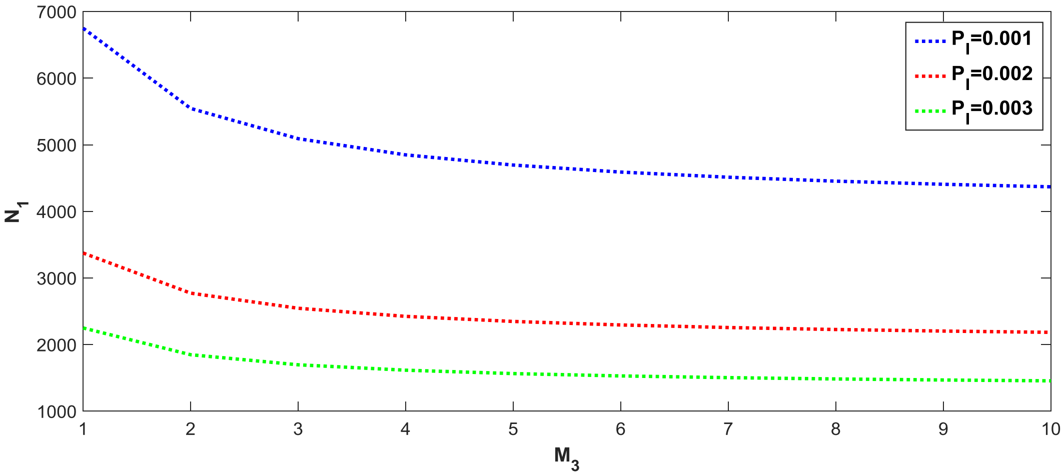

Figure 6: Variation of N_1 with M_3 for $P_l = 0.001$, $P_l = 0.002$, $P_l = 0.003$

In Figure 5, N_1 is plotted against P_l for $M_3 = 1$, $M_3 = 2$, $M_3 = 3$. It is clear that with the increase in magnetic permeability P_l, the magnetic thermal Rayleigh number N_1 decreases, indicating that the magnetic permeability P_l has destabilizing effect on the system. In Figure 6, N_1 is plotted against M_3 for

$P_l = 0.001$, $P_l = 0.002$, $P_l = 0.003$. It is clear that with the increase in non-buoyancy magnetization parameter M_3, the magnetic thermal Rayleigh number N_1 decreases, thereby the non-buoyancy magnetization parameter M_3 showing the destabilizing effect on the system.

8. Principle of Exchange of Stabilities

To examine the possibility of oscillatory modes, we put $\omega = i\omega_i$ in equation (30). Equating the imaginary part of equation (30), we get

$$\omega_i \begin{bmatrix} -\frac{1}{\varepsilon}\left(\pi^2 + a^2\right)\left[\left(\pi^2 + a^2 M_3\right) G P_r - \pi^2 P_r' M_2 G\right]\omega_i^2 + \\ \left[\frac{1}{\varepsilon}\left(\pi^2 + a^2\right)^2\left(\pi^2 + a^2 M_3\right) + \frac{1}{k_1}\left(\pi^2 + a^2\right)\left(\pi^2 + a^2 M_3\right) P_r \right. \\ \left. - \frac{1}{k_1}\pi^2\left(\pi^2 + a^2\right) P_r' M_2 - a^2 R\left(1 - M_2\right)\left(\pi^2 + a^2\left(1 + M_1\right) M_3\right) G\right] \end{bmatrix} = 0 \tag{40}$$

It is evident from equation (41) that ω_i may be either zero or non-zero, meaning that the modes may be either non-oscillatory or oscillatory. In the absence of stress relaxation parameter G, we get

$$\omega_i \begin{bmatrix} \frac{1}{\varepsilon}\left(\pi^2 + a^2\right)^2\left(\pi^2 + a^2 M_3\right) + \frac{1}{k_1}\left(\pi^2 + a^2\right)\left(\pi^2 + a^2 M_3\right) P_r \\ - \frac{1}{k_1}\pi^2\left(\pi^2 + a^2\right) P_r' M_2 \end{bmatrix} = 0 \tag{41}$$

Here the quantity inside bracket is positive definite if

$$\left[\frac{1}{\varepsilon}\left(\pi^2 + a^2\right) + \frac{1}{k_1} P_r\right]\left(\pi^2 + a^2 M_3\right) > \frac{1}{k_1}\pi^2 P_r' M_2 \tag{42}$$

which implies that $\omega_i = 0$. Hence, the principle of exchange of stabilities is valid and the oscillatory modes are not allowed.

III. Conclusion

In this paper, thermal convection in a Maxwell ferromagnetic fluid layer through a porous medium is studied and the linear stability analysis and normal mode technique is used. An exact solution is obtained for a flat Maxwell ferromagnetic fluid layer contained between two free boundaries using Galerkin method. The thermal Rayleigh number and the magnetic thermal Rayleigh number have been obtained. We have investigated the effects of stress relaxation parameter G, buoyancy magnetization parameter M_1, non-buoyancy magnetization parameter M_3 and the medium permeability parameter P_l. It is found that all the three parameters, buoyancy magnetization parameter M_1, non-buoyancy magnetization parameter M_3 and the medium permeability parameter P_l have destabilizing effect on the system. The stress relaxation parameter G vanishes in stationary convection. It is also observed that the in the absence of stress relaxation parameter G, the principle of exchange of stabilities is valid for the certain conditions.

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