

## Web mathematica on Chromatic Graphs

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**Abstract.** A graph  $G$  is a mathematical structure consisting of two sets  $V(G)$  (vertices of  $G$ ) and  $E(G)$  (edges of  $G$ ). Proper coloring of a graph is an assignment of colors either to the vertices of the graphs, or to the edges, in such a way that adjacent vertices / edges are colored differently. This paper discusses coloring and operations on graphs with Mathematica and webMathematica. We consider many classes of graphs to color with applications. We draw any graph and also try to show whether it has an Eulerian and Hamiltonian cycles by using our package ColorG.

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### I. Introduction

Graph theory would not be what it is today if there had been no coloring problems. In fact, a major portion of the 20th-century research in graph theory has its origin in the four color problem [1]. A graph  $G$  is a mathematical structure consisting of two sets  $V(G)$  (vertices of  $G$ ) and  $E(G)$  (edges of  $G$ ). Proper coloring of a graph is an assignment of colors either to the vertices of the graphs, or to the edges, in such a way that adjacent vertices / edges are colored differently. Vertex coloring is a hard combinatorial optimization problem.

We apply several operations which act on graphs to give different graphs. In addition to apply graph operations, we color vertices of these obtained graphs properly. Also we developed ColorG package to color the vertices and edges of graphs and to find the Eulerian and the Hamiltonian cycles with webMathematica. Many of these graphs are truly beautiful when drawn properly, and they provide a wide range of structures to manipulate and study.

Before concluding this introduction, we recall some basic definitions.

A complete graph is a simple graph such that every pair of vertices is joined by an edge. A nontrivial closed path is called a cycle. A graph which is obtained by joining a new vertex to every vertex of a cycle is called a wheel. A connected acyclic graph is called a tree [4].

### II. Graph Coloring with Web Mathematica

One of the most exciting new technologies for dynamic mathematics on the World Wide Web is a webMathematica. This new technology developed by Wolfram research enables instructors to create web sites that allows users to compute and visualize results directly from a web browser. webMathematica is based on a standard java technology called servlets. It allows a site to deliver HTML pages that are enhanced by the addition of Mathematica commands [5]. When a request is made for one of these pages the Mathematica commands are evaluated and the computed result is placed in the page. People who access webMathematica sites do not have to know how to use Mathematica [11].

In this section, we give applications of ColorG package to color the vertices and the edges of the graphs with webMathematica

#### 2.1 Vertex Coloring

The most applications involving vertex coloring are concerned with determining the minimum number of colors required under the condition that the end points of an edge cannot have the same color. A proper vertex coloring of a graph is an assignment from its vertex set to a color set that the end points of each edge are assigned two different colors. The chromatic number of a graph  $G$ , denoted by  $\chi(G)$ , is the minimum number of different colors required for a proper vertex coloring of  $G$ . Applications of vertex coloring include scheduling, assignment of radio frequencies, separating combustible chemical combinations, and computer optimization. We use some commands in the Combinatorica package with Mathematica to color the vertices of graphs and to give web-based examples with webMathematica as in The Fig. 1 [9].

Fig. 2. Edge Coloring of a Graph with webMathematica

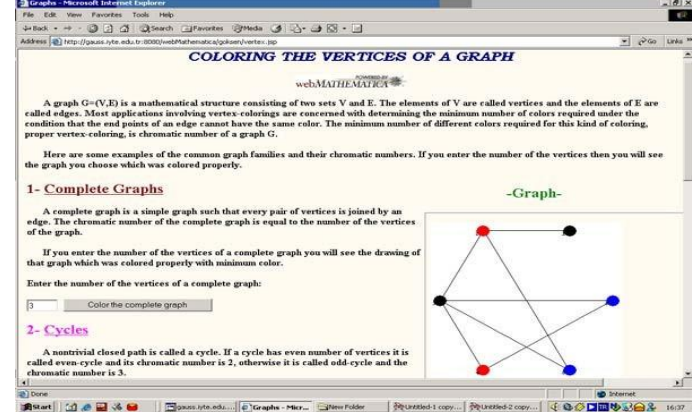


Fig. 1. Vertex Coloring of a Graph with webMathematica

If the users press the CompleteGraph, Cycle, Wheel, Star, RandomTree, and any graph and enter the number of vertices they can get the vertex-colored graph.

### 2.2 Edge Coloring

Edge coloring is an optimization problem: An edge-coloring of a graph  $G$  is an assignment of colors to the edges of  $G$  such that edges with a common endpoint

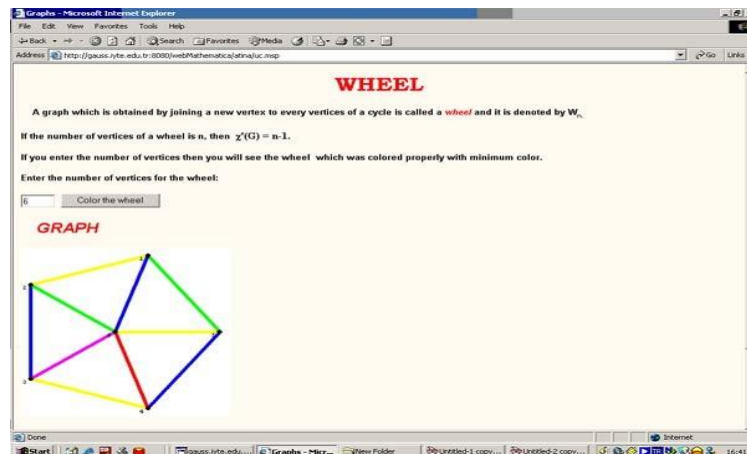


Fig. 2. Edge Coloring of a Graph with webMathematica

have different colors. Let  $\chi(G)$  denote the chromatic index of  $G$ , that is the minimum number of colors necessary to color the edges of  $G$ . Vizing [10] proved that  $\chi(G)$  is either  $\Delta(G)$  or  $\Delta(G) + 1$  for each graph  $G$ , where  $\Delta(G)$  denotes the maximum degree of a vertex in  $G$ . Then the graph  $G$  belongs to one of two classes; either to class 1 or to class 2. This classification problem is NP-complete, and this implies that there are no polynomial-time algorithms for this problem. We use some commands in the ColorG package with Mathematica to color the edges of graphs and to give web-based examples with webMathematica as in The Fig. 2 [9].

If the user press the Wheel button and enter the number of vertices, he/she can get the edge-colored graph.

### III. Cycle Structure in Graphs with WebMathematica

A cycle in a graph is a simple closed path. We will represent a cycle in  $G$  as a list of vertices  $C = v_1, v_2, \dots, v_1$  such that there is an edge of  $G$  from each vertex to the next in  $G$ .

#### 3.1 Eulerian Cycle

Euler initiated the study of graph theory in 1736 with the famous Seven Bridges of Königsberg problem. The town of Königsberg straddled the Pregel River with a total of seven bridges connecting the two shores and two

islands. The townsfolk were interested in crossing every bridge exactly once and returning to the starting point. An Eulerian cycle is a complete tour of all the edges of a graph. The term circuit is often used instead of cycle, since each vertex can be visited more than once.

We use ColorG package with Mathematica to find the Eulerian cycle and to give web-based examples with webMathematica. If the number of the vertices is entered, it is possible to see the Eulerian cycle in that graph if there exists.

### 3.2 Hamiltonian Cycle

A Hamiltonian cycle of a graph  $G$  is a cycle which visits every vertex in  $G$  exactly once, as opposed to an Eulerian cycle which visits each edge exactly once. A Hamiltonian path is like a Hamiltonian cycle, except that it is a path. The problem of computing a Hamiltonian cycle or a Hamiltonian path is fundamen-

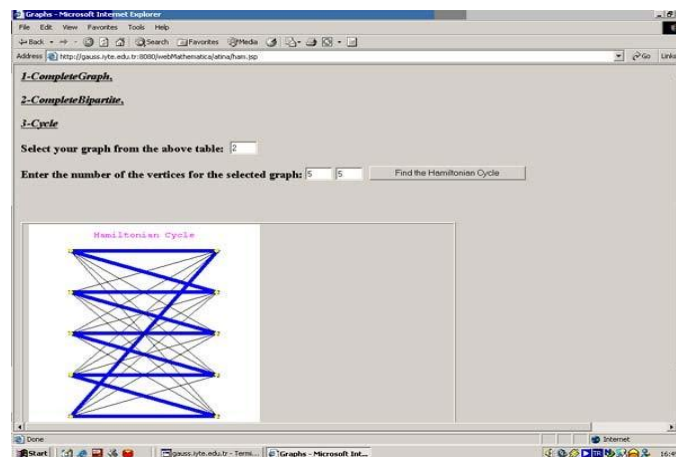


Fig. 3. A Hamiltonian Cycle in a Complete Bipartite Graph with webMathematica

tally different from the problem of computing an Eulerian cycle, because testing whether a graph is Hamiltonian is NP-complete.

We use ColorG package with Mathematica to find the Hamiltonian cycle and to give web-based examples with webMathematica. If the number of the vertices is entered, it is possible to see the Hamiltonian cycle in that graph if there exists. The Fig. 4. shows the Hamiltonian Cycle for the Complete Bipartite Graph.

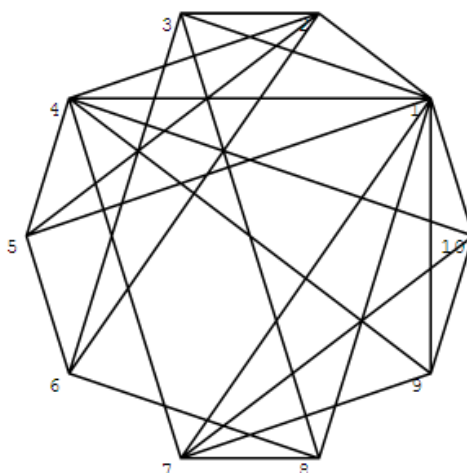
### 3.3 An Application

Some scheduling problems induce a graph coloring, i.e., an assignment of positive integers (colors) to vertices of a graph. We discuss a simple example for coloring the vertices of a graph with a small number  $k$  of colors and present computational results for calculating the chromatic number, i.e., the minimal possible value of such a  $k$ . Draw up an examination schedule involving the minimum number of days for the following problem.

**Example:** Set of students:  $S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9$  Examination subjects for each group: {algebra, real analysis, and topology}, {algebra, operations research, and complex analysis}, {real analysis, functional analysis, and topology}, {algebra, graph theory, and combinatorics}, {combinatorics, topology, and functional analysis}, {operations research, graph theory, and coding theory}, {operations research, graph theory, and number theory}, {algebra, number theory, and coding theory}, {algebra, operations research, and real analysis}.

Let  $S$  be a set of students,  $P = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$  be the set of examinations respectively algebra, real analysis, topology, operational research, complex analysis, functional analysis, graph theory, combinatorics, coding theory, and number theory.  $S(p)$  be the set of students who will take the examination  $p \in P$ .

Form a graph  $G = G(P, E)$ , where  $a, b \in P$  are adjacent if and only if  $S(a) \cap S(b) \neq \emptyset$ . Then each proper vertex coloring of  $G$  yields an examination schedule with the vertices in any color class representing the schedule on a particular day. Thus  $\chi(G)$  gives the minimum number of days required for the examination schedule. The Mathematica commands for this solution are as follows:



**Fig. 4.** The colored graph of the example

5 days are required and you can see below the lessons in the same parenthesis which are on the same day  
 $\{\{1, 6\}, \{2, 8, 9\}, \{3, 4\}, \{5, 7\}, \{10\}\}$

It was very exciting to take 100-year old ideas, simple as they are, and implement them in *Mathematica* and *webMathematica* for anybody. But, there is more work to be done, both of a theoretical and practical nature. If we consider a coloring problem posed as a two-person game, with one person (Alice) trying to color the graph, and the other (Bob) trying to prevent this from happening. Alice and Bob alternate turns, with Alice having the first move. A move consisting of selecting an uncolored vertex  $x$  and assigning it a color from the color set  $X$  distinct from the colors assigned previously to neighbors of  $x$ . If after  $n = V(G)$  moves, the graph  $G$  is colored, Alice is the winner. Bob wins if an impasse is reached before all vertices in the graph are colored. The game chromatic number of a graph  $G$ , denoted by  $\chi_g(G)$ , is the least cardinality of a color set  $X$  for which Alice has a winning strategy [3]. We believe that it is possible to play this game online with *webMathematica*. Of course, this leads to the rich area of game coloring and the many difficult and intriguing questions there.

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