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ϕ* Closed Set In Fuzzy Topological Spaces

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Abstract
In this paper, we study a fuzzy sets, namely, fuzzy ϕ- closed sets for fuzzy topological space, and some of there properties have been proved. Further we discuss fuzzy ϕ*-closed (open) functions, fuzzy ϕ*- continuous, fuzzy ϕ*- irresolute function as application of fuzzy sets, fuzzy $T_{1/5}$-spaces, fuzzy $T_{1/5}{}^{\phi^*}$ - spaces and fuzzy ϕ$T_{1/5}$-spaces.*
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I. Introduction:

In 1965, zadeh [1] introduced the fundamental concept of fuzzy operation and fuzzy sets. Subsequently, many researchers have worked on various concepts from general topology using fuzzy sets and developed the theory of fuzzy topological spaces [3-7]. Muthu kumaraswamy [8] introduced (briefly fgα – closed and fαg- closed) set in fuzzy topological space in 2004.

In this paper we introduce a fuzzy generalized closed sets called fuzzy ϕ* closed sets, which is properly placed in between the class of fuzzy α- closed set, and fuzzy generalized α- closed sets. In the " fuzzy ϕ* -closed set in Fuzzy Topological Space " section , We introduce the definition of fuzzy ϕ* -closed set, ϕ* continuous, and fuzzy ϕ* - irresolute function in Fuzzy Topological Space ". further, fuzzy $T_{1/5}$-spaces, fuzzy $T_{1/5}{}^{\phi^*}$ -spaces, and fuzzy ϕ* $T_{1/5}$-spaces, are introduced in "Application of F_{ϕ^*}- closed sets" section .

II. Preliminaries:

This paper, (G, τ) and (H, σ) (or G and H) always mean fuzzy topological space. The members of τ are called fuzzy open sets, and their complements are fuzzy closed sets.
(ie) Ψ : (G, τ) → (H, σ) (or Ψ : G→H) denotes a mapping Ψ from Fuzzy Topological Space G to Fuzzy Topological Space H.
For a fuzzy set D of (G, τ), fuzzy closure and fuzzy interior of D denoted by Cl(D) and int(D) resp., Defined by Cl(D) = ∧{ E : E is fuzzy closed set of G, E≥D, i – E ∈ τ} and int D = V{S : S is fuzzy open set of G, S≤D, S ∈ τ } [10].

Definition : 1

A fuzzy set D of Fuzzy Topological Space 'G' is called fuzzy generalized αCl(D)≤ U where D≤ U and U is fuzzy α-open in (G, τ). The complement of Fgα- closed sets is called Fgα-open set.

Definition: 1.2

A fuzzy set D of F is 'G' is called fuzzy generalized α-open [4] if D≤ int (Cl(intD)) and fuzzy α-closed if D≥Cl(int(Cl(D))) the intersection of all fuzzy α-closed set of (G, τ) containing D is called α-closure of a fuzzy subset D of G and is denoted by αCl(D).

Definition: 1.3

A function Ψ : (G, τ)→ (H, σ) is said to be fuzzy open (fuzzy closed) [2], if the image of every fuzzy open (fuzzy closed) set in G is fuzzy open (fuzzy-closed) set in H.

Definition: 1.4

Let (G, τ) and (H, σ) be two fuzzy topological spaces (Fuzzy Topological Space). A function : (G, τ) → (H, σ) is as follows,

(i) F_α- continuous [10] of ϕ^{-1} (v)is F_α-closed in G, for each V ∈ F C(H)
(ii) $F_{g\alpha}$- continuous [8] if ϕ^{-1} (v) is $F_{g\alpha}$- closed in G, for each V ∈ F C(H)
(iii) F-irresolute [11] if ϕ^{-1}(v) is F- closed in G, for each V ∈ F C (H)

III. Fuzzy ϕ*-closed in Fuzzy Topological Space

In this section, we introduce a relationship between fuzzy ϕ*-closed sets in fuzzy topological space and some of its characterizations.

Definition (2.1)

A fuzzy set D in (G, τ) is called fuzzy ϕ*- closed (F ϕ*- closed) if α Cl (D) $\leq$ U where D$\leq$ U and U is $F_{g\alpha}$- open in (G, τ).

 The complement of F ϕ*- closed set is called F ϕ*- open set.

Note:

The class of fuzzy ϕ*- closed sets of Fuzzy Topological Space (G, τ) is denoted by F ϕ* C(G).

Example (2.1)

Let G={a,b,c} with fuzzy topology

τ ={0,1,{$a_{0.5}$,$b_{0.2,}$ $c_{0.7}$ },{$a_{0.7,}$ $b_{0.8,}$ $c_{0.3}$ }, {$a_{0.5,}$ $b_{0.2,}$$c_{0.3}$}, {$a_{0.7}$, $b_{0.8}$, $c_{0.7}$}}.

 Fuzzy subset D= {$a_{0.4}$, $b_{0.8}$, $c_{0.7}$} is F ϕ*- closed set in (G, τ) but not F_α-closed set since Cl (in t (Cl (D))) = {$a_{0.5}$, $b_{0.8}$, $c_{0.7}$}.

Preposition: 2.1

Every fuzzy α- closed set is fuzzy ϕ*- closed.

Proof:

Let D be a F_α-closed set in (G, τ), and since every F_α-closed set is $F_{g\alpha}$- closed. Then, α Cl (D)$\leq$ U, where D $\leq$ U and U is F_α- open in (G, τ) and since every F_α- open set is $F_{g\alpha}$- open. So, α Cl (D) $\leq$ U, where D $\leq$ U and U is $F_{g\alpha}$- open in (G, τ). Thus, D is F ϕ*- closed.

Converse of need not be true as seen from the above example.

Preposition: 2.2

Every fuzzy ϕ*- closed set is fuzzy g_α-closed set.

Proof:

Every F_α- open set is $F_{g\alpha}$- open from the fact.

Converse need not be true as seen from the following example.

Example 2.2

In Eg (2.1) the fuzzy set D={ $a_{0.5,}$ $b_{0.3,}$$c_{0.7}$} $F_{g\alpha}$- closed set in (G, τ) but not F ϕ*- closed set.

Proposition 2.3

If D is F ϕ*- closed set in (G, τ) and D $\leq$ E $\leq$ α Cl (A), then E is F ϕ*- closed set of (G, τ).

Proof:

Let U be a $F_{g\alpha}$- open subset of (G, τ) such that E $\leq$ U. Then, D $\leq$ U and since D ∈ F ϕ* C (G), then α Cl (D) $\leq$ U. (ie) α Cl (E) $\leq$α Cl (D) $\leq$ U. Then E ∈ F ϕ* C (G).

Proposition: 2.4

If D and E are F ϕ*- closed set in (G, τ), then DUE is also F ϕ*- closed set in (G, τ).

Proof:

If D V E $\leq$ U and U are $F_{g\alpha}$- open then D $\leq$ U and E $\leq$ U. Since D and E are F ϕ*- closed, α Cl (D) $\leq$ U, and α Cl (E) $\leq$ U and hence α Cl (D V E) = α Cl (D) V α Cl (E) $\leq$ U. Thus, DVE is F ϕ*- closed set in (G, τ).

Corollary 2.1

If D is F ϕ*- open set in (G, τ) and α in t (D)$\leq$ E $\leq$ D, then E is F ϕ*- open set.

Proof:

Let D ∈ F ϕ* O (G), and α int (D)$\leq$ E $\leq$ D. Then 1- D ∈ F ϕ* C (G), and 1- D $\leq$1-E $\leq$α Cl (1-D). By prop 2.3, 1-B ∈ F ϕ* C (G). Hence E F ϕ* O (G).

Proposition: 2.5

If D is $F_{g\alpha}$- open set and fuzzy ϕ*- closed set in (G, τ) then D is fuzzy α- closed set in (G, τ).

Proof:

Since D $\leq$ D and D is $F_{g\alpha}$- open set and F ϕ*- closed, α Cl (D) $\leq$ D. since D $\leq$ α Cl (D) , then D= α Cl (D) , and thus D is F_α- closed set in (G, τ).

Proposition: 2.6

Every fuzzy ϕ*- open set in fuzzy g_α- open.

Proof:

Let D ∈ F ϕ* O (G), then 1-D F ϕ* O (G) and hence $F_{g\alpha}$- closed set in (G, τ) by prop 2.2 D is $F_{g\alpha}$- open set in (G, τ). Hence every F ϕ*- open set in G is $F_{g\alpha}$- open set in G.

Definition: 2.2

Let (G, τ) be a fuzzy topological space. Then for a fuzzy subset D of G, the fuzzy ϕ*- closure of D (Ψ*- Cl (D)) is the intersection of all fuzzy ϕ*- closed set of G containing D. (ie) ϕ * Cl(D) = ∧ {E:E ≥ D, E is fuzzy ϕ *- closed m G }.

Definition: 2.3

For any fuzzy set D in Fuzzy Topological Space G, we have the fuzzy ϕ * interior of D (ϕ * - int (D)) is the union of all fuzzy ϕ * -open set of G □ D.

(ie) ϕ * - int(D)=∨ {E:E ≤ D, E is F ϕ * - open in G}

Proposition: 2.7

For any fuzzy sets D and B in a Fuzzy Topological Space G, we have as follows:

(i) ϕ *- int (D) ≤ D

(ii) D is F ϕ * - open ⇔ ϕ * - int (D)= D

(iii) ϕ *- int (ϕ *- int (D)) = ϕ * - int (D)

(iv) If D ≤ B, then ϕ * - int (D) ≤ ϕ * - int (B)

Proof:

(i) Follows from definition 2.3

(ii) Let D∈ F ϕ* O (G), Then D ≤ ϕ * - int (D). By using (1), we get D= ϕ * - int (D).

Conversely assume that D= ϕ * - int (D). By using definition (2.3) D ∈ F ϕ* O (G).

(iii) By using (ii), we get ϕ * - int (ϕ * - int(D)) = ϕ * - int (D)

(iv) Since D ≤ E, by using (i), ϕ * - int (D) ≤ D ≤ E.

(ie) ϕ * - int (D) ≤ E, by using (iii), ϕ * - int (ϕ * - int (D)) ≤ ϕ * - int (E). Thus ϕ * - int (D)≤ ϕ * - int (E).

Proposition: 2.8

For any fuzzy set D in a Fuzzy Topological Space G, we have as follows:

(i) (ϕ * - int (D))^c = ϕ *- Cl (D^c)

(ii) (ϕ *- Cl (D))^c = ϕ * - int (D^c)

Proof:

(i) By using definition (2.3) ϕ *- int (D) = ∨ {E:E ≤ D, E ∈ F ϕ* O (G)}. Taking complement on both sides, we get

(ϕ * - int (D))^c = (sup {E:B ≤ D, E is F ϕ*- open in G}^{c}

= in f {E^c: E^c ≥ D^c, E^c is F ϕ*- closed in G}

Replacing E^c by c, we get

(ϕ * - int (D))^c = ∧ {C: C ≥ D^c, C is F ϕ*- closed in G}

By definition: 2.4, (ϕ * - int (D))^c = ϕ *- Cl (D^c).

(ii) By using (i), (ϕ * - int (D^c))^c = ϕ *- Cl (D^c)^c= ϕ *- Cl (D).

Taking complement on both side, we get

ϕ * - int (D^c) = (ϕ *- Cl (D))^c

Proposition: 2.9

Let D be a fuzzy set in Fuzzy Topological Space G, Then D ∈ F ϕ*c (G) if f D^c is F ϕ* - open

Proposition: 2.10

For any fuzzy set D and E in Fuzzy Topological Space. G, we have

(i) D ≤ ϕ *- Cl (D)

(ii) D is F ϕ* closed ⇔ ϕ *- Cl (D) = D

(iii) ϕ *- Cl (ϕ *- Cl(D)) = ϕ *- Cl (D)

(iv) If D ≤ E, then ϕ *- Cl(D) ≤ ϕ *- Cl(E).

Proof:

(i) Follows from definition (2.5)

(ii) Let D ∈ F ϕ* C (G), by using proposition (2.9) D^C ∈ F ϕ* O (G). By using proposition (2.10) (ii) ϕ * - int (D^c) = D^c ⇔ (ϕ *- Cl (D))^c = D^c ⇔ ϕ *- Cl (D) = D.

(iii) By using (ii), we get ϕ *- Cl (ϕ *- Cl (D) = ϕ *- Cl (D)

(iv) If D∧E≤ D and D∧E≤ E by using proposition (2.8)

(v) ϕ * - int (E^c) ≤ ϕ * - int (D^c). Taking complement on both side, we get (ϕ * - int (E^c))^c ≥ (ϕ * - int (D^c))^c
By using proposition (3.9) (ii), ϕ *- Cl (E)≥ϕ *- Cl (D).

Proposition 2.11

Let D be a fuzzy set in Fuzzy Topological Space G, Then int(D)≤ α-int(D)≤ ϕ * - int (D)≤ ϕ *- Cl (D)≤ α-Cl(D)≤ Cl (D).

Proof:

If follows from the definition of corresponding operator.

Proposition 2.12

For any fuzzy set D and E in a Fuzzy Topological Space G, we have

(i) ϕ *- Cl (DVE) = ϕ *- d (D)V ϕ *- Cl (E)

(ii) ϕ *- Cl (D∧E) = ϕ *- d (D) ∧ ϕ *- Cl (E)

Proof:

Since ϕ *- Cl (DVE) = ϕ *- Cl $(DVE)^c$, by using proposition (2.8)(i) we have ϕ *- Cl (DVE) = ϕ * - (int $(DVE)^c))^c$ = (ϕ * - int $(D^c \wedge E^c))^c$ By using proposition we have ϕ *- Cl (DVE) = (ϕ * - int (D^c) ∧ ϕ * - int $(E^c)^c$ = (ϕ * - int $(D^c)^c$ V (ϕ * - int $(E^c)\,)^c$, By using proposition (2.8) (i) we have ϕ *- Cl (DVE) = ϕ *- Cl $(D^c)^c$ V ϕ * - Cl $(E^c)^c$ = ϕ *- Cl (D) V ϕ *- Cl (E).

(ii) Since D∧E$\leq$ D and D∧E$\leq$ E , by using proposition (2.10) (iv) we have ϕ *- Cl (D∧E)$\leq$ ϕ *- Cl (D) and ϕ *- Cl (D∧E)$\leq$ ϕ *- Cl (E). This implies that ϕ *- Cl (D∧E)$\leq$ ϕ *- Cl (D) ∧ ϕ *- Cl (E).

Proposition 2.13

For any fuzzy set D and E in a Fuzzy Topological Space G, we have as follows

(i) ϕ *- Cl (D) $\geq$ D ∨ ϕ *- Cl (ϕ * - int (D))

(ii) ϕ * - int (D)$\leq$ D ∨ ϕ *- int(ϕ *- Cl (D))

(iii) int(ϕ *- Cl (D)$\leq$ int (Cl (D))

(iv) int(ϕ *- Cl (D) $\geq$ int (ϕ *- Cl(ϕ * - int (D))

Proof:

(i) By proposition (2.10) D$\leq$ ϕ *- Cl (D). Again, using proposition (2.7) ϕ * - int (D)$\leq$ D. Then, we have D ∨ ϕ *- Cl (ϕ * - int (D))$\leq$ ϕ *- Cl (D).

(ii) By proposition (2.7) (i) ϕ * - int (D)$\leq$ D. Again by proposition (2.10) (i) D$\leq$ ϕ *- Cl (D). Then ϕ * - int (D)$\leq$ ϕ *- int(ϕ *- Cl (D)).Then, ϕ * - int (D)$\leq$ D ∨ ϕ *- int(ϕ *- Cl (D)).

(iii) By proposition (2.12) ϕ *- Cl (D)$\leq$Cl (D). we get int(ϕ *- Cl (D)) $\leq$ int (Cl (D)).

(iv) By (i), ϕ *- Cl (D) $\geq$ D ∨ ϕ *- Cl (ϕ * - int (D)) $\geq$ D ∨ ϕ *- Cl (ϕ * - int (D)). Then we have int(ϕ *- Cl (D)) $\geq$ int(D ∨ ϕ *)- Cl (ϕ * - int (D)). Since int(D ∨ E) $\geq$ int(D) ∨ int(E), int(ϕ *- Cl (D)) $\geq$ int(D) ∨ int (ϕ *- Cl(ϕ * - int (D)) $\geq$ int(D) ∨ int (ϕ *- Cl(ϕ * - int (D)).

IV. Fuzzy ϕ *- irresolute and fuzzy ϕ *-continuous functions in Fuzzy Topological Space

Here, we introduce some types of fuzzy functions and there application of closed - ϕ * sets (fuzzy) as well as some of their properties.

Definition3.1

A fun Ψ: (G, τ)→ (H, σ) is said to be fuzzy ϕ *- continuous (F ϕ *- continuous) if Ψ^{-1} (V) is F ϕ *- closed in G, for each fuzzy closed set V in H.

Proposition 3.1

Every F_α-continuous function is F ϕ *- continuous.

Proof:

Let V∈FC(H). Since Ψ is F_α-continuous then Ψ^{-1} (V) is F ϕ *-closed in G. Since every F_α-closed set is F ϕ *-closed, then Ψ^{-1} (V) ∈ F ϕ *C(G). Thus Ψ is F ϕ *- continuous.

Converse of proposition (3.1) need not be true as seen from the following example.

Example 3.1

Suppose that G = { a, b, c } with fuzzy topology τ = {0,1,{$a_{0.5}$, $b_{0.2}$, $c_{0.7}$}, {$a_{0.7}$, $b_{0.8}$, $c_{0.3}$}, {$a_{0.5}$, $b_{0.2}$, $c_{0.3}$}, {$a_{0.7}$, $b_{0.8}$, $c_{0.7}$} and H ={x, y, z} with fuzzy topology σ = {{0,1} {$x_{0.8}$, $y_{0.2}$, $z_{0.3}$}}. Let Ψ: (G, τ)→ (H, σ) be defined Ψ(a) = x, Ψ(b) = y and Ψ(c) = z, $\phi$is F ϕ* - continuous functions but it is not a F_α-continuous function, since V = {$x_{0.2}$, $y_{0.8}$, $z_{0.7}$} ∈ FC(H) but Ψ^{-1} (V) ∉ $F_\alpha C(G)$.

Definition 3.2

A function Ψ: (G, τ)→ (H, σ) is said to be F ϕ *- irresolute (F ϕ *- irresolute) if Ψ^{-1} (V) ∈ F ϕ *C(G), for each $F_{\phi *}$-closed set V in +1.

Proposition 4.2

Every F ϕ *- irresolute function is F ϕ *- continuous.

Proof:

It follows from the definition, the converse proposition 3.3 need not be true as from the following example.

Example:

In example 3.1 let Ψ: (G, τ)→ (H, σ) be defined by Ψ(a)=x, Ψ(b)=y and Ψ(c)=z. Ψ is F ϕ *- continuous function, but it is not a F ϕ *- irresolute function, since V = {$x_{0.2}$, $y_{0.7}$, $z_{0.4}$}∈ F ϕ *C(H) but Ψ^{-1} (V) ∈ F ϕ *C(G).

Proposition:

Let Ψ: G→H and γ:H→W be any two function. Then

(i) γ□ Ψ is F ϕ *- continuous if g is fuzzy continuous and Ψ is F ϕ *- continuous.

(ii) γ□ Ψ is F ϕ *- irresolute if both Ψ and g are F ϕ *- irresolute.

(iii) $\gamma\Box\ \Psi$ is F ϕ *- continuous if g is F ϕ *- continuous, and Ψ is F ϕ *- continuous and Ψ is F ϕ *- irresolute.

Proof:

Let V∈FC(W). Since γ is fuzzy continuous, then $\gamma^{-1}(V)$ ∈FC(H). Since ϕ is F ϕ*- continuous, then we have $\Psi^{-1}(\gamma^{-1}(V) \in$ F ϕ *C(G). consequently $\gamma\Box\ \Psi$ is F ϕ *- continuous.

By similarly (ii) – (iii).

Application of F ϕ *- closed sets

In application of F ϕ *- closed sets, three fuzzy spaces, namely fuzzy $T_{1/5}$- spaces, fuzzy $T_{1/5}$ ϕ *spaces, and fuzzy ϕ * $T_{1/5}$- spaces are introduced.

Here, we introduce the following definitions

Definition 4.1

A fuzzy topological space (G, τ) is as follows,

(i) Fuzzy $T_{1/5}$- space of every $F_{g\alpha}$-closed set in G. G is F_α-closed set in G.

(ii) Fuzzy $T_{1/5}$ ϕ * - space of every F ϕ *- closed set in G is a F_α-closed set in G.

(iii) Fuzzy ϕ * $T_{1/5}$ - space of every F ϕ *- closed set in G.

Proposition 4.1

If ϕ: G → H is F ϕ *- irresolute and G is fuzzy ϕ * $T_{1/5}$ – space, then Ψ is F ϕ *- continuous.

Proof:

By see next theorem 4.2.

Proposition 4.2

If Ψ: G → H is F ϕ *- continuous and G is fuzzy $T_{1/5}$ ϕ *– space, then Ψ is F ϕ *- continuous.

Proof:

Let V ∈ FC(H); since f is F ϕ *- continuous. Then $\Psi^{-1}(V) \in F_{\phi^*}C(G)$. Since G is F $T_{1/5}$ ϕ *– space, then $\Psi^{-1}(V)$ is F_α-closed set in G. Thus Ψ is F_α-continuous.

Proposition 4.3

If Ψ : G → H is $F_{g\alpha}$- continuous and G is fuzzy $T_{1/5}$ ϕ *– space, then Ψ is F ϕ *-continuous.

Proof:

Let V ∈ FC(H); since Ψ is $F_{g\alpha}$- continuous and $\Psi^{-1}(V)$ is $F_{g\alpha}$- closed set in G. Since G is F ϕ * $T_{1/5}$ – space then $\Psi^{-1}(V) \in F_{\phi^*}C(G)$. Thus Ψ is $F_{g\alpha}$- continuous.

Proposition 4.4

Let G, H, W be Fuzzy Topological Space, and Ψ : G → H, γ : H → W and $\gamma\Box\ \Psi$: G → W be function, then if Ψ is F_α- irresolute function and is F ϕ * - continuous function, such that H is fuzzy $T_{1/5}$*- space. Then $\gamma\Box\ \Psi$ is F_α- continuous function.

Proof:

Let u ∈ FC(W), since γ is F ϕ * - continuous, then $\gamma^{-1}(u)$ ∈ F ϕ *C(H). Since H is fuzzy $T_{1/5}$*- space, then $\gamma^{-1}(u)$ is F_α- closed set in H. But ϕ is F_α-irresolute function. Then $\Psi^{-1}(\gamma^{-1}(w))$ is F_α- closed set in H. But $\Psi^{-1}(\gamma^{-1}(u)) = (\gamma\Box\ \Psi^{-1})(u)$.

$\gamma\Box\ \Psi$ is F_α- continuous function.

Proposition 4.5

Let Ψ : G → H be onto F ϕ * - irresolute, and F_α- closed. If G is fuzzy then $T_{1/5}$ ϕ *-Space, then H is also a fuzzy $T_{1/5}$ ϕ *-Space.

Proof:

Let V ∈ F ϕ *C(H); since f is F ϕ * - irresolute, then $\Psi^{-1}(V) \in$ F ϕ *C(G). Since G is $F_{T1/5}$ ϕ *-Space, then $\Psi^{-1}(V)$ is F_α- closed set in G. Since Ψ is F_α- closed and onto, then we have V is F_α- closed.

H is also a $F_{T1/5}$ ϕ *-Space.

Definition 4.2

A map Ψ : (G, τ) → (H, σ) is said to be F ϕ *- open (F ϕ *-closed) if the image of every open (closed) fuzzy set in G is F ϕ *- open (closed) set in H.

Proposition 4.6

Every fuzzy open-map is fuzzy ϕ *-open map.

Proof:

The proof follows from the definition (4.2) The converse of proposition 4.5 need not be true as seen in the following example.

Example 4.1

Suppose G = {a, b, c} with fuzzy topology τ = {0,1 $\{a_{0.8}, b_{0.2}, c_{0.3}\}$}, and H ={x, y, z} with fuzzy topology σ = {0,1, $\{x_{0.5}, y_{0.2}, z_{0.7}\}$, $(x_{0.7}, y_{0.8}, z_{0.3}\}$, $\{x_{0.5}, y_{0.2}, z_{0.3}\}$, $\{x_{0.7}, y_{0.8}, z_{0.7}\}$. Let Ψ : (G, τ) → (H, σ) be defined by Ψ(a)=x, Ψ(b)=y and Ψ(c)=z. Ψ is F ϕ *- open map, but it is not a F – open map, since G = { $a_{0.8}, b_{0.2}, c_{0.3}$}∈ FO(G) but Ψ(G) ∉ F□ (H).

Proposition 4.7

Every fuzzy- closed map is F ϕ *- closed map

Proof :

The proof follows from the definition (4.2)

The converse of proposition (4.7) need not be true as seen from the following example.

Example 4.2

In example 4.1, let $\Psi : (G, \tau) \rightarrow (H, \sigma)$ be defined by $\Psi(a)=x$, $\Psi(b)=y$ and $\Psi(c)=z$. Ψ is F ϕ *- closed map, but it is not an F – closed map, since $V= \{ a_{0.2}, b_{0.8}, c_{0.7}\} \in FC(G)$ but $\Psi(V) \notin FC(H)$.

Proposition 4.8

If Let $\Psi : G \rightarrow H$ is F – closed map and $\gamma : H \rightarrow W$ is F ϕ *- closed map, then $\gamma \square \Psi : G \rightarrow W$ is F – closed map.

V. Conclusion

In this paper, we have defined a new class of fuzzy sets, namely, fuzzy ϕ*-closed sets for fuzzy topological spaces, which is properly placed in between the class of fuzzy α-closed sets and the class of fuzzy generalized α-closed sets. We also investigated some properties of these fuzzy sets. fuzzy ϕ*- continuous, fuzzy ϕ*- irresolute functions, and fuzzy ϕ*- closed (open) function have been introduced. We have proved that every Fϕ*-continuous function is Fg α- continuous, but the Converse need not be true and the composition of two Fϕ*- irresolute function is Fϕ*-irresolute. Fuzzy $T_{1/5}$-spaces and $\phi^*_{T1/5}$- spaces have been established as application of fuzzy ϕ*-closed set. In future, we will generalize this class of fuzzy set in bi-topological spaces.

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References:

[1]. Zadeh, L.A.: Fuzzy sets. Inf. Control. 8, 338–353 (1965)
[2]. Azad, K.K.: On fuzzy semi continuity, fuzzy almost continuity, and fuzzy weakly continuity. J. Math. Anal. Appl. 82(1), 14–32 (1981)
[3]. Chang.C.L.Fuzzy topological space, J.Math. Anal. App.24(1),182-190(1968)
[4]. Ravi.O and Ganesan.S "α-closed sets in topology" International Journals of computer science and engineering technology 23, Pag 330-337(2010).
[5]. Shahna, A.S.B.: On fuzzy strong semicontinuity and fuzzy precontinuity. Fuzzy Sets Syst. 44(2), 303–308 (1991)
[6]. Thakur, S.S., Singh, S.: On fuzzy semi-preopen sets and fuzzy semi-precontinuity. Fuzzy Sets Syst. 98(3), 383–391 (1998)
[7]. Balasubramanian, G., Sundaram, P.: On some generalizations of fuzzy continuous functions. Fuzzy Sets Syst. 86(1), 93–100 (1997)
[8]. El-Shafei, M.E., Zakari, A.: Semi-generalized continuous mappings in fuzzy topological space. J. Egyptian Math. Soc. 15(1), 57–67 (2007)
[9]. K. Muthukumaraswamy and R. Devi "On fuzzy α-generalized closed sets" Acta Ciencia Indica. Mathematics, 30(2004).
[10]. Abd Allah, M.A., Nawar, A.S.: "ψ*-closed sets in topological spaces" Wulfenia Journal 21(9), 391–401(2014).
[11]. Signal, M.K., Rajvansi, N.: Fuzzy alpha-sets and alpha-continuous maps. Fuzzy Sets Syst. 48(3), 383–390 (1992)
[12]. T. H. Yalvac "Semi-interior and semiclosure of a fuzzy set", J. Math. Anal. Appl.. 132(2), 356–364(1988).