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"Observance of Multiple Spring Constants in Rubber Bands following Hooke's Law"

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Abstract
In this article, we are showing that rubber band follows Hooke's law and shows band's extension is proportional to the force applied on it. It is also observed that rubber bands do not follow Hooke's law with one linearity perfectly if all data points are included for linear fit. This means that bands do not follow one linear trend while force is applied to them. In this paper, our interest is to show that rubber bands can show more than one linearity constant or spring constant following Hooke's Law. Some rubber bands were tested for their extension under limit of force applied from 0 N to 7.85 N (800 g mass) at room temperature under identical environmental and experimental conditions. It is observed that all of them follow more than one linearity under Hooks' law applicability and represents multiple spring constants. It is possible that rubber bands may have multiple spring constants based on their molecular arrangements and molecular movements under conditions of force.

Keywords: *Force, extension, spring constant, multiple proportionality constants, Hooke' law, molecular arrangement, molecular movement, Physics.*

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I. INTRODUCTION

Rubber bands are easily available material at home or stores and comes with some items as free product like produce or fruits or plastic bags. Rubber bands are the material that people use in their daily life very often for various purposes example tying objects, plastic bags or packets, tying hair or in shape of wrist bands, hair bands, toys, and elastic strings. Since people use it very often in their daily life, it can be material of interest of study. Our interest is to study nature of rubber bands following Hooke's law. In addition, we are also interested analyzing rubber bands' nature in depth to see how bands follow concept of spring constant and can be compared with springs. During COVID 19, it was hard for a high school student to go out and do some research, so, it brought our attention to do research on a material like rubber band that can be found at home easily where concept of Physics can be applied on it by doing hands on experiments at home.

Rubber bands are material that is made of polymers. Polymers are big molecules and called macromolecules those are made up of so many small molecules called monomers. The polymers are made of long chain of molecules or can be said as having a long molecule. Since rubber band has very long molecule, it is possible that the rubber band molecule tangles up easily or coiled up when they are at rest or at room temperature. So, when rubber band is stretched, the applied force makes them to untangled or uncoiled from their original shape and rubber bands shows extension while force is applied. It is very similar to a spring when a force (F) is applied to a spring, the spring gets stretched and shows an extension ($\wedge$L) and follows Hooke's law as F = -k* $\wedge$L. Similar process can be seen in rubber bands as they also follow concept of extension and follow Hooke's Law. Some work on rubber band on stretching can be seen in one of our previously published articles in European Journal of Applied Physics (EJAS). [1] This paper shows the effect of applied force on extension of variety of rubber bands following Hooke's Law.

Comparing rubber bands to springs, it can be seen in the reference [1] that when applied force is removed, the bands go back to its original shape similarly as spring behaves. The ability to stretch is called elasticity. Hence, rubber bands and spring both show elasticity. The main difference between bands and springs is that the chemical bonds that hold molecules of rubber bands together can store energy in the form of tension and allow the rubber to stretch or un-stretch and move back to its original shape, but springs have coils in their structure that extends and compresses when force is applied. Some published material related to this topic can be seen in these articles and websites. [1- 9] One of our other published articles on free fall can be seen in this reference that also shows how experiments can be performed during COVID 19 time at home. [10]

Comparing to our work published in 2021 [1], our interest in this paper is to see if further research and detailed analysis can be done on rubber bands to find out if they follow more than one linear trend. [1] This paper is focusing on depth of the nature of rubber bands those vary in their width and cross-sectional area to see how their linear trend changes within range of force and how many linear trend can be found.

Figure 1 shows the typical physical behavior of the rubber bands. This picture is taken from internet using reference (3). Typically, rubber bands are coiled with each other as they have long molecule at rest position or at room temperature but when some force is applied on them, they get stretched and show untangling and then if more force is applied, then they show complete untangle stage. These stages can be seen in sub-parts of Figure 1.

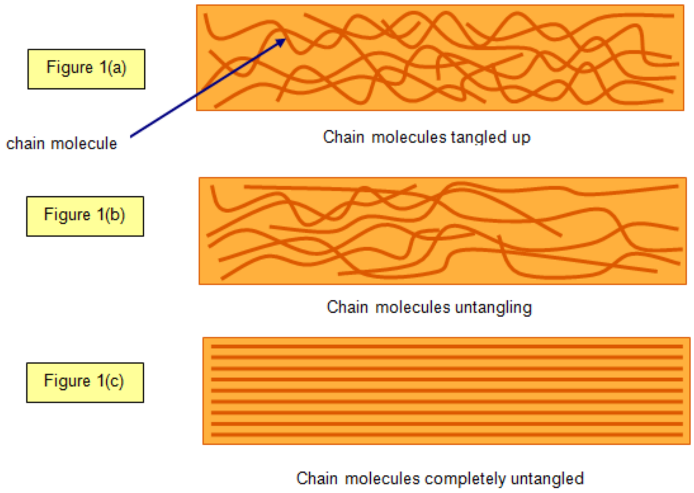

Figure 1: This image is taken from Reference (3) to show how rubber bands can tangle and untangle while force is applied.

II. THEORY AND EXPERIMENTS

Theory:

The main theory used in this experimental work is Hooke's Law. The Hooke's Law can be written in various ways and these ways are shown below. Where F is force in N, k is spring constant in N/m, ʌL (or dL) is extension is m, Lo is initial length of band in m, L is final length of the band in m, M is mass in kg, g is gravity in m/s^2, the negative sign in the equation shows that F and ʌL are opposite in nature. The negative sign can be omitted just to see numerical values as well. The basic equation of the Hooke's Law can be seen in equation (1). The form of equation used in these experiments can be seen in equation (3) and the spring constant is found using slope of the linear trend of the graph that is plotted between F and ʌL. k_avg is the average spring constant that is found by adding all spring constants (i.e., k1, k2, k3, k4) and dividing by the number of spring constants (ks), k_t is total spring constant when all data is considered as one linear trend where as k1, k2, k3, k4 are the multiple spring constants of one band based on how many linear trends they follow.

F = - k*ʌL --- 1
M*g = - k*ʌL --- 2 (where F = M*g)
M*g = k* ʌL --- 3 (where ʌL = L – Lo)
k = (M*g)/ ʌL --- 4 (k can be found from the slope of the graph plotted between F vs ʌL)
k_avg = (k1+k2+k3+k4)/4 --- 6 (average spring constant)
% error in k = (k_avg – k_t)*100/k_avg --- 7 (percent error between average and total spring constants)

Experiments:

To perform hands on experiments at home, following procedure was followed:
Five types of rubber bands were taken based on their thickness, width, cross-sectional area, and initial length, as shown in Figure 2. This figure shown the picture of all five rubber bands in the order of increasing thickness and cross-sectional area up to rubber band # 4, then rubber band # 5 is longest but have intermediate value of thickness.

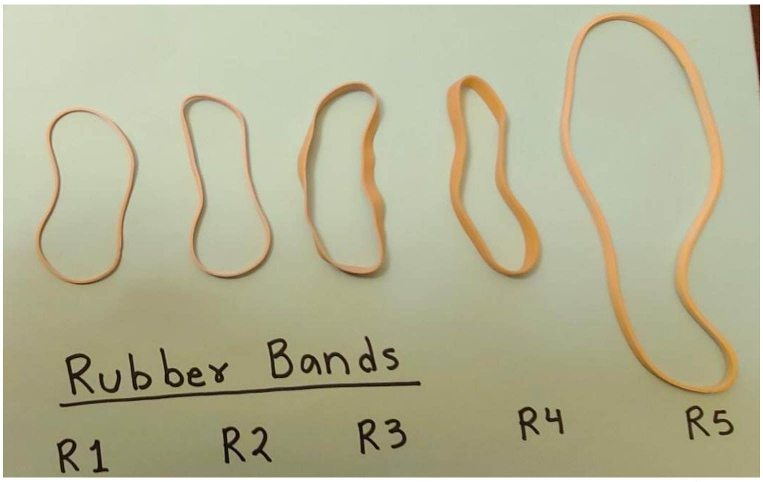

Figure 2: Picture of Five rubber bands those are used in this research work in order of increasing thickness and cross-sectional area.

Their initial length, width, thickness, and cross-sectional area were measured at the beginning of the experiments. Then each rubber band was hung on a stand and measured their initial length again. Then keep putting masses on it one by one in an increment of either 50 g or 20 g from 50 g to 800 g and recorded their respective extended length. Total, five rubber bands were tested for their extension at room temperature of 22 oC as shown in the Figure 2. At the end of the measurements, final length of all bands was measured to see if their length got changed from beginning to find if there is any deformation took place, but all rubber bands were found in their original length as no more loads were loaded beyond their limit that make them to go beyond Hooke's Law. Change in length per applied force was calculated for each band and then a graph is plotted on excel for each rubber band as Force applied on Y axis verses their extended length on X axis and slope was measured from Excel including all data in the graph. These graphs can be seen in result section. The slope shown by taking all data at once is called total spring constant, k_t and similar graphs are plotted for all bands from R1-R5 following Hooke's law. But it was observed that not a single rubber band shows single linear data. It seems that each band shows more than one trend of linearity and that made us curious to study these bands more in depth. Since Excel doesn't give an option to do linear fit of the obtained data in parts, we used Logger Pro to analyze these data in depth and more detailed results are found which can be seen in the result section.

III. RESULTS

All five rubber bands were tested for their initial dimensions and then followed the methods as mentioned above to perform experiments with each band to find out their extension with corresponding applied forces. Then linear fit was found for each band by plotting graph between Force (F) and extension (∧L or dL) including all data found for extension following Hooke's law.

Data Table # 1: Details of length (L), cross-sectional area (A) and total spring constant (k_t) for five rubber bands.

Band #	L (m)	A (m^2)	k_t (N/m)
R1	0.07	2.0*10^-6	19.5
R2	0.08	3.8*10^-6	39.8
R3	0.08	12.2*10^-6	149.6
R4	0.08	17.7*10^-6	165.6
R5	0.18	4.9*10^-6	35.5

The Data Table # 1 shown above shows all initial data of all five bands where band #, L, A, and k_t means rubber band order as shown in the Figure 2, initial length (m), cross-sectional area (m^2), and total spring constant including all data of extension (N/m) for each rubber band. It is clear to see that the data table shows how spring constant of each band is varying based on size and dimension of each rubber band. It can be seen from this Data Table # 1 that thicker bands have higher spring constants than thin bands.

Figure 3 shows an excel graph plotted for rubber band # 1 as Force versus extension to show how it follows Hooke's Law of linearity and how spring constant can be found from linear fit of the data points. But one thing is noticeable here that if all data point of extension is counted for linearity, the total spring constant (k_t) is found 19.5 N/m which includes all data points those do not show single linear trend.

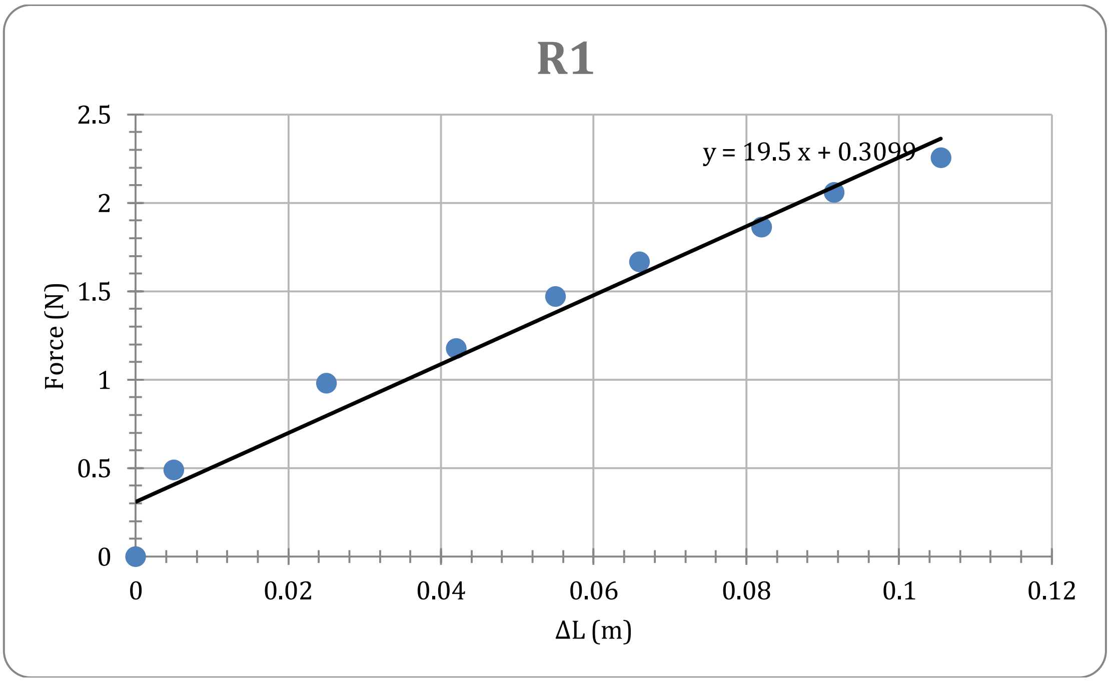

Figure 3: This graph is plotted between Force (F) and extension (ΔL) for rubber band #1. It can be seen the total spring constant (k_t) is 19.5 N/m. All data points do not follow one linear trend.

It can be seen clearly from Figure 3 that There are two different linearity this graph follows where 1st linearity is followed by first two points in the graph at the beginning and then rest of the points follow the 2nd linearity. To do detailed analysis of linear fit, Logger pro is used for the data of rubber band # 1 and replotted. Figure 4 shows a Logger Pro graph for R1 where different linearity is counted separately, and two linear fits are done as 1st and 2nd linearity and can be seen in the Figure 3 that it shows two slopes at the top which are slopes of the two linear trends for R1 as k1, k2. Then total linearity is also shown as k_t which is like the excel graph. Then, all slopes found from 1st and 2nd linearity is used to find average linearity as k_avg and then compared with total linearity k_t and then a percent error between k_avg and k_t is found which shows that rubber band # 1 follows Hooke's law but it has more than one spring constants as k1 and k2 and if we compared these two spring constants with its total spring constant we found in excel graph as Figure 2, a huge percent error comes which indicates that rubber band # 1 has more than one linearity and more than one spring constants not just one.

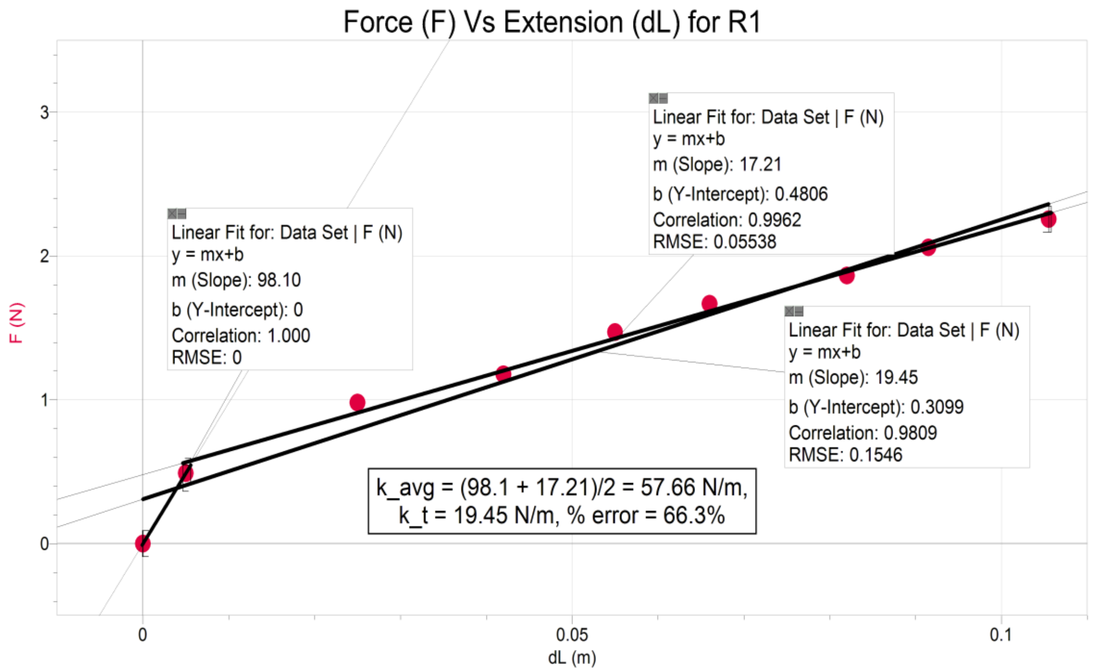

Figure 4: This graph is plotted in Logger Pro between Force (F) and extension (dL) for rubber band #1. It shows two spring constants for R1 as two separate linear fits.

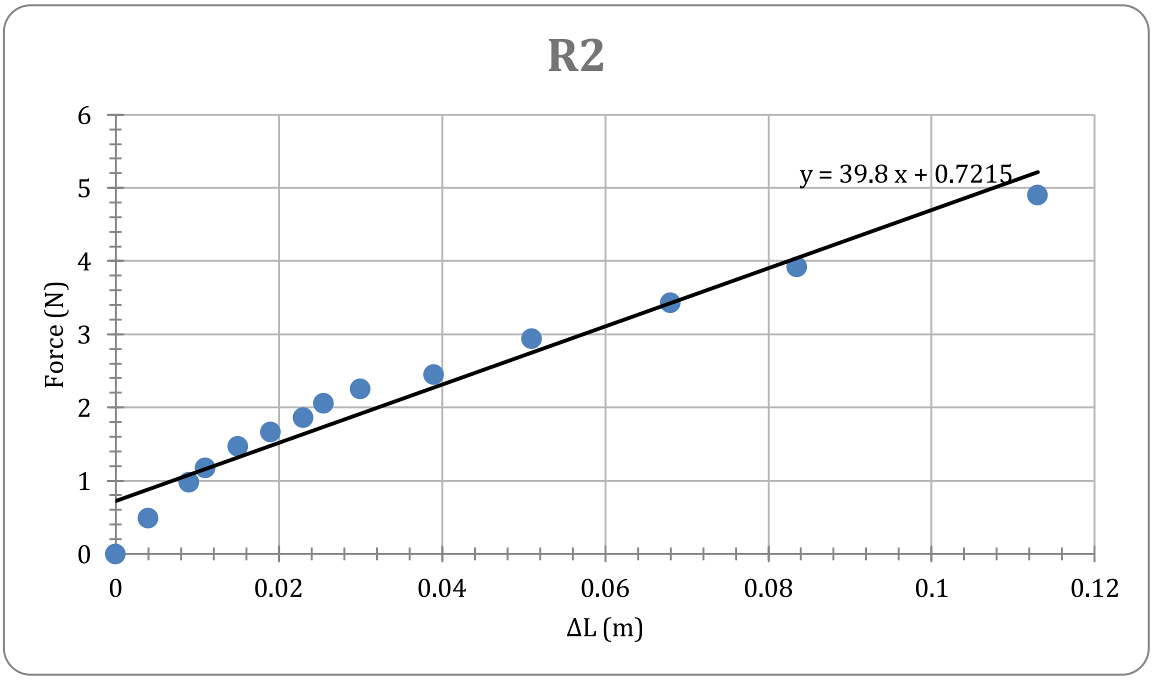

Figure 5: This graph is plotted between Force and extension for rubber band #2. It can be seen the total spring constant (k_t) is 39.8 N/m. Data points do not follow one linear trend.

Figure 5 shows total spring constant k_t for rubber band # 2. It is plotted in excel and all data points are included whereas not all data points follow one linear trend. Hence, similar type of data analysis is done using Logger pro for rubber band # 2 and graph is plotted as Figure 6. In Figure 6, three linear trends can be seen, and they are plotted separately in Figure 6 that shows three spring constants as k1, k2, k3 and k_t for all data points. Then average of all three spring constants k1, k2, k3 are taken as k_avg. Then average linearity k_avg is compared with total linearity k_t and then a percent error between k_avg and k_t is found which shows that rubber band # 2 also follows Hooke's law but it has three spring constants as k1, k2, k3 and it also shows huge percent error in between average and total spring constants. This indicates that rubber band # 2 also has more than one linearity and more than one spring constants not just one.

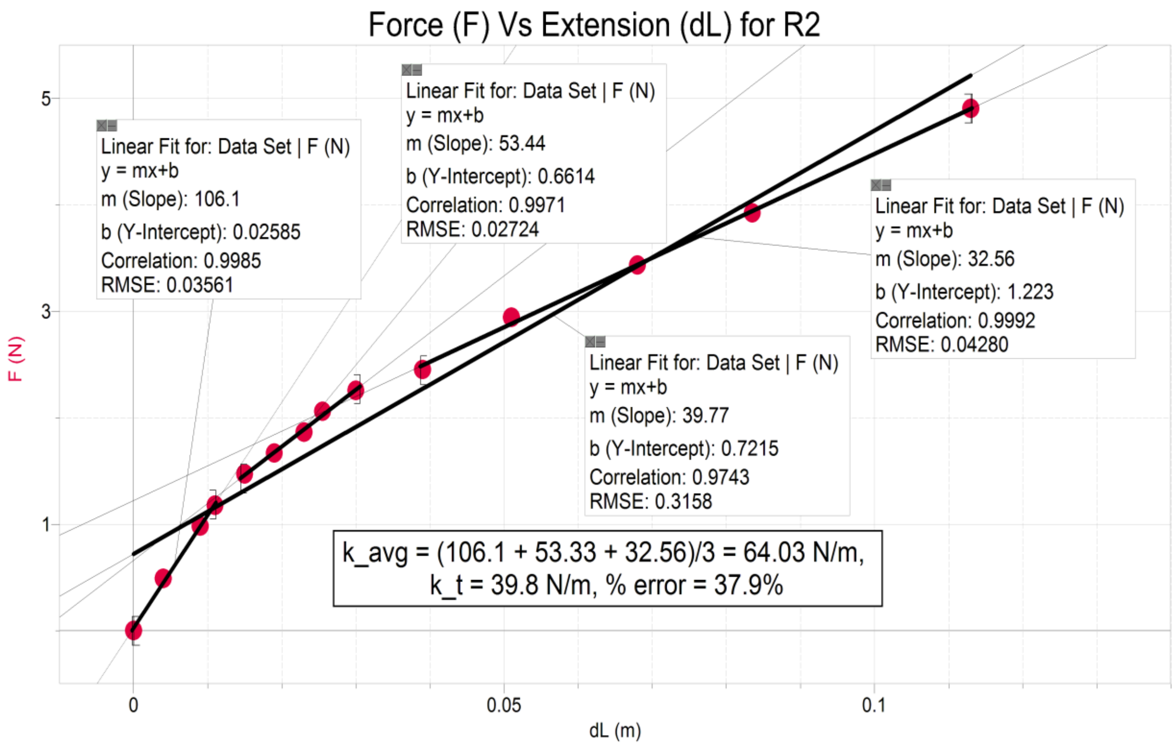

Figure 6: This graph is plotted in Logger Pro for rubber band # 2 between Force (F) and extension (dL). It shows three spring constants for rubber band # 2 as three separate linear fits.

Similar graphs are plotted for Rubber band # 3 in excel and then in logger pro and these Figures are shown as Figure 7 and Figure 8 respectively below. Rubber band # 3 has four spring constants as k1, k2, k3, k4 not just one.

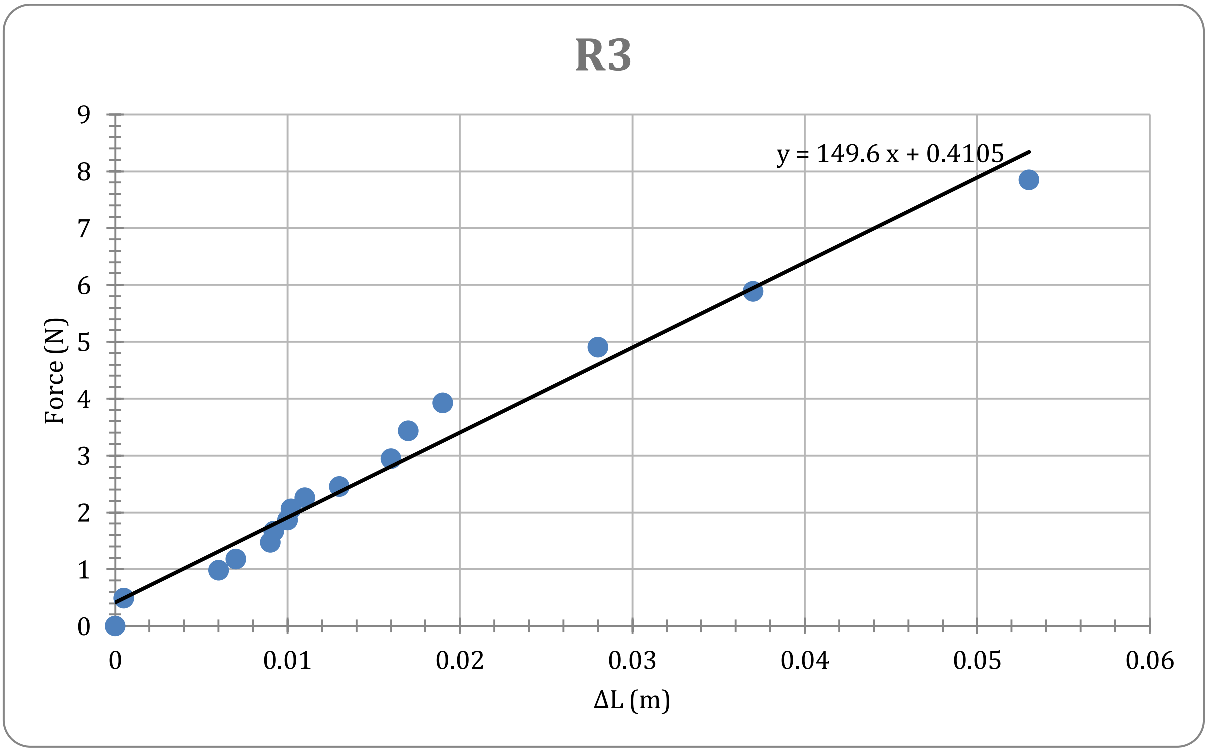

Figure 7: This graph is plotted between Force and extension for rubber band # 3. It can be seen the total spring constant (k_t) is 149.6 N/m. Data points do not follow one linear trend.

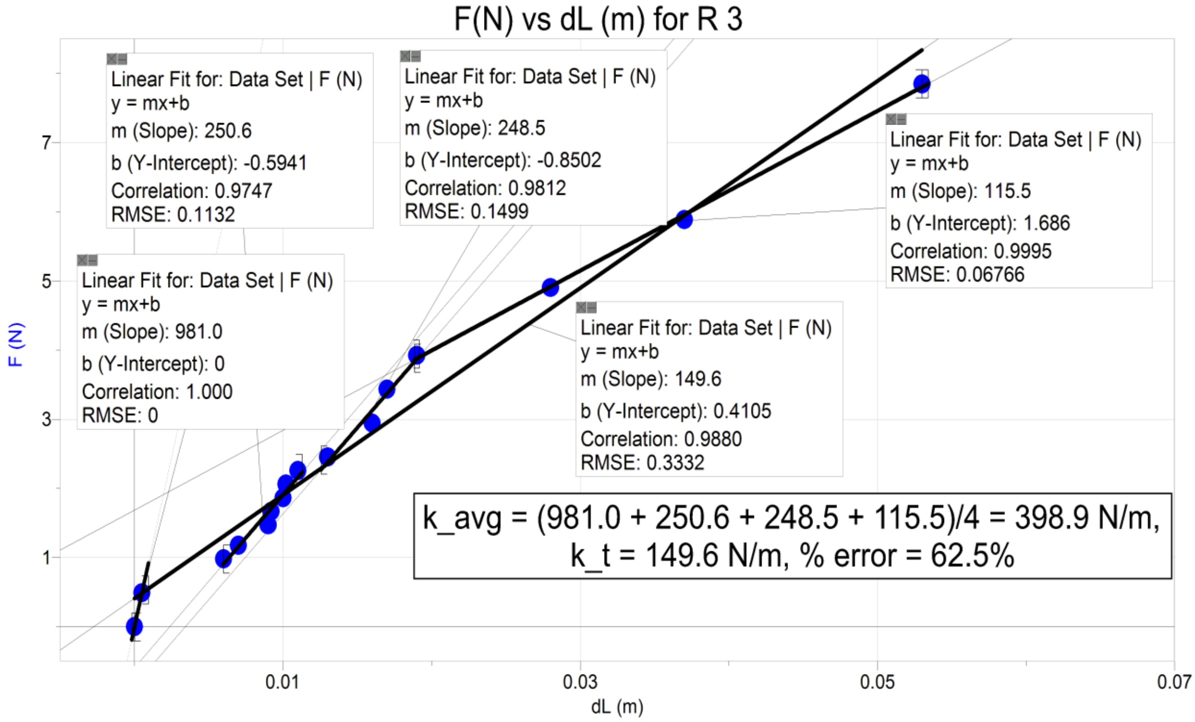

Figure 8: This graph is plotted in Logger Pro for rubber band # 3 between Force (F) and extension (dL). It shows four spring constants for rubber band # 3 as four separate linear fits.

Following same steps for Rubber band # 4, two graphs are plotted one in excel and then one in logger pro and these Figures are shown as Figure 9 and Figure 10 respectively below. It is seen that rubber band # 4 has four spring constants as k1, k2, k3, k4 not just one.

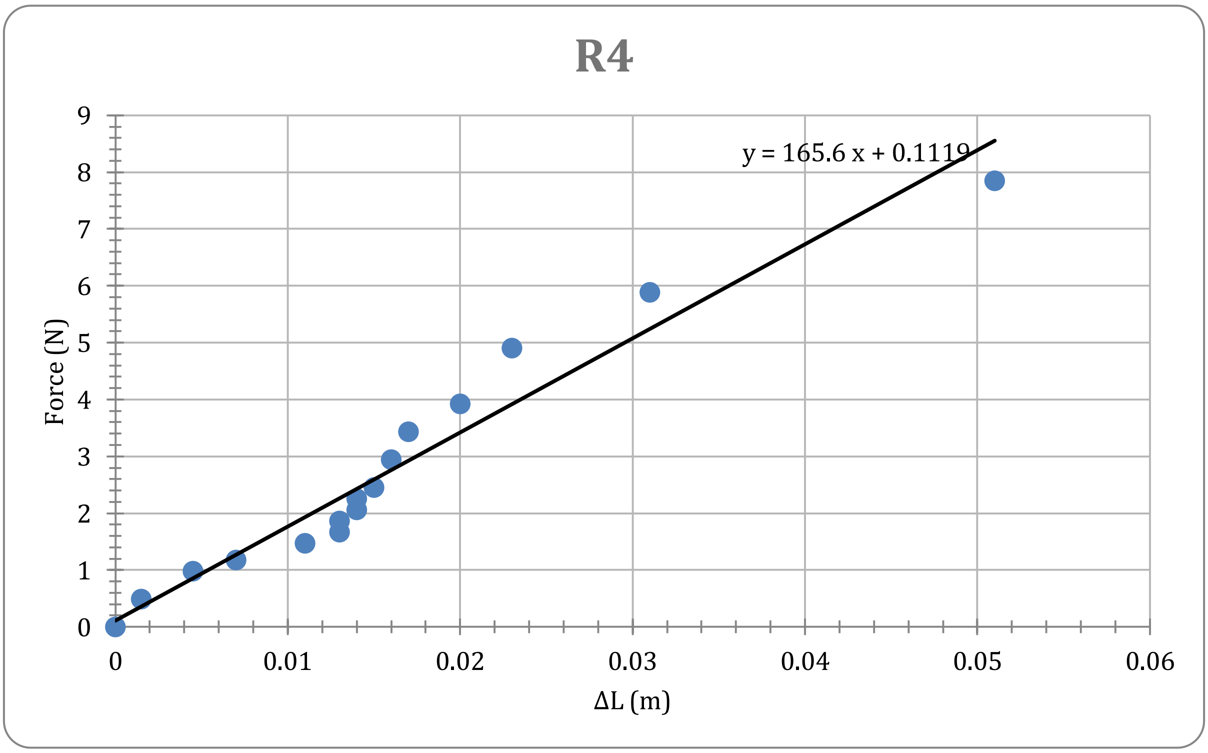

Figure 9: This graph is plotted between Force and extension for rubber band # 4. It can be seen the total spring constant (k_t) is 165.6 N/m. Data points do not follow one linear trend.

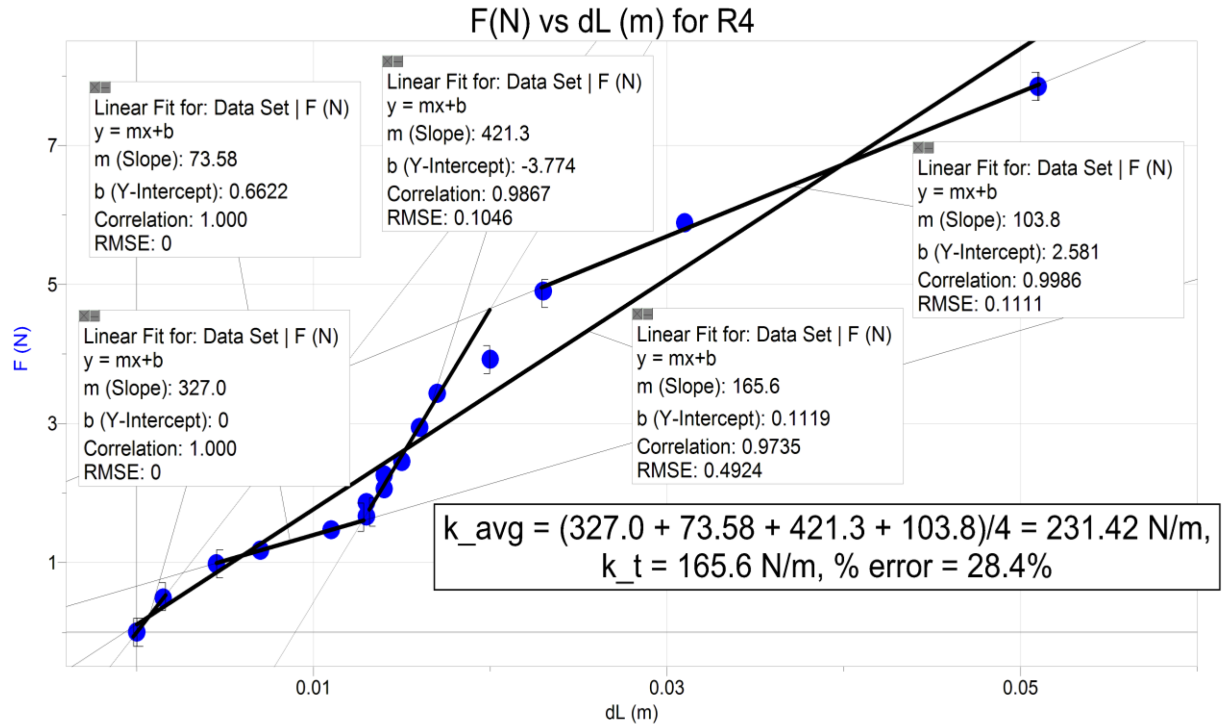

Figure 10: This graph is plotted in Logger Pro for rubber band # 4 between Force (F) and extension (dL). It shows four spring constants for rubber band # 4 as four separate linear fits.

Now the last rubber band # 5 is used which is longest but has intermediate thickness. Figure 11 shows k_t for band # 5 including all data points and rubber band # 5 has k_t = 35.5 N/m and doesn't follow one linear trend as all other bands do.

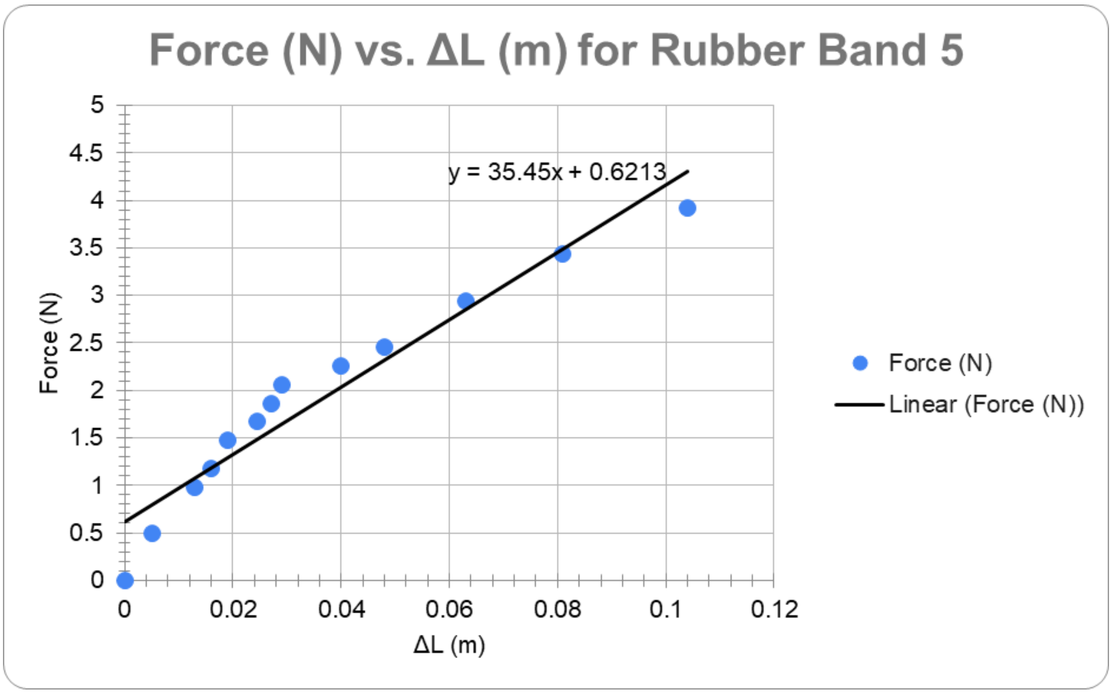

Figure 11: This graph is plotted for rubber band # 5. It has total spring constant (k_t) is 35.45 N/m. Data points do not follow one linear trend.

The data found for rubber band 5 are then analyzed using logger pro in depth to see how many linear trends it follows, and it can be seen in Figure 12 that it follows three linear trend and has three spring constants.

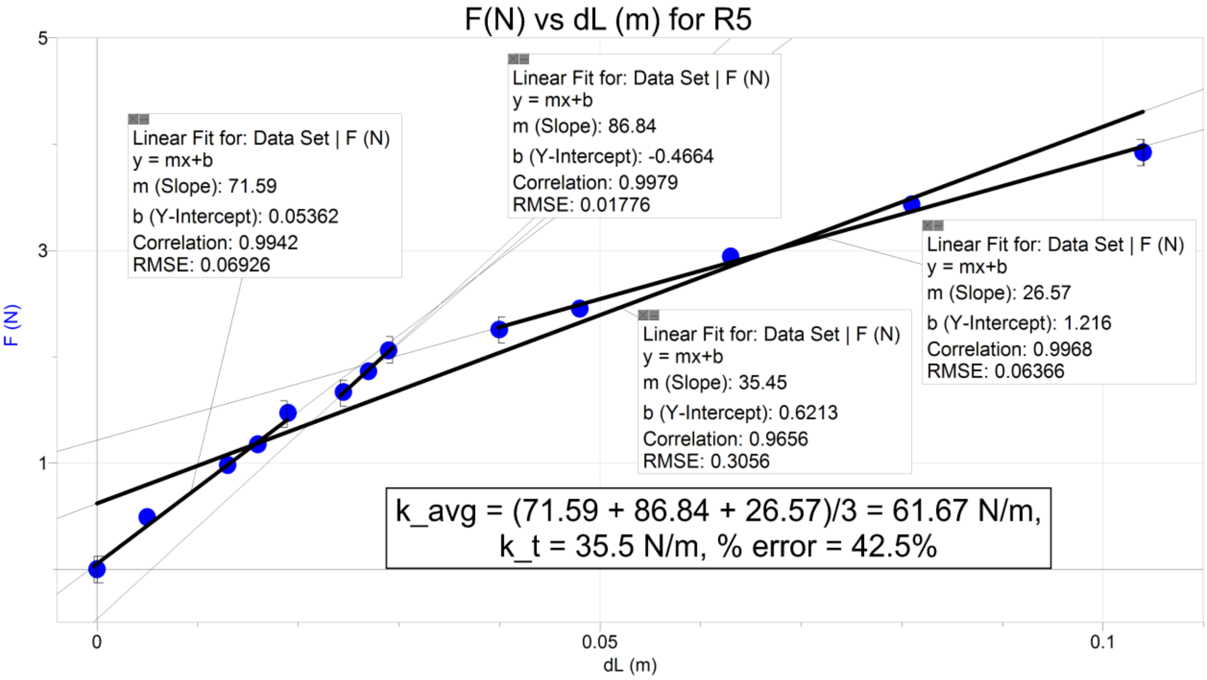

Figure 12: This Logger Pro graph, plotted for rubber band # 5 shows three separate linear fits and three spring constants in this band.

IV. DISCUSSION

Based on graphs shown above, it is clear that all rubber bands from R1-R5 show multiple linear trends not just one and follows multiple spring constants. The thickness also play important role to each rubber band and give them space to stretch more differently as thickness increases from R1-R5. Thickness and cross-sectional area increased from R1-R4 but R5 has similar thickness and cross-sectional area to R2 and R5 is the longest. After all these detailed analyses, it can be said that all rubber bands follow Hooke's Law, but they do not show one linear trend. Instead of that, they follow multiple linear trend and show multiple spring constants. R1 has two spring constants as k1, k2. R2 and R5 have three spring constants as k1, k2, and k3. R3 and R4 have four which is maximum number of spring constants as k1, k2, k3 and k4. These details can be seen in Data Table # 2.

Data table # 2 shows details of number of spring constants (# ks) for each rubber band as k1-k4. It also shows the average spring constant (k_avg), total spring constant (k_t) and percent error between k_avg and k_t is. It also shows the width (w) of each band. The thinnest rubber band (R1) has only two spring constants. The thickest rubber bands (R3, R4) have four spring constants. The rubber bands that have intermediate thickness (R2, R5), show three spring constants.

Data Table # 2: Details of Multiple Spring Constants for Five Bands:

Band #	k1 (N/m)	k2 (N/m)	k3 (N/m)	k4 (N/m)	k_avg (N/m)	k_t (N/m)	% error	# ks	w (m)
R1	98.1	17.21	*	*	57.66	19.45	66.3	2	1.8*10^-3
R2	106.1	53.33	32.56	*	64.03	39.77	37.9	3	3.2*10^-3
R3	981	250.6	248.5	115.5	398.9	149.6	62.5	4	6.8*10^-3
R4	327	73.58	421.3	103.8	231.42	165.6	28.4	4	6.8*10^-3
R5	71.59	86.84	26.57	*	61.67	35.5	42.5	3	3.5*10^-3

As thickness is increasing number of spring constants are increasing. The first spring constant (k1) for each rubber band is always highest and shows that rubber band behaves stiffest in this range. It can be explained from the Figure 1(a) to Figure 1(b) as bands stay coiled when they have no force applied and as soon as force is applied, they start uncoiling and it takes more force and gives less extension and that is why k1 is always highest for every rubber band. After first stretch, the band starts uncoiling and then after when more force is applied, they show their significant extension with increase of force and hence the 2^{nd} spring constant for each rubber band (k2) is smaller than first spring constant (k1).

As thickness of the band increases from R2 -R5, the band's molecular structure follows different trend of extending. Figure 13 shows a typical molecule of a natural rubber band. A natural rubber band is an organic molecule that is made with several atoms and contains C-H group. It is a very long molecule, a polymer. As rubber band gets thicker, it means the matter in that band increases and hence number of molecules also increases. Since rubber band is made of a very big molecule and contains C-H group, it shows more stretchability and stretches differently under different range of forces. For thicker bands (R2, R5), three spring constants are observed. Third spring constant of rubber bands (k3) is smaller than k1 and k2 which indicates that as band gets used to bear more force under its limit of extension, it starts extending more and more than how much force is applied and show higher flexibility with lower spring constants.

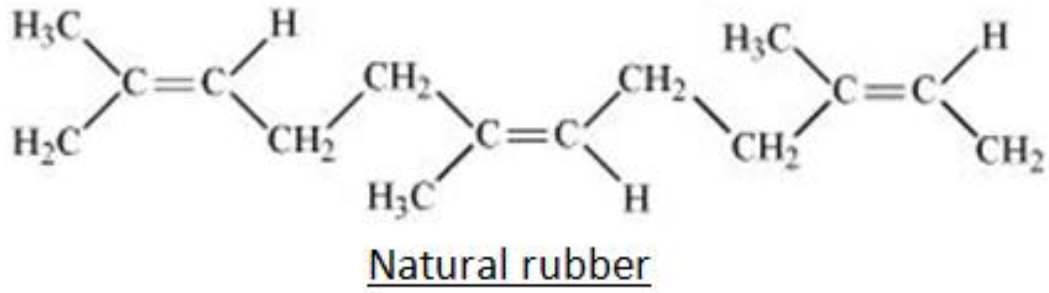

Figure 13: This figure shows a picture of a molecule of a natural rubber band that is made of C-H group and is a polymer as well as an organic molecule. This picture is taken from reference (6).

For thickest rubber bands (R3, R4), bands contain more molecules and when more force is applied, it shows more extension and extension goes faster than the rate how force is increased and hence the fourth spring constant (k4) shows least value in terms of highest flexibility. When all forces applied on all bands are removed, the structure of rubber band goes back to coiled state and hence it follows the same initial length of the rubber band that each band has at the beginning.

When spring constant is calculated as total spring constant (k_t) including all data points during experiment whether all data points follow linear trend or not, huge error in slope can be seen in between average spring constant and total spring constant for each rubber band.

V. CONCLUSION

This work shows extensive data analysis for variety of rubber bands based on their thickness and length that can be found at home easily or can be bought easily from stores to analyze and see how hands on experiments can be done at home using knowledge of Physics during COVID 19 time. In this work, five rubber bands are tested for their extension by applying force from 0 N to 7.85 N (0 g to 800 g) to find their spring constant using Hooke's Law under same environmental and experimental conditions. It is found that each band follows Hooke's Law and show spring constant in the form of slope in the graph that is plotted force verses extension. But it is also observed that each rubber band follows multiple linear trends not just one. Based on thickness of rubber bands, as thickness increases, the rubber band follows more linear trends and show more spring constants. The minimum number of spring constants can be seen as two and the maximum number of spring constants can be seen as four. The explanation can be given for multiple spring constants for each band based on molecular structure of rubber band. The rubber band molecule is made of polymer, a long chair organic compound with C-H group. As rubber bands get thicker, the number of molecules in the band increases and let the band expand more and more with more amount of force in different fashion and follow different linear fits and show multiple spring constants.

ACKNOWLEDGEMNET

This experimental work is done by a high school student during Pandemic time of COVID 19. The goal of this work is to see how some hands-on experiments can be conducted at home with the materials found easily at home during COVID 19 time and how the concepts of Physics can be used in it. Dr. Sharma supervised student for data analysis and writing this journal article.

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