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Effects of Relativistically Degenerate Electrons and Positrons on Ion- Acoustic Double Layers in Warm Ion Plasma

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Abstract: *Ion- acoustic double layers are studied in a plasma having relativistically degenerate electrons and positrons, and nondegenerate warm ions using pseudo potential approach. Expression of Sagdeev potential is derived and from the nonlinear equation the solution of double layers are obtained. The profiles of double layers are drawn and discussed for different parameters in degenerate plasma. It is seen that double layers are compressive in nature and the presence of relativistically degenerate electrons and positrons has significant contribution on the excitation of double layers in relativistically degenerate plasma. The results on double layers are new which are not obtained by any author in degenerate (or nondegenerate) plasma consisting of non-Maxwellian (or Maxwellian) electrons and positrons.*

Keywords: Ion acoustic wave . Relativistically degenerate electrons and positrons , Pseudo potential method, Double layers.

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I. INTRODUCTION

Nonlinear waves in electron-ion-positron (e-i-p) plasma behave differently in the plasma consisting of electrons and ions. Due to relevance on the wave processes in galactic nuclei [1], pulsar magnetosphere [2], polar caps of neutron stars [3] etc. many authors have considered e-i-p plasma for their studies on nonlinear propagation of waves [4-8]. The relativistic effects on solitary waves in e-i-p plasma is found to be interesting. Following the works of Das and Paul [9], Mushtaq and Shah [10], Gill et al. [11] other authors [12-18] have shown that both positrons and relativistic factor causes to increase the amplitude of soliton and to shrink the width of soliton. But in the plasmas, where the density is quite high and temperature is very low, the thermal de-Broglie wavelength may become comparable to the interparticle distances. In such situations, quantum effects become important due to the overlapping of wave functions of the neighbouring particles. The quantum effects can significantly modify the linear and nonlinear characteristics of waves in quantum (degenerate) plasma [19-23]. Considering an unmagnetized two-species relativistic quantum plasma system comprised of electrons and ions Sahu [24] have derived the Korteweg–de Vries (dKdV) equation by reductive perturbation method using one-dimensional quantum hydrodynamic model (QHD).

But, in relativistically-degenerate dense plasma, propagation of waves are generally studied following the works of Chandrashekhar [25-26] instead of Das and Paul [9] . The degenerate electron equation of state suggested by Chandrasekhar is $P_e \propto n_e^{5/3}$ for the non-relativistic limit, and $P_e \propto n_e^{4/3}$ for the ultra-relativistic limit, where P_e is the degenerate electron pressure and n_e is the degenerate electron number density. Shah et al [27] have undertaken investigation on the effect of trapping on the formation of solitary structures in relativistic degenerate (RD) plasmas. They have used the relativistic Fermi-Dirac distribution to describe the dynamics of the degenerate trapped electrons by solving the kinetic equation. The Sagdeev potential approach has been employed to obtain the arbitrary amplitude solitary structures both when the plasma has been considered cold and when small temperature effects have been taken into account. The theoretical results obtained have been analyzed numerically for different parameter values, and the results have been presented graphically. Masood and Eliasson [28] have studied electrostatic solitary waves in a degenerate (quantum) plasma with relativistically degenerate electrons and cold ions. The inertia is given by the ion mass while the restoring force is provided by the relativistic electron degeneracy pressure, and the dispersion is due to the deviation from charge neutrality. A nonlinear Korteweg–de Vries (KdV) equation is derived for small but finite amplitude waves and is used to study the properties of localized ion acoustic solitons for parameters relevant for dense astrophysical objects such as white dwarf stars. Later, Chandra et al. [29] have studied electron-acoustic solitary waves in a relativistically degenerate quantum plasma with two-temperature electrons using the QHD model and degenerate electron pressure $P_e \propto n_e^{4/3}$ .They have shown that degeneracy parameter significantly influences the conditions of formation and properties of solitary structures.

Since double layers in relativistically degenerate plasma has not been studied , we are interested here to study the ion-acoustic double layers (IADLs) in a unmagnetized nonlinear plasma with relativistically degenerate electrons and positrons and nondegenerate warm ions using pseudo potential approach. We have derived the expression of Sagdeev potential and from the nonlinear equation we have obtained the double layer solution of ion acoustic wave in such plasma. The profiles of IADLs are drawn and discussed for different values of the plasma parameters. The effect of degenerate electrons and positrons on the IADLs are shown graphically.

II. FORMULATION

We consider an unmagnetized plasma whose constituents are relativistically degenerate electrons and positrons,and nondegenerate warm ions. The dynamics of such plasma are governed by the following equations,

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i u_i)=0 \tag{1}$$

$$\frac{\partial u_i}{\partial t}+\frac{1}{2}\frac{\partial u_i^2}{\partial x}+\frac{\partial \varphi}{\partial x}+\frac{\sigma_i}{2}\frac{\partial n_i^2}{\partial x}=0 \tag{2}$$

$$en_e\frac{\partial \varphi}{\partial x}-\frac{\partial P_e}{\partial x}=0 \tag{3}$$

$$en_p\frac{\partial \varphi}{\partial x}+\frac{\partial P_e}{\partial x}=0 \tag{4}$$

$$\frac{\partial^2 \varphi}{\partial x^2}-\beta n_e+n_i+\alpha n_p=0 \tag{5}$$

Where,

$\alpha = n_{p0}/\left|Z_i\right| n_{i0}, \beta = n_{e0}/\left|Z_i\right| n_{i0}, \sigma_i = 3T_i/T_{Fe}$, All other physical quantities are dimensionless. The number density of electrons, ions positrons are represented by n_e ,n_i and n_p respectively and rescaled by their equilibrium values. u_i is the ion velocity, T_i and T_{Fe} are the ion temperature and electron Fermi temperature respectively. Other parameters have their usual meanings. The equilibrium condition of charge neutrality is $\beta = 1 + \alpha.$

Following Chandrasekhar [25,26] the degeneracy pressure of electron and positron in fully degenerate and relativistic configuration can be expressed in the form:

$$P_j = \left(\pi m_e^4 c^5 / 3h^3\right)\left[R_j(2R_j^2 - 3)\sqrt{1+R_j^2} + 3\sinh^{-1}(R_j)\right] \tag{6}$$

in which $R_j = p_{Fj}/m_e c = \left[3h^3 n_j / 8\pi m_e^3 c^3\right]^{\frac{1}{3}} = R_{j0} n_j^{\frac{1}{3}}$ (7)

where $R_{j0} = \left(n_{j0}/n_0\right)^{1/3}$ with $n_0 = 8\pi m_e^3 c^3 / 3h^3 \approx 5.9\times10^{29} cm^{-3}$, ‘c’ being the speed of light in vacuum. p_{Fj} is the electron Fermi relativistic momentum; the index j will denote either electrons (e) or positrons (p) in the plasma. It is to be noted that the parameter R_{j0} is a measure of the relativistic effects and may be called relativistic degeneracy parameter. For ultra- relativistic case $R_{j0} >> 1$ and for weakly relativistic case $R_{j0} << 1$. The parameter R_{j0} can also be related to mass density as $\rho(gr/cm^3) = 1.97\times10^6 \cdot R_{j0}^3$. The density of white dwarfs can be in the range $10^5 < \rho < 10^9$. So in this case the relativity parameter R_{j0} can be in the range $0.37 < R_{j0} < 8$.

In the limits of very small and very large values of relativity parameter R_j, we obtain:

$$P_j = \frac{1}{20}\left(\frac{3}{\pi}\right)^{\frac{2}{3}} \frac{h^2}{m_e} n_j^{\frac{5}{3}} \quad \text{(For } R_j \to 0\text{)}, \tag{8}$$

$$P_j = \frac{1}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}} hc n_j^{\frac{4}{3}} \quad \text{(For } R_j \to \infty\text{)} \tag{9}$$

Note that the degenerate electron (positron) pressure depends only on the electron (positron) number density but not on the temperature. Now, from Eqs.(1) - (9) the densities of degenerate electrons and positrons are obtained as

$$n_e = R_{e0}^{-3}[R_{e0}^2 + 2\varphi(1+R_{e0}^2)^{1/2} + \varphi^2]^{3/2} \tag{10}$$

$$n_p = p^{-1}R_{e0}^{-3}[p^{2/3}R_{e0}^2 - 2\varphi(1+p^{2/3}R_{e0}^2)^{1/2} - \varphi^2)]^{3/2} \tag{11}$$

where, $p = n_{p0}/n_{e0} = \alpha/\beta$.

To obtain localized solutions of Eqs. (1) –(5) using the Sagdeev potential formalism [30] we consider a new moving frame of a single variable $\eta = x - Mt$, which is the moving frame with velocity (Mach number) M. Using this transformation and equilibrium quasi-neutrality condition as well as the boundary conditions i) $n_i \to n_{i0}$, ii) $u_i \to 0$, iii) $\varphi \to 0$ at $|x| \to \infty$ we obtain from Eqs.(1) and (2) the non-degenerate ion density as

$$n_i = \frac{1}{2\sqrt{\sigma_i}}[\{(M+\sqrt{\sigma_i})^2 - 2\varphi\}^{1/2} \pm \{(M-\sqrt{\sigma_i})^2 - 2\varphi\}^{1/2}] \tag{12}$$

Now, using Eqs.(10),(11) and (12) in Eq.(5) we get

$$
\begin{aligned}
\frac{d^2\varphi}{d\eta^2} &= \beta R_{e0}^{-3}\left[R_{e0}^2 + 2\varphi(1+R_{e0}^2)^{1/2} + \varphi^2)\right]^{3/2} \\
&-\alpha p^{-1} R_{e0}^{-3}\left[p^{2/3}R_{e0}^2 - 2\varphi(1+p^{2/3}R_{e0}^2)^{1/2} - \varphi^2)\right]^{3/2} \\
&-\frac{1}{2\sqrt{\sigma_i}}[\{(M+\sqrt{\sigma_i})^2 - 2\varphi\}^{1/2} - \{(M-\sqrt{\sigma_i})^2 - 2\varphi\}^{1/2}] = -\frac{d\psi}{d\varphi}
\end{aligned}
\tag{13}
$$

(Taking negative branch of ion density).

Where, ψ is the Sagdeev potential and it is given by

$$
\begin{aligned}
\psi &= -\int \beta R_{e0}^{-3}\left[R_{e0}^2 + 2\varphi\sqrt{1+R_{e0}^2} + \varphi^2\right]^{3/2} d\varphi \\
&+\int \alpha p^{-1} R_{e0}^{-3}\left[p^{2/3}R_{e0}^2 - 2\varphi\sqrt{1+p^{2/3}R_{e0}^2} + \varphi^2)\right]^{3/2} d\varphi \\
&+\int\left[\frac{[(\sigma_i+M^2)/2-\varphi)] - \sqrt{[(\sigma_i+M^2)/2-\varphi]^2 - \sigma_i M^2}}{\sigma_i}\right]^{1/2} d\varphi
\end{aligned}
\tag{14}
$$

After simplification ψ becomes

$$
\begin{aligned}
\psi &= \frac{\beta_1}{8}\left[-2(\varphi+\gamma_1)(\varphi^2+2\varphi\gamma_1+\gamma_1^2-1)^{3/2} + 3(\varphi+\gamma_1)(\varphi^2+2\varphi\gamma_1+\gamma_1^2-1)^{1/2} - 3\ln\{\frac{(\varphi+\gamma_1)(\varphi^2+2\varphi\gamma_1+\gamma_1^2-1)^{1/2}}{(R_{eo}+\gamma_1)}\}\right] \\
&+\frac{\alpha_1}{8}\left[2(\varphi-\gamma_2)(\varphi^2-2\varphi\gamma_2+\gamma_2^2-1)^{3/2} - 3(\varphi-\gamma_2)(\varphi^2-2\varphi\gamma_2+\gamma_2^2-1)^{1/2} + 3\ln\{\frac{(\varphi-\gamma_2)(\varphi^2-2\varphi\gamma_2+\gamma_2^2-1)^{1/2}}{(p^{1/3}R_{eo}-\gamma_2)}\}\right] \\
&+\beta_2\gamma_1(2R_{eo}^2+3) + \alpha_2\gamma_2(2p^{2/3}R_{eo}^2-3) + (M^2+\frac{\sigma_i}{3}) - \frac{2^{1/2}\sigma_i}{3}(M_1-M_2)^{1/2}(2M_1+M_2)
\end{aligned}
\tag{15}
$$

where

$\gamma_1 = (1+R_{e0}^2)^{1/2}, \gamma_2 = (1+p^{2/3}R_{e0}^2)^{1/2}, \beta_1 = \beta/\mathrm{R}_{eo}^3, \alpha_1 = \alpha/pR_{eo}^3, \beta_2 = \beta/8\mathrm{R}_{eo}^2, \alpha_2 = \alpha/8p^{2/3}R_{eo}^2,$
$M_1 = 1+M^2/\sigma_i - 2\varphi/\sigma_i, M_2 = (M_1^2 - 4M^2/\sigma_i)^{1/2}, \beta = 1+\alpha, p = \alpha/\beta.$

Now, integrating (14) we obtain an equation (the Energy law) in terms of a single independent variable

$$
\frac{1}{2}(\frac{d\varphi}{d\eta})^2 + \psi(\varphi) = 0 \tag{16}
$$

where ψ is the Sagdeev potential and we have used the boundary conditions $\varphi \to 0, d\varphi/d\eta \to 0$ and $d^2\varphi/d\eta^2 \to 0.$

Eq. (16) describes the motion of a pseudoparticle of unit mass with velocity $d\varphi/d\eta$ and position φ in a potential $\psi(\varphi)$. The first term on the right-hand side of Eq. (16) can be regarded as the kinetic energy of the pseudo particle. Since the kinetic energy is always nonnegative, it follows that $\psi(\varphi) \le 0$ for the entire motion, zero being the maximum value of $\psi(\varphi)$. Eq. (16) shows that $d\psi/d\varphi$ is the force on the particle at the position φ . Eq. (16) 8may also be regarded as an anharmonic oscillator equation, provided that we interpret φ and η as space and time coordinates, respectively. For the double layer solution of Eq. (13), the Sagdeev potential $\psi(\varphi)$ should be negative between $\varphi = 0$ and φ_m, where φ_m is some extremum value of the potential φ, called the amplitude of the DL. For the double layer solutions, $\psi(\varphi)$ must additionally satisfy the following boundary conditions:

(i) $\psi(\varphi) = 0, d\psi/d\varphi = 0$ and $d^2\psi/d\varphi^2 < 0$ at $\varphi = 0$, (17a)

(ii) $\psi(\varphi) = 0, d\psi/d\varphi = 0$ and $d^2\psi/d\varphi^2 < 0$ at $\varphi = \varphi_{\max}(\neq 0)$, (17b)

(iii) $\psi(\varphi) < 0,$ for $0 < |\varphi| < |\varphi_m|$ (17c)

When the above conditions are satisfied, $\varphi = 0$ is an unstable equilibrium position. The pseudo particle is not reflected at $\varphi = \varphi_m$ because the pseudo force and the pseudo velocity are vanished . Instead, it goes to another state producing an asymmetrical double layer with a net potential drop φ_m .

III. SOLUTION FOR THE DOUBLE LAYERS

Now we expand the right hand side of (13) and (14) in power series of ϕ and we obtain

$$\frac{d^2\varphi}{d\eta^2} = A_1\varphi + A_2\varphi^2 + A_3\varphi^3 + A_4\varphi^4 + \cdots\cdots \tag{18}$$

Where,

$$A_1 = \alpha R_{e0}^{-3}\left[\begin{array}{l} p[\{3(1+R_{e0}^2) - \frac{3}{8}(1+R_{e0}^2)^{-1} - \frac{3}{16}(1+R_{e0}^2)^{-2} - \frac{15}{128}(1+R_{e0}^2)^{-3}\}] \\ -p^{-1}[\{3(1+p^{2/3}R_{e0}^2) - \frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-1} - \frac{3}{16}(1+p^{2/3}R_{e0}^2)^{-2} \\ -\frac{15}{128}(1+p^{2/3}R_{e0}^2)^{-3}\}] \end{array}\right] \tag{19a}$$

$$-\frac{1}{2\sqrt{\sigma_i}}\left[\{(M+\sqrt{\sigma_i})^{-1}\} - \{(M-\sqrt{\sigma_i})^{-1}\}\right]$$

$$A_2 = \alpha R_{e0}^{-3}\left[\begin{array}{l} p[\{3(1+R_{e0}^2)^{1/2} + \frac{3}{8}(1+R_{e0}^2)^{-3/2} + \frac{3}{8}(1+R_{e0}^2)^{-5/2} + \frac{45}{128}(1+R_{e0}^2)^{-7/2}\}] \\ -p^{-1}[\{3(1+p^{2/3}R_{e0}^2)^{1/2} + \frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-3/2} + \frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-5/2} \\ +\frac{45}{128}(1+p^{2/3}R_{e0}^2)^{-7/2}\}] \end{array}\right]$$

$$-\frac{1}{4\sqrt{\sigma_i}}\left[\{(M+\sqrt{\sigma_i})^{-3}\} - \{(M-\sqrt{\sigma_i})^{-3}\}\right] \tag{19b}$$

$$A_3 = \alpha R_{e0}^{-3}\begin{bmatrix} p[\{1-\frac{3}{8}(1+R_{e0}^2)^{-2}-\frac{5}{8}(1+R_{e0}^2)^{-3}-\frac{105}{128}(1+R_{e0}^2)^{-4}\}] \\ -p^{-1}[\{1+\frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-2}+\frac{5}{8}(1+p^{2/3}R_{e0}^2)^{-3}+\frac{105}{128}(1+p^{2/3}R_{e0}^2)^{-4}\}] \end{bmatrix}$$

$$-\frac{1}{8\sqrt{g_i}}\left[\{(M+\sqrt{g_i})^{-5}\}-\{(M-\sqrt{g_i})^{-5}\}\right] \tag{19c}$$

$$A_4 = \alpha R_{e0}^{-3}\begin{bmatrix} p[\{\frac{3}{8}(1+R_{e0}^2)^{-5/2}+\frac{15}{16}(1+R_{e0}^2)^{-7/2}+\frac{105}{64}(1+R_{e0}^2)^{-9/2}\}] \\ -p^{-1}[\{\frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-5/2}+\frac{15}{16}(1+p^{2/3}R_{e0}^2)^{-7/2}+\frac{105}{64}(1+p^{2/3}R_{e0}^2)^{-9/2}\}] \end{bmatrix}$$

$$-\frac{5}{32\sqrt{\sigma_i}}\left[\{(M+\sqrt{\sigma_i})^{-7}\}-\{(M-\sqrt{\sigma_i})^{-7}\}\right] \tag{19d}$$

and

$$\psi(\varphi) = -\frac{1}{2}A_1\varphi^2-\frac{1}{3}A_2\varphi^3-\frac{1}{4}A_3\varphi^4-\frac{1}{5}A_4\varphi^5-\frac{1}{6}A_5\varphi^6-\cdots\cdots \tag{20}$$

Assuming small amplitude waves $\varphi < 1$ and taking the terms up to φ^3 of Eq.(18) we get the solution of IADLs in plasma. Applying boundary conditions (17a), (17b) and (17c) we obtain

$$\varphi_m = -\frac{2A_2}{3A_3} \tag{21}$$

and

$$\psi(\varphi) = -\frac{A_3}{4}\varphi^2(\varphi_m-\varphi)^2 \tag{22}$$

The solution of small amplitude IADLs of (18) satisfying (20) is obtained as

$$\varphi_D = \frac{1}{2}\varphi_m\left(1-\tanh[\sqrt{(A_3/8\varphi_m)}\eta]\right) \tag{23}$$

The width of IADLs is given by

$$W_{dL} = 2\left(\frac{A_3}{8|\phi_m|}\right)^{1/2} \tag{24}$$

It is seen that width of IADLs depends on the plasma parameters e.g. relativistically degenerate parameter, density of degenerate electrons, density of degenerate positrons , ion temperature etc. Eq. (21) relates the amplitude φ_m of the IADL to the ratio between the coefficients A_2 and A_3 of the quadratic and cubic terms on the right-hand side of Eq. (18). It is important to be noted that the IADLs in plasma can only exist for positive value of A_3 i.e. $A_3 > 0$. To satisfy the condition , the plasma parameters should have some critical values. The degenerate electron density β must satisfy the condition $\beta < \beta_c$, β_c is the critical value of β. For the existence of IADLs , the parameter β_c obtained from (19c) is

$$\beta_c = (p^{-1}C_2 + D)/C_1 \tag{25}$$

Where,

$$C_1 = R_{e0}^{-3}[\{1-\frac{3}{8}(1+R_{e0}^2)^{-2}-\frac{5}{8}(1+R_{e0}^2)^{-3}-\frac{105}{128}(1+R_{e0}^2)^{-4}\}$$
$$-p^{-1}\{\{1+\frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-2}+\frac{5}{8}(1+p^{2/3}R_{e0}^2)^{-3}+\frac{105}{128}(1+p^{2/3}R_{e0}^2)^{-4}\}]$$
$$C_2 = \{1+\frac{3}{8}(1+p^{2/3}R_{e0}^2)^{-2}+\frac{5}{8}(1+p^{2/3}R_{e0}^2)^{-3}+\frac{105}{128}(1+p^{2/3}R_{e0}^2)^{-4}\}]$$
$$D = \frac{1}{8\sqrt{\sigma_i}}[\{(M-\sqrt{\sigma_i})^{-5}\}-\{(M+\sqrt{\sigma_i})^{-5}\}]$$

Now, if IADLs are excited in plasma for $A_3 > 0$, we see that the sign of A_2 will determine the nature of IADLs. If A_2 is positive IADL will be rarefactive and for negative value of A_2 the IADL will be compressive. It is seen that A_2 depends on the parameters R_{e0}, α , β , p , σ_i and M .

IV. RESULTS AND DISCUSSIONS

From the solution of ion-acoustic double layers (IADLs) (21) in a relativistically degenerate electron-ion-positron plasma it seen that excitation of IADLs depends on i) relativistic parameter (R_{e0}), ii) degenerate positron density(α), iii) degenerate electron density(β) and iv) ion temperature (σ_i). The role of these parameters on the IADLs are shown in Figs. 1-4.

A) **Profiles of Ion-Acoustic Double Layers**

i) **Effect of relativistic degeneracy parameter**

To see the effects of relativistic degeneracy parameter (R_{e0}) on the IADLs in degenerate plasma having fixed values of p = 0.1, σ_i =0.01, α =0.4 , M =2; we have followed the conditions for the existence of DLs, i.e. the coefficient A_3 must be positive for the values of the plasma parameters. Moreover, to understand the nature of IADLs, the amplitude of DLs given by (21) is first numerically estimated using the values of parameters of our model plasma. It is observed from the numerical estimation that amplitude of IADLs is positive giving rise to compressive IADLs. Using (23) of the electrostatic potential of IADLs the profiles of DLs are drawn as shown in Fig.1. It is seen from Fig.1 that the coefficient A_2 is negative so compressive IADLs are excited in plasma having R_{e0} = 10.5, 12, 13.5 and 15 with fixed values of p = 0.1, σ_i =0.01, α =0.4 , M =2. It is also observed that the amplitudes of compressive IADLs are decreased with increase of R_{e0}.

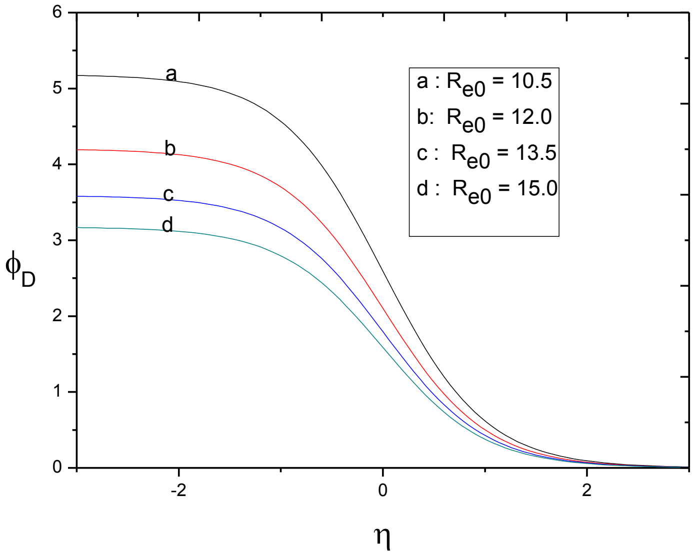

Fig. 1. Profiles of IADLs for different values of R_{e0} in degenerate plasma with fixed values of p =0.1, σ_i =0.01, α =0.4 , M =2; Curves a, b, c and d represent four values of R_{e0} = 10.5, 12, 13.5 and 15 respectively.

ii) **Effect of degenerate positrons**

Similarly, to find the effects of relativistically degenerate positron density (p) on the IADLs in degenerate plasma having fixed values of α =0.3, σ_i =0.01, M =2 , R_{e0} =10; we have followed the conditions for the existence of IADLs, i.e. the coefficient A_3 must be positive for the values of the plasma parameters. Moreover, to understand the nature of IADLs, the amplitude of DLs given by (21) is first numerically estimated using the values of parameters of our model plasma. It is observed from the numerical estimation that amplitude of IADLs is positive giving rise to compressive IADLs. Using (23) of the electrostatic potential of IADLs, the profiles of DLs are drawn as shown in Fig.2. It is observed from Fig.2 that the coefficient A_2 is negative so compressive IADLs are excited in plasma having p = 0.017, 0.021, 0.025, 0.029 with fixed values of α = 0.3, σ_i =0.01, M =2 , R_{e0} =10. It is also observed that the amplitudes of compressive IADLs are decreased with increase of p .

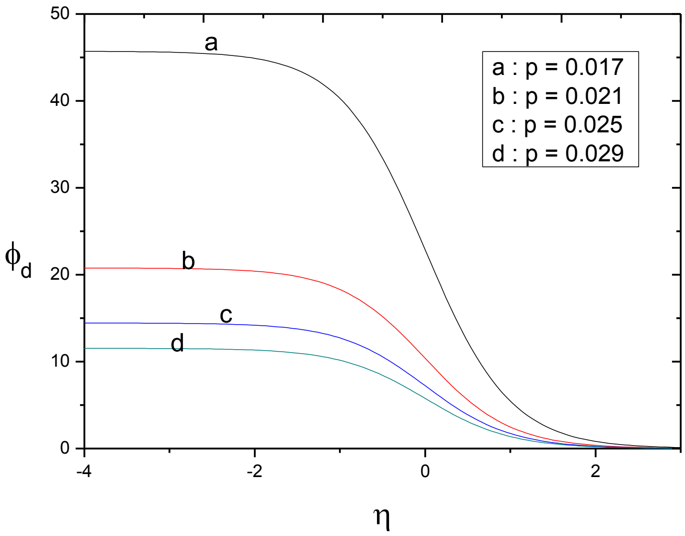

Fig. 2. Profiles of IADLs for different values of p in degenerate plasma with fixed values of α = 0.3, σ_i =0.01, M =2 , R_{e0} =10; Curves a, b, c and represent four values of p = 0.017, 0.021, 0.025, 0.029 respectively.

iii) **Effect of degenerate electrons**

In similar way , to see the effects of relativistically degenerate positron density (α) on the IADLs in relativistically degenerate plasma having fixed values of σ_i =0.01, R_{e0} =11, M =1.9, p =0.1; we have followed the conditions for the existence of DLs, i.e. the coefficient A_3 must be positive for the values of the plasma parameters. Moreover, to understand the nature of IADLs, the amplitude of DLs given by (21) is first numerically estimated using the values of parameters of our model plasma. It is observed from the numerical estimation that amplitude of IADLs is positive giving rise to compressive IADLs. Using (23) of the electrostatic potential of IADLs the profiles of DLs are drawn as shown in Fig.3. It is observed from Fig.3 that the coefficient A_2 is negative so compressive IADLs are excited in plasma having α = 0.15, 0.3, 0.45 and 0.6 with fixed values of σ_i =0.01, R_{e0} =11, M =1.9, p =0.1. It is seen that the amplitudes of compressive DLs are decreased with increase of α .

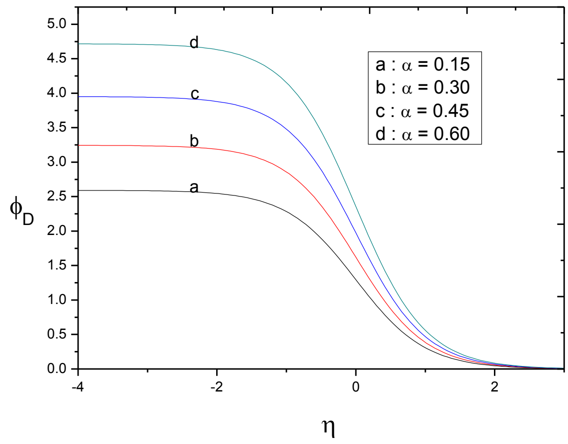

Fig. 3. Profiles of IADLs for different values of α in degenerate plasma with fixed values of σ_i =0.01, R_{e0} =11, M =1.9, p =0.1; Curves a, b, c and d represent four values of α = 0.15, 0.3, 0.45 and 0.6 respectively.

iv) **Effect of ion temperature**

Similarly , to see the effects of ion temperature (σ_i) on the IADLs in relativistically degenerate plasma having fixed values of α = 0.3, p =0.01, M =2, R_{e0} =10; we have followed the conditions for the existence of DLs, i.e. the coefficient A_3 must be positive for the values of the plasma parameters. Moreover, to understand the nature of IADLs, the amplitude of DLs given by (21) is first numerically estimated using the values of parameters of our model plasma. It is observed from the numerical estimation that amplitude of IADLs is positive giving rise to compressive IADLs. Using (23) of the electrostatic potential of IADLs the profiles of DLs are drawn as shown in Fig.4. It is observed from Fig.4 that the coefficient A_2 is negative so compressive IADLs are excited in plasma having σ_i = 0.3, 0.4, 0.5 and 0.6 with fixed values of α = 0.3, p =0.01, M =2, R_{e0} =10. It is seen that the amplitudes of compressive DLs are decreased with increase of σ_i .

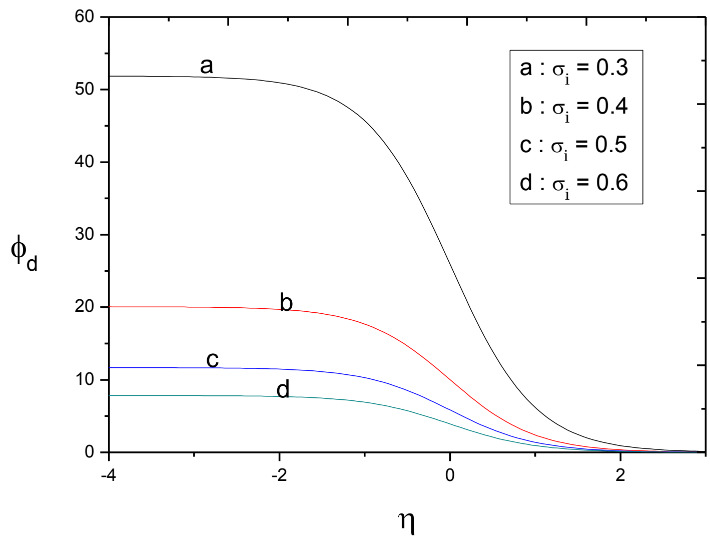

Fig. 4. Profiles of IADLs for different values of σ_i in degenerate plasma with fixed values of α = 0.3, p =0.01, M =2, R_{e0} =10. Curves a, b, c and d represent four values of σ_i = 0.3, 0.4, 0.5 and 0.6 respectively.

B). **Width of Ion-Acoustic Double Layers**

It is known that the width (or thickness) of DLs is important for the study of DLs in plasma because it is important for experimental studies in laboratory and in space plasma. We have numerically estimated the width of IADLs using (24) and its variation for different parameters of an ion beam plasma is graphically discussed. It is to be remembered that for the numerical estimation we have followed the conditions for the existence of IADLs (i.e. A_3 must be positive).

Now, we have plotted the thickness of IADLs with the relativistic degenerate parameter (R_{e0}) in plasma for different values of relativistically degenerate positron density (p) shown in Fig.5. It is seen that the width of IADLs increases with increase of R_{e0} for p = 0.01, 0.014, 0.016, 0.018 for the fixed values of σ_i =0.01, α =0.12, M =2. More over, the thickness of Dl increases with increase of p .

To see the effect of relativistically degenerate electrons (α) and relativistic degenerate parameter (R_{e0}) we have plotted Fig.(6). It is seen that the width of IADLs increases with increase of α for R_{e0} =9, 11, 13, 15 for the fixed values of σ_i =0.01, p =0.1, M =2. More over, the thickness of DL increases with increase of R_{e0} .

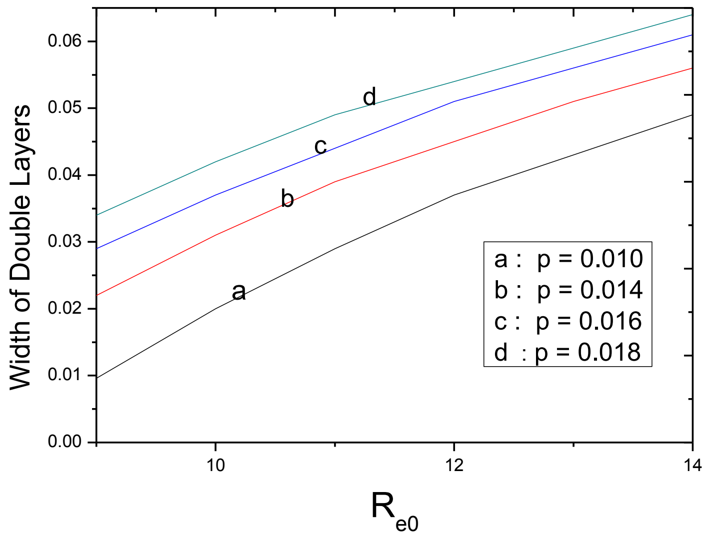

Fig.5. Variation of Width of IADL with R_{e0} for different values of p in degenerate plasma with fixed values of σ_i = 0.01, α =0.12, M =2; Curves a, b, c and d represent four values of p = 0.01, 0.014, 0.016 and 0.018 respectively.

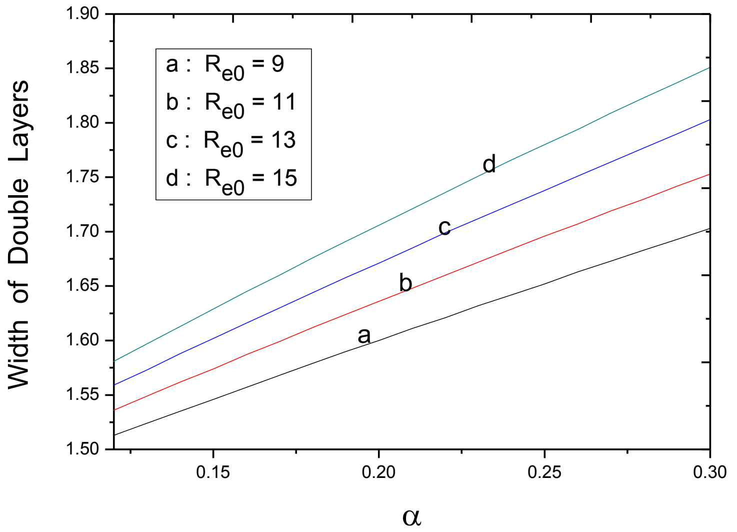

Fig.6. Variation of width of IADLs with α for different R_{e0} in degenerate plasma with fixed values of σ_i =0.01, p =0.1, M =2; Curves a, b, c and d correspond to four values of R_{e0} =9, 11, 13 and 15 respectively.

V. CONCLUSIONS

Considering a relativistically degenerate plasma composed of degenerate electrons and positrons with nondegenerate warm ions we have theoretically studied the excitation of ion- acoustic double layers (IADLs) in the plasma using the pseudo-potential method. It has been shown that the IADLs in such plasma depends on the plasma parameters. From numerical estimation the profiles of IADLs are drawn. The IADLs in relativistically plasma are compressive and the amplitudes depend on the values of the plasma parameters.

Our main findings are :

i) The IADLs in relativistically degenerate e-i-p plasma are compressive in nature for different values of plasma parameters e.g. relativistic degenerate parameter, degenerate positron, degenerate electrons and ion temperature.

ii) The amplitudes of IADLs s are small for large values the plasma parameters.

iii) The width of IADLs increases with increase of R_{e0} for p = 0.01, 0.014, 0.016, 0.018 with the fixed values of σ_i =0.01, α =0.12, M =2. More over, the width of IADL increases with increase of p .

iv) The width of IADLs increases with increase of α for R_{e0} =9, 11, 13, 15 with the fixed values of σ_i =0.01, p =0.1, M =2. More over, the thickness of DL increases with increase of R_{e0} .

However, the effect of ion drift has not been considered in the present investigation of IADLs. It is known that ion acoustic wave will propagate through the plasma with two modes , fast-mode and slow- mode, in presence of drifting (streaming) ions. Considering the ion drift some new results on nonlinear propagation of waves in degenerate plasma may be obtained from which the causes of some phenomena in astrophysical objects may be understood.

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