International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 10 Issue 11 ǁ November 2022 ǁ PP. 104-110

Generation of Satin and Sateen Weaves from Circulant Matrices

Yumnam Kirani Singh
C-DAC Kolkata, Plot E2/1, Block-GP, Saltlake Electronics Complex, Kolkata-91 India

Abstract
Proposed here are some new methods for generating satin and sateen weaves from right circulant matrix. Satin and sateen weaves are the third type ofbasic weave patternsused in weaving besides plain and twill weaves. These two weave patterns are can considered the same as one is the reverse of the other. Both weaves are designed in the similar way. The designing of these weaves is quite a bit different from the way how plain and twill weaves are designed. They are specified by repeat size and move number. The move numbers are to be chosen correctly for a given repeat size to get true or valid satin weaves. So, drawing satin or sateen weave on a graph paper is more complex than drawing plain or twill weaves and hence is time consuming and quite erroneous. The same is true for creating a satin or sateen weave in a computer by clicking on the grid of a digital sheet. We propose here some methods of automatic generation of satin and sateen weaves. These two weaves can be generated from the left or the right circulant matrices. For generating satin or sateen weaves with move numbers in the horizontal direction, right circulant matrices are more suitable. For generating satin or sateen weaves with move numbers in the vertical direction, left circulant matrices are more suitable. Using these methods, satin or sateen weave patterns can be generated automatically in a much easier and faster way without needing to draw manually using mouse in a software application.

Keywords: *Circulant matrix, Left circulant matrix, Right Circulant matrix, Plain weaves, Twill weaves, Satin weaves, Sateen weaves, Weave patterns,Automatic Weave Pattern Generation.*

Date of Submission: 04-11-2022 Date of acceptance: 16-11-2022

I. INTRODUCTION

Textile weaves are created by using the methods of interlacement of warps and wefts to increase the look and feel of a fabric. There are three different basic weave patterns we generally use to make a fabric. They are plain weave, twill weave and Satin weaves. Many other different interesting weave patterns are formed by combination of these basic weave patterns.In this paper, we will consider mainly the third weave patterns i.e., satin weaves. As compared with the plain and twill weaves, which can be generated using two and three harness, satin weaves are more difficult to design and requires minimum of 5 harness to produce regular satin weave fabric. The distinguishing characteristic of the twill weave is a diagonal rib pattern. The important characteristic of a satin weave is that in a given or chosen repeat size, there is only one interlacement point in every warp or weft line. As there are fewer interlacement point in a given repeat size, the fabric becomes smooth and lustrous. Satin weaves are of two types depending on whether the floats along the warp lines are more or floats along the weft lines are more. Accordingly, they are called warp face satin and weft face satin or simply sateen. Satin and sateen weaves are the reverse of each other. They are generated or designed in the same way. The main difference is that in satin weaves warps are more visible and they are made with good quality warp yarns. Commonly used warp yarns for satin weaves are as silk, nylon and polyester.On the other hand, in sateen weaves weft yarns are more visible and hence they are fabricated with good quality weft yarns. Commonly used weft yarns for sateen weaves are cotton and rayon. More about satin and sateen weaves designs and characteristics can be found in [1,4,5,6]. Satin and sateen weaves can also be obtained by rearranging twill weaves in warp-wise and weft-wise direction and hence they are also respectively known as warp faced rearranged twill weave and weft faced rearranged twill weaves. In [7], some methods for automatic generation of twill weaves basic from left circulant matrices are given in which circulant matrices have only move number 1, circular shift of 1 position. In this paper, we will extend the use of circulant matrices for automatic generation of satin and satin weaves by extending the circular shifts of more than 1 position.

It may be noted that circulant matrices have certain regular patterns which we can conveniently be used to generate interesting patterns in fabric to enhance the look and feel of the fabric. Circulant matrices are of two types – left circulant matrix and right circulant matrixdepending on in which directions the successive rows are shifted to generate the circular matrix. In left circulant matrices, successive rows are obtained by left circular shift of the previous row. On the other hands, right circulant matrices are generated by circularly shifting previous rows by 1 position. A left circulant matrix and its corresponding right circulant matrix are inter-related, that is, from a left circulant matrix, a right circulant matrix can be generated. Both types of circulant matrices have many different applications in many fields [2,3]. In this paper, we will be dealing both the circulant matrices in generating satin or sateenweave patterns by incorporating move numbers. In section II, we perform the analysis of satin and sateen weaves so as to understand how move numbers introduce the change in weave patterns in satin and satin weaves. In Section III, generation of satin and sateen weaves from both the left and right circulant matrices are given. In Section IV, different types of satin and sateen weaves of repeat size 7 and 8 at all possible move numbers are given to demonstrate which move numbers give valid regular satin and sateen weaves for a specific repeat size. A brief conclusion is given in Section V.

II. Analysis of Satin and Sateen Weave Structure

Satin weaves are classified into two types – regular and irregular. In regular satin weaves, the weave structure follows a continuous regular pattern in a repeat size. However, in irregular satin weave, there is no uniform weave pattern. Half of the repeat size has a pattern and the remaining half has a different weave pattern. But the main property of a satin weave i.e., every column or row of a satin weave in a given repeat size must have an up point of a warp. In this analysis, we will explore mainly the characteristics of the regular satin or sateen weaves.

A regular satin weave is specified by two parameters i.e., repeat size and move number. The repeat size of a satin weave must be greater than its move number. There is a specific relation between the two which must be satisfied to create a satin weave. The move number must be greater than 1 and less than repeat size. Also, there should not be any common factor between the move number and the repeat size. That is, for a satin weave of repeat size N, and move number N, it holds the following conditions.

$1< M <N-1$ and $GCD(M,N)=1$
Where GCD indicates the Greatest Common Divisor.

The minimum repeat size for a satin weave having valid move number is 5. This is because, any repeat size less than 5 does not have a valid move number. Also, repeat size 6 does not have any valid move number because all numbers between 1 and 5 have a common factor with 6. All other numbers have greater than 6 have at least two valid move numbers. For a given repeat size, the number of valid move numbers is always even. This is because for each move number, there is a reciprocal move number. If M is a valid move number of repeat size N, then N-M is also a valid move number. For example, 2 is a valid move number for repeat size 5, then 5-2=3 is also a valid move number for the repeat size 5. i.e., 2 and 3 are the reciprocal move numbers of repeat size 5. If we generate a satin weave of repeat size 5 with move number applied horizontally, we get the same satin weave with move number 3 applied vertically and vice versa.

Figure-1(a) shows the satin weave of repeat size 5 with move number 2 and Figure-1(b) shows the corresponding graph. Figure-2(a) shows the weave matrix of satin weave with size 5 and move number 3 applied horizontally. Figure-2(b) shows the corresponding graph of Figure-2(a).

1.	1.	1.	0.	1.
1.	0.	1.	1.	1.
1.	1.	1.	1.	0.
1.	1.	0.	1.	1.
0.	1.	1.	1.	1.

Figure-1(a): Weave matrix of Satin weave (5,2)

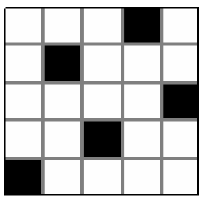


Figure-1(b): Weave graph of Satin weave (5,2)

1.	1.	0.	1.	1.
1.	1.	1.	1.	0.
1.	0.	1.	1.	1.
1.	1.	1.	0.	1.
0.	1.	1.	1.	1.

Figure-2(a): Weave matrix of Satin weave (5,3)

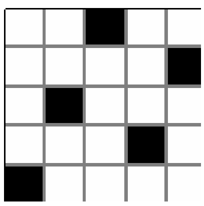


Figure-2(b): Weave Graph of Satin (5,3)

If we closely observe the weave matrices of Figure-1(a) and Figure-2(a), rows in one weave matrix are related to columns in the other weave matrix. If we transpose the weave matrix of satin(5,2) and then flip row-wise and column-wise, we get the weave matrix of satin (5,3). In general, we can say that, if we transpose the weave matrix of satin(N,M), followed by row-wise and column-wise flipping, we get the weave matrix of satin (N,N-M). That is,

Satin(N,N-M)=FhFv(Satin(N,M)T) =FvFh(Satin(N,M)T)

Where Fh denotes horizontal (column-wise) flipping, Fv denotes the vertical (row-wise) flipping and the operator ( )T denotes transposition operation.

The above relation can be used to find the reciprocal satin weave of given size and move number. So, if we know the satin(5,2), we can find its reciprocal satin weave i.e., satin(5, 3) as

satin(5,3)=FhFv(satin(5,2)T

Similarly, if we know the satin weave of repeat size 11 and move number 4, i.e. satin(11,4), we can find its reciprocal satin weave satin(11,7) as
satin(11,7)=FhFv(satin(11,4)T

III. SATIN AND SATEEN WEAVE GENERATION

Satin and sateen weaves have regular patterns obtained by circularly shifting a particular binary vector or array in the left or right by certain position specified as move number. So, Satin weave can be conveniently generated either from left circulant or right circulant matrix. As left circulant and right circulant matrices are interrelated, once we know the generation of left circulant matrix, right corresponding circulant matrix can be easily generated. In [7], different methods of getting or generating left circulant matrices, which are basically left circulant matrices of move number 1. For generating satin or sateen weaves, we require circulant matrices having move number greater than 1. To understand how circulant matrices having move higher numbers can be generated, we must have enough knowledge on the generation of circulant matrices having move number 1, i.e., how successive rows or columns of a circulant matrices are interrelated.

Let us consider an array x=[1 2 3 4 5 6 7] having 7 elements. We will generate first see how left circulant matrix is generated from this array. As we will be dealing with circulant matrices with different move numbers, we will refer a circulant matrix along with size and move number. i.e. left circulant matrix (7,1) would mean a left circulant matrix with move number 1 as shown in Figure-3.

1.	2.	3.	4.	5.	6.	7.
2.	3.	4.	5.	6.	7.	1.
3.	4.	5.	6.	7.	1.	2.
4.	5.	6.	7.	1.	2.	3.
5.	6.	7.	1.	2.	3.	4.
6.	7.	1.	2.	3.	4.	5.
7.	1.	2.	3.	4.	5.	6.

Figure-4(a): Left circulant Matrix (7, 1)

1.	2.	3.	4.	5.	6.	7.
7.	1.	2.	3.	4.	5.	6.
6.	7.	1.	2.	3.	4.	5.
5.	6.	7.	1.	2.	3.	4.
4.	5.	6.	7.	1.	2.	3.
3.	4.	5.	6.	7.	1.	2.
2.	3.	4.	5.	6.	7.	1.

Figure-4(b): Right Circulant Matrix(7,1)

If we carefully observe, we will find that all rows in the matrix are obtained by left circular shifting of the first row i.e., given array x by certain positions. Row-2 is obtained by shifting x on the left by 1 position., Row-3 is obtained by shifting x by 2 position, Row-4 is obtained by shifting x by 3 positions and so on. In general, Row-m is obtained by shifting x or Row-1 by (m-1) positions.

In the similar way, we can get the right circulant matrix (7,1) shown in Figure-4(b) from the array x by right circular shifting by 1, 2, 3, …, 6 positions. If we compare the left circulant matrix and right circulant matrix, we see that first rows of the two matrices are the same. The other remaining rows of the two matrices are in reverse order, the 2nd row in left circulant matrix is the 7th row in the right circulant matrix, or 2nd row in the right circulant matrix becomes the 7th row in the left circulant matrix. The similar relationship holds between left and right circulant matrices for move number greater than 1 as shown in Figure-4(c) and 4(d).

1.	2.	3.	4.	5.	6.	7.
3.	4.	5.	6.	7.	1.	2.
5.	6.	7.	1.	2.	3.	4.
7.	1.	2.	3.	4.	5.	6.
2.	3.	4.	5.	6.	7.	1.
4.	5.	6.	7.	1.	2.	3.
6.	7.	1.	2.	3.	4.	5.

Figure-4(c): Left circulant Matrix (7,2)

1.	2.	3.	4.	5.	6.	7.
6.	7.	1.	2.	3.	4.	5.
4.	5.	6.	7.	1.	2.	3.
2.	3.	4.	5.	6.	7.	1.
7.	1.	2.	3.	4.	5.	6.
5.	6.	7.	1.	2.	3.	4.
3.	4.	5.	6.	7.	1.	2.

Figure-4(d): Right Circulant Matrix (7,2)

This relation between rows of left and right circulant matrices of same size n and same move number k can be expressed as

Left shift by k in a left circulant matrix = Right shift by n-k in the corresponding right circulant matrix
Right shift by k in a right circulant matrix = Left shift by n-k in the corresponding left circulant matrix.

Using these relations, we can easily find or generate right circulant from left circulant matrix and vice versa. Considering L as left circulant matrix of n and move number k, we can generate the circulant matrix as

```
For i=1 to n
        For j=1 to n
                L(i,j)=1+ (k*(i-1)+j-1)mod n
        End
End
```

Generation of right circulant from the left circulant matrix can be done in the following way.
Step-1: Generate the left circulant matrix L having size n and move number k.
Step-2: Assign the first row of L to the first row of right circulant R having size n and move number k
Steo-3: Modify the remaining rows as follows.

```
For i=2 to n
        For j=1 to n
                R(i,j)=L(n-(i-1), j);
        End
End
```

We can also generate right circulant matrix first and then generate left circulant matrix from it. From Figure-4(b) and Figure-4(d), the relation between rows and the corresponding move numbers can be written as

ith row=1+ (n-1)*k*(i-1)

y(i,j)=1+<u>modulo</u>((n-1)*k*(i-1)+(j-1),n);

Generation of left circulant matrix from the right circulant matrix can be done in the following way.

Step-1: Generate a right circulant matrix R of size n and move number k
Step-3: Assign the first row of the right circulant matrix to the left circulant matrix L.
Step-3: Compute the remaining rows of the left circulant matrix as

```
For i=2:n
        For j=1:n
                L(n,j)=R(n+1-i,j);
        End
End
```

Another interesting property of left and right circulant matrices is that left circulant matrix of size n and move number k is exactly the same as right circulant matrix of size n and move number n-k. The reverse is also true, that is, the right circulant matrix of size n and move number k is exactly the same as the left circulant matrix of size n and mover number k. That is,

L(n, k)=R(n,n-k)
R(n,k)=L(n,n-k)

Figure-5(a) shows the left circulant matrix of size 7 and move number 2. Figure-5(b) shows the right circulant matrix of size 7 and move number 5. It may be seen that the two figures give the same circulant matrix. Similarly, right circulant matrix of size 7 and move number 2 and the left circulant matrix of size 7 and move number result in the same circulant matrix as shown in Figure-5(c) and Figure-5(d).

1.	2.	3.	4.	5.	6.	7.
3.	4.	5.	6.	7.	1.	2.
5.	6.	7.	1.	2.	3.	4.
7.	1.	2.	3.	4.	5.	6.
2.	3.	4.	5.	6.	7.	1.
4.	5.	6.	7.	1.	2.	3.
6.	7.	1.	2.	3.	4.	5.

Figure-5(a): Left circulant Matrix(5,2)

1.	2.	3.	4.	5.	6.	7.
3.	4.	5.	6.	7.	1.	2.
5.	6.	7.	1.	2.	3.	4.
7.	1.	2.	3.	4.	5.	6.
2.	3.	4.	5.	6.	7.	1.
4.	5.	6.	7.	1.	2.	3.
6.	7.	1.	2.	3.	4.	5.

Figure-5(b): Right Circulant Matrix (7,5)

1.	2.	3.	4.	5.	6.	7.
6.	7.	1.	2.	3.	4.	5.
4.	5.	6.	7.	1.	2.	3.
2.	3.	4.	5.	6.	7.	1.
7.	1.	2.	3.	4.	5.	6.
5.	6.	7.	1.	2.	3.	4.
3.	4.	5.	6.	7.	1.	2.

Figure-5(c): Right Circulant Matrix (7,2)

1.	2.	3.	4.	5.	6.	7.
6.	7.	1.	2.	3.	4.	5.
4.	5.	6.	7.	1.	2.	3.
2.	3.	4.	5.	6.	7.	1.
7.	1.	2.	3.	4.	5.	6.
5.	6.	7.	1.	2.	3.	4.
3.	4.	5.	6.	7.	1.	2.

Figure-5(d): Left Circulant Matrix (7,5)

Algorithm for generation of Satin Weave:

1. Get the repeat size n and move number k of a Satin weave
2. Check if k is valid for given repeat size
3. GenerateC right circulant matrix (n,k) or left circulant matrix (n, n-k)
4. Subtract 1from each element of C so that minimum number in C becomes 0.
5. Find W, the ceiling of C when divided by n or n-1.
6. Flip the binary matrix W row-wise (i.e., up down direction).
7. Display the matrix in Step-6 as image.

The above algorithm is a generalized one in the sense that it can be used for generating balanced and unbalanced twill weaves. In the above algorithm, when we divide the circulant matrix by a small down number, some of the quotients become greater than 1 in step-5 which when subtracted from 1 result in negative values in step-6 which correspond to 0 lines. So, it is necessary to make zeros for all negative values in Step-6. Such negative values do not occur when the down number is greater than or equal to ups numbers. Following is the Scilab code for generating twill weave.

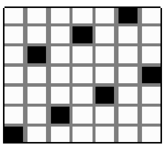


Figure-6(a): Satin(7,2),

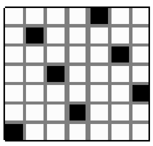


Figure-6(b): Satin(7,3)

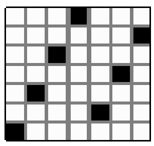


Figure-6(c): Satin(7,4)

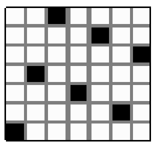


Figure-6(d): Satin(7,5)

IV. EXPERIMENTAL RESULTS

To test the methods of generating satin and sateen weaves from the right or left circulant matrix, we apply the method on various repeat sizes and move numbers. It may be noted that for a given repeat size, there are specific number of move numbers which can give true or regular satin or Sateen weaves. For a repeat size of a composite number, the number of possible moves is very few. For a repeat size of prime number, the number of possible move numbers is the chosen prime number minus 3. So, we can safely choose a prime number as a repeat size, so that we can more numbers of choice for move numbers. For the purpose of study, we choose generate weaves of all move numbers for a given repeat size. By observing the generated weaves patterns, we find that there are missing rows or columns in the generated satin or sateen weave when a move number is having a common factor with the chosen repeat size.

Table-1 shows the weave graphs of satin and sateen weaves ofrepeat sizes 7. First row of the table shows the weave graphs generated from the right circulant matrices of all move numbers, i.e., move numbers 1 to 7. It may be noted that out of these seven weave graphs generated from circulant matrix, the first and the last two graphs are not considered as regular satin weaves. The first weave graph is a 1/6 twill weave, the 6th w weave graph is a broken twill weave 1-up and 6-down while the last i.e., the seventh weave graph is not a valid weave graph which can be woven. In other words, 1, 6, and 7 move numbers cannot be used for generation of satin or sateen weaves. In general, for a repeat size of a prime number p, the move numbers 1, p-1 and p cannot be used to generate regular satin weaves. Similar is the case for the weave graphs in the second row. In these weave graphs, which are reverse of the weave graphs of the first row, only the weave graphs corresponding to the move numbers 2, 3, 4, and 5 are valid sateen weaves.

Table-2 shows the weave graphs of repeat size 8 generated fromright circulant matrix for all possible move numbers. The first row contains valid satin weaves and the second row contains valid sateen weaves.

Table-1: Weave graphs for a circulant matrix of repeat size 7 at different move numbers (i.e., circular shifts)

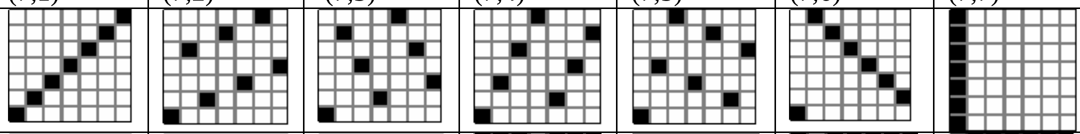
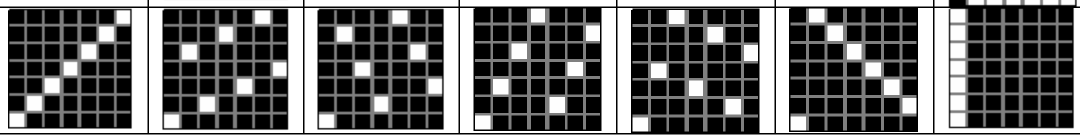
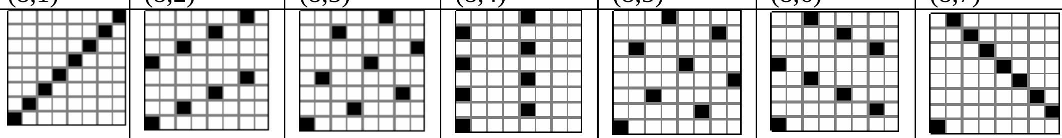
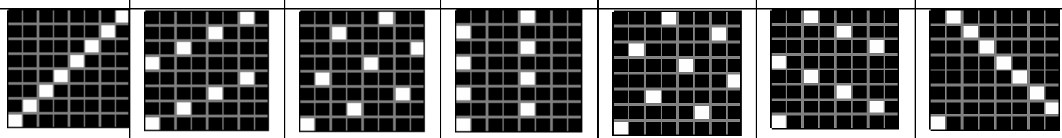
Type	(7,1)	(7,2)	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)
Statin							
Sateen							

Table-2: Weave graphs for a circulant matrix of repeat size 8 at different move numbers (i.e., circular shifts)

Type	(8,1)	(8,2)	(8,3)	(8,4)	(8,5)	(8,6)	(8,7)
Satin							
Sateen							

From Table-2, it may be seen that the weave graphs of column numbers 2, 4, and 6 are not valid weave graphs as they contain no interlacing points. If we carefully observe, we see that these move numbers, 2, 4, and 6 have common factor with the repeat size 8. That is the reason why move numbers are not chosen for numbers having a common factor with repeat size. The weave graphs of the first column and the last columns are the twill weaves and they are not considered as satin or sateen weaves. So, the valid satin and sateen weaves are weave graphs corresponding to move numbers 3 and 5 only.

.

V. CONCLUSION

In this paper, we describe about left and right circulant matrices for generating satin and satin weaves. Both left and the right circulant matrix can be used to generate circulant matrices of specific size and move numbers. Left circulant matrix is more convenient to generate satin or sateen weaves if the movement or shifting of the interlacement point in the bottom to top or top to bottom direction. Right circulant matrix is more convenient to generate satin or satin weaves if the shifting direction of interlacement point is along the horizontal i.e., left right direction which is commonly followed by majority of the weave designers. However, as left and right circulant matrices are interrelated, satin or sateen weave of any specific move number can be generated.

REFERENCES

[1]. Saharon D. Alderman, "Mastering Weave Structures: Transforming Ideas into great fabrics", Interweave Press, 2004.
[2]. P.J. Davis, Circulant Matrices, AMS Chelsea Publishing, 1994
[3]. G.M. Gray, "Toeflitz and Circulant Matrices – A Review", https://ee.stanford.edu/~gray/toeplitz.pdf
[4]. Vasant R. Kothari, "Satin and Sateen Weave", http://vasantkothari.com/content/view_presentation/425/16-Satin-and-Sateen-Weave
[5]. Priyank Goyal, "Sateen and Satin Weaves", https://www.scribd.com/document/85351769/Satin-and-Sateen-Weave
[6]. Satin and other weaves, https://www2.cs.arizona.edu/patterns/weaving/monographs/ics_507.pdf
[7]. Y. K. Singh, "Generation of Plain and Twill Weaves from Left Circulant Matrices", International Journal of Research in Engineering and Sciences, Volo.10, No. 10, pp. 283-291, 2022.