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# Generation of Plain and Twill Weaves from Left Circulant Matrices

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***Abstract***
*Proposed here are some new methods for generating plain and twill weaves from left circulant matrix. Generally, weave patternsare designed on a graph paper which is time consuming and requires a lot of efforts and patience. With the development of computer graphics, many software applications are available in which the designs are made by clicking on the square grids resembling grid on a graph paper. Developing weave patterns in a software application is more convenient than developing them on a graph paper as editing or error corrections can be done quite easily. However, drawing manually on computerusing mouse still takes time and requires knowledge on weave designs. In this paper, we describe how these weave patterns can be automatically generated from a left circulant matrix. First, the method of generating a left circulant matrix of any size desired size is given. Then, methods for generating plain and twill weaves from the circulant matrix are described. Using these methods, weave patterns can be generated automatically in a much easier and faster way needing to draw manually using mouse in a software application.*

***Keywords:*** *Circulant matrix,Left circulant matrix, Right Circulant matrix, Plain weaves, Twill weaves, Weave patterns,Automatic Weave Pattern Generation.*

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## I. INTRODUCTION

Textile weaves are created by using the methods of interlacement of warps and wefts to increase the look and feel of a fabric. There are three different basic weave patterns we generally use to make a fabric. They are plain weave, twill weave and Satin weaves. Many other different interesting weave patterns are formed by combination of these basic weave patterns.Of the three basic weave patterns, plain and twill weaves are quite popularly used in fabric designs. Plain weave is the simplest weave which can be easily designed in a loom having only two harnesses. For twill weaves, at least three harnesses looms are required. Twill is one of the three major types of textile weaves, along with satin and plain weaves. The distinguishing characteristic of the twill weave is a diagonal rib pattern. Twill weaves generally have high thread count, which make the fabric opaque, thick, and durable.Twill weaves find a wide range of application such as drill cloth, khaki uniforms, denim cloth, blankets, shirtings, hangings and soft furnishings.In [4,6], basic twill weaves and their applications are given. Traditionally, these weave patterns are created by filling the grid or cells on a graph paper. But with the development of computer graphics applications, creating weave patterns can be done in a computer. There are many software applications available to enable a user to create weave design. As compared to designingon a graph paper, designing using a software application is more flexible, error free and easy to edit and update. Some of the popularly used software applications for creating weave patterns are given in [1, 5, 7]. However, creating weave patterns in a computer still requires time and efforts to manually fill the grids of a digital sheet by manually clicking them correctly. In this paper, we will be discussing on how to automatically generate plain and twill weaves from circulant matrices.

Circulant matrices are matrices which are generated from a single row or a column vector. Usually, circular matrices have certain regular patterns as we gave patterns in weaves. There are two types of circular matrices depending on in which directions the successive rows are shifted to generate the circular matrix. If successive rows are sifted towards left then they are called left circulant matrices. On the other hands, if the successive rows are shifted towards right, they are called right circulant matrices. These two circulant matrices can be generated from each other. Right circulant matrices are obtained by flipping the row elements left-right direction. Similarly, left circulant matrices are obtained from left circulant matrices by flipping the rows in left right directions. Both types of circulant matrices have many different applications in many fields. More on circulant matrices and their applications can be obtained from [2,3]. In this paper, we will be dealing mainly with the left circulant matrices in generating twill weave patterns. In section II, different ways of generating left circulant matrices are given. Also, the relationship between the left and right circulant matrices are given. In Section II, generation of various types of Twill weaves from left circulant matrices are given along with algorithm and Scilab code. In Section IV, different types of twill weaves generated from the left and right circulant matrices areprovided as experimental resultsand Conclusions is given in Section V.

## II. GENERATION OF LEFT CIRCULANT MATRIX

Circulant matrices are matrices which are generated from a given row or column by shifting it towards left (anti-clockwise) or right (clock-wise) direction. Usually, a circulant matrix is a square matrix generated by successive shifting of a row vector in each row. First row is the given row vector, second row is obtained by circular shifting the first row by one element. Third row is obtained by circular shifting of the second row by one element. That is, each row is obtained by circular shifting of the previous row by one-element.

Suppose, $\boldsymbol{c} = [c_0, c_1, c_2, c_3, \dots, c_{N-1}]$is a row vector having N elements. Then, a left circulant matrix C can be generated from the row vector $\boldsymbol{c}$by circularly shifting each element by one position in each successive row. Following is the left circulant matrix generated from the row vector $\boldsymbol{c}$.

$$C = \begin{bmatrix} c_0 c_1 c_2 c_3 \dots \; c_{N-1} \\ c_1 c_2 c_3 \dots \; c_{N-1} c_0 \\ c_2 c_3 \dots \; c_{N-1} c_0 c_1 \\ c_3 \dots \; c_{N-1} c_0 c_1 c_2 \\ \dots \quad \dots \quad \dots \\ c_{N-1} c_0 c_1 c_2 \; \dots \; c_{N-2} \end{bmatrix}$$

From the left circulant matrix, it may be observed that all elements parallel to the off-diagonal elements are the same. These patterns are very much similar to z-twill weave patterns. If we carefully observe, it can be seen that elements in a particular row are the same as the elements in the corresponding column. That is, the elements in the first row are the same as elements in the first column, elements in the second row are the same as the elements in the second column and so on. Columns in a circulant matrix have the same relation as the rows in a circulant matrix. In short, a circulant matrix can be generated by circularly shifting the elements in clockwise or anti-clockwise direction. As the rows and columns are the same, the transpose of a circulant matrix is the circulant matrix itself.

The relation between the elements of row vector $\boldsymbol{c}$and circulant C can be established as follows.

$$C[i,j]=\boldsymbol{c}[(i + j)\%N]$$

Where i, j=0, 1, 2, …, N-1 and % denotes the modulus operator.

The above relation can be easily implemented in a programming language supporting 0-offset arrays using two For loops.

```
For i=0 to N-1
        For j=0 to N -1
                C(i,j)=c((i+j)%N);
        End
End
```

To implement the same in the programming languages that support 1-offset array, some modification needs to be done.

For 1-offset arrays, the algorithm to generate left circulant matrix is given below.

```
For i=0 to N-1
        For j=0 to N -1
                C(i+1,j+1)=c(1+((i+j)%N));
        End
End
```

For designing weave patterns, we are interested to have a circulant matrices whose elements are a range natural number such as 0 to 9, 0-15 etc. Such a circulant matrices can be generated from an array or row vector whose elements are x=[0,1,2,3,4,5,6,7,8,9]. As the content of the arrays are known, we can simply specify the largest element in the array, such as 9 which indicates a circulant matrix of size 10x10 with elements 0 to 9. So, we can have another algorithm to generate a circulant matrix of a specific size given a positive number as its input N.

Following is the algorithm to generate circulant matrix of size N whose elements are 0 to N-1.

```
For i=0 to N-1
        For j=0 to N-1
                C(i,j)=(i+j)%N;
        End
End
```

Similarly, the algorithm for 1-offet arrays can be modified accordingly.

These two algorithms, i.e., algorithm to generate a circulant matrix from a given array and the algorithm to generate a circulant matrix from a given number, can be combined to generate a left circulant matrix given regardless of whether an array is given as input or just a number as input. Scilab code for generating left circulant matrix of any desired size is given in Figure-1 in which the input can be row vector or a scaler. If a row vector is given as input, the left circulant matrix is given by the circularly shifting the vector in each successive rows as given in the algorithm. If the input is a scalar number n, then, it generates left circulant matrix from elements 0 to n-1.The outputs of the Scilab function leftcirc for generating a left circulant matrix of size 8 given a number and a vector as input are shown respectively in Figure-2 and Figure-3.

```
functiony=leftcirc(x)
// x is a row  vector or a number
n=length(x);
ifn==1then
fori=0:(x-1)
forj=0:(x-1)
y(i+1,j+1)=modulo(i+j,x);
end
end
else
fori=0:n-1
forj=0:n-1
y(i+1,j+1)=x(1+modulo(i+j,n));
end
end
end
endfunction
```

Figure-1: Code for generating Left circulant matrix

```
--> y=leftcirc(8)
 y  =

   0.   1.   2.   3.   4.   5.   6.   7.
   1.   2.   3.   4.   5.   6.   7.   0.
   2.   3.   4.   5.   6.   7.   0.   1.
   3.   4.   5.   6.   7.   0.   1.   2.
   4.   5.   6.   7.   0.   1.   2.   3.
   5.   6.   7.   0.   1.   2.   3.   4.
   6.   7.   0.   1.   2.   3.   4.   5.
   7.   0.   1.   2.   3.   4.   5.   6.
```

Figure-2: Output of a leftcirc function given 8 as input

```
--> y=leftcirc([0 1 2 3 4 5 6 7])
 y  =

   0.   1.   2.   3.   4.   5.   6.   7
   1.   2.   3.   4.   5.   6.   7.   0
   2.   3.   4.   5.   6.   7.   0.   1
   3.   4.   5.   6.   7.   0.   1.   2
   4.   5.   6.   7.   0.   1.   2.   3
   5.   6.   7.   0.   1.   2.   3.   4
   6.   7.   0.   1.   2.   3.   4.   5
   7.   0.   1.   2.   3.   4.   5.   6
```

Figure-3: Output of leftcirc function given a 1-d array as input.

**Alternative Method of generating left Circulant matrix:**

It may be seen that the method for generating left circulant matrix based on the shifting of each elements requires using for loops. In this section, we describe a different method for generating left circulant method without using for loops which is very convenient and fast for array-based languages such as Scilab.

**Algorithm for generating Circulant matrix without using for loops**
Step-1: create a ones matrix X of desired size
Step -2: Compute Xr -->Cumulative sum of X along rows.
Step-3: Compute Xc -->Cumulative sum of X along columns, which is the same as transpose of Xr.
Step-4: Add Xr and Xc, to get symmetric matrix Y
Step-5: Subtract 2 from the rest of the elements in Y
Step-6: Compute the remainder of Y when divided by the size of X

The steps of the algorithm for generating a left circulant matrix of size 8 are shown with numerical examples for as shown below.

$$X = \begin{bmatrix} 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1 \end{bmatrix}$$

Step-1

$$Xr = \begin{bmatrix} 1&2&3&4&5&6&7&8\\ 1&2&3&4&5&6&7&8\\ 1&2&3&4&5&6&7&8\\ 1&2&3&4&5&6&7&8\\ 1&2&3&4&5&6&7&8\\ 1&2&3&4&5&6&7&8\\ 1&2&3&4&5&6&7&8\\ 1&2&3&4&5&6&7&8 \end{bmatrix}$$

Step-2

$$Xc = \begin{bmatrix} 1&1&1&1&1&1&1&1\\ 2&2&2&2&2&2&2&2\\ 3&3&3&3&3&3&3&3\\ 4&4&4&4&4&4&4&4\\ 5&5&5&5&5&5&5&5\\ 6&6&6&6&6&6&6&6\\ 7&7&7&7&7&7&7&7\\ 8&8&8&8&8&8&8&8 \end{bmatrix}$$

Step-3

It may be noted that we can get $Xr$ and $Xc$ in the following ways as well.

$$Xr = \begin{bmatrix} 1\\1\\1\\1\\1\\1\\1\\1 \end{bmatrix} * [1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8] \qquad Xc = \begin{bmatrix} 1\\2\\3\\4\\5\\6\\7\\8 \end{bmatrix} * [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1]$$

This method of generating Xr and Xc is based on matrix multiplication. As one of the multiplicands is an array of ones, the multiplication process becomes simpler. The two matrices Xr and Xc are transpose of each other. Once we compute one, the other can be obtained by transposing it. The remaining steps of the algorithm i.e., Steps 4, 5 and 6 are shown below.

$$Y = \begin{bmatrix} 2&3&4&5&6&7&8&9\\ 3&4&5&6&7&8&9&10\\ 4&5&6&7&8&9&10&11\\ 5&6&7&8&9&10&11&12\\ 6&7&8&9&10&11&12&13\\ 7&8&9&10&11&12&13&14\\ 8&9&10&11&12&13&14&15\\ 9&10&11&12&13&14&15&16 \end{bmatrix}$$

Step-4

$$Y = (Y-2) = \begin{bmatrix} 0&1&2&3&4&5&6&7\\ 1&2&3&4&5&6&7&8\\ 2&3&4&5&6&7&8&9\\ 3&4&5&6&7&8&9&10\\ 4&5&6&7&8&9&10&11\\ 5&6&7&8&9&10&11&12\\ 6&7&8&9&10&11&12&13\\ 7&8&9&10&11&12&13&14 \end{bmatrix}$$

Step-5

$$Y = (Y\%8) = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 0 \\ 2 & 3 & 4 & 5 & 6 & 7 & 0 & 1 \\ 3 & 4 & 5 & 6 & 7 & 0 & 1 & 2 \\ 4 & 5 & 6 & 7 & 0 & 1 & 2 & 3 \\ 5 & 6 & 7 & 0 & 1 & 2 & 3 & 4 \\ 6 & 7 & 0 & 1 & 2 & 3 & 4 & 5 \\ 7 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

Step-6

The matrix obtained in Step-6 is the left circulant matrix of size 8. Also, we find that the square matrix obtained in Ste-p-5 can also be used to generate Plain andtwill weave patterns. It is found that it is more robust than left circulant matrix for generating repeated twill weave matrix.

All the six steps given in the algorithm for generating left circulant matrix can be implemented in a single line statement in Scilab. However, we will implement the algorithm in two-line statements as we are interested to get the matrix in step-5 as well. Following is the Scilab function to generate the left circulant matrix from positive number given as input.

```
function[y, y1]=leftcirca(n)
y1=ones(n,1)*[1:n]+[1:n]'*ones(1,n)-2;
y=modulo(y1,n);
endfunction
```

## Plain Weave Generation from a Left Circulant Matrix

Plain weave is the weave in which warp and weft threads are interlaced alternately. In matrix form, it is represented by a matrix having elements of alternate 0s and 1s. So, plain weaveof desired size can be obtained by finding the remainder of 2 of left circulant matrix or natural circulant matrix. Figure-4(a) shows the plain weave matrix generated from a left circulant of size 8 and the corresponding weave graph is shown in Fig. 4(b).

$$P = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Figure-4(a): Plain weave matrix

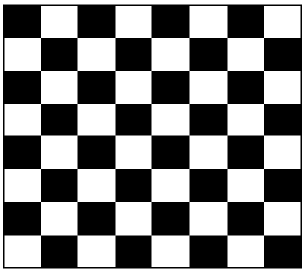


Figure-4(b): Plain weave graph of Figure-4(a).

### III. TWILL WEAVE PATTERN GENERATION FROM LEFT A CIRCULANT MATRIX

Twill weave is one of the popular basic weave patterns used in fabric designs. It has many desired characteristics to users. Every weave pattern has a repeat size replicating which multiple times generates a fabric. Twill Weaves have specific number of ups and down numberssum of which determines the repeat size. The pattern in a twill weave depends on the number of ups and downsin a given repeat size. The minimum repeat size for a twill weave is 3 corresponding to 2/1 (2 up and 1 down) or 1/2 (1 up 2 down) twill weave. Figure-5(a) shows the 2/1 twill weave matrix of size 9x9 in which 0s' represent the ups and 1s' represent the downs of warps. The repeat size is 3 marked in orange color. The corresponding weave graph is shown in Figure-5(b) in which the ups are marked in black (0s) and downs are in white (1s). For repeat size of 3, only two different twill weaves are possible i.e., 2/1 and twill weaves. These two weaves are interrelated, i.e., one once a 2/1 weave matrix is obtained, the 1/2 weave matrix can be obtained from it. Figure-6(a) shows a 1/2 twill weave matrix and its corresponding weave graph is shown in Figure-6(b). For a twill weave of larger repeat size, there are three or more variants of twill weave having the same repeat size. For example, for a twill weave repeat size of 8, the possible variants are 1/7, 7/1, 2/6 , 6/2, 3/5, 5/3, 4/4, i.e., there are seven variants of twill weaves having the repeat size 8. Twill weave patterns having the different number of ups and downs numbers are known as unbalanced twill weaves. If the number of ups and downs are the same, the twill weave are known as balanced twill weaves. The unbalanced twill weaves have different back and front patterns in the fabric. For balanced twill weaves both sides of the fabric have similar weave patterns.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |

Figure-5(a): 2/1 twill weave of size 9x9

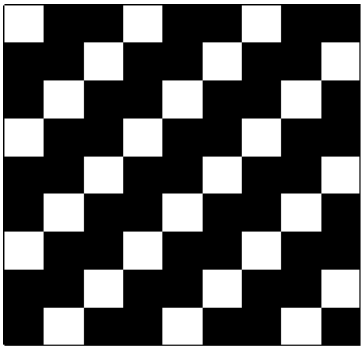


Figure-5(b):weave graph of 2/1 twill weave

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |

Figure-6(a): 1/2 Twill weave

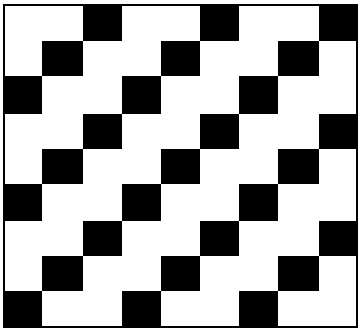


Figure-6(b): Graph of 1/2 Twill Weave

It may be seen that from figures 5 and 6 that twill weaves have black and white rib lines parallel to the off-diagonal line. The same is true for a left circulant matrix. So, it is convenient to generate the twill weaves from the left circulant matrix of appropriate size. To generate a twill weave of specific ups and down lines from a left circulant matrix, we need to appropriately make zeros corresponding to the up numbers and ones corresponding to the down numbers. To understand the process of generation twill weave from a left circulant matrix, let us generate a balanced twill weave of repeat size 4 having 2 ups and 2 downs, i.e., 2/2 twill weave of size 8. For this, we need to create a left circulant matrix X of size 8x8. Then, we need to make two zeros' lines followed by two ones' lines parallel to the off-diagonal line. The elements in off-diagonal line of X are 7. So, to make zero lines parallel to off-diagonal line starting from it, we need to make elements 6 and 7 zeros and elements 4 and 5 ones for two lines of ones parallel to the off-diagonal line. But the repeat size of twill weave is 4, so the elements in the matrix can be made 0 to 3 by finding the remainder of 4 to get matrix X'. Make the elements 2 and 3 zeros and 0 and 1 ones to get the 2/2 twill weave matrix T. When we divide X' by 2, we will get 0s as quotients for 0 and 1 elements and 1s as quotients for 2 and 3 elements. This can be achieved by flooring operation. Inverting the result will give us the twill matrix T. Figure-7 shows the steps for generating a twill weave from a left circulant matrix.

$$X = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 0 \\ 2 & 3 & 4 & 5 & 6 & 7 & 0 & 1 \\ 3 & 4 & 5 & 6 & 7 & 0 & 1 & 2 \\ 4 & 5 & 6 & 7 & 0 & 1 & 2 & 3 \\ 5 & 6 & 7 & 0 & 1 & 2 & 3 & 4 \\ 6 & 7 & 0 & 1 & 2 & 3 & 4 & 5 \\ 7 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix} \quad X' = (X\%4) = \begin{bmatrix} 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 \end{bmatrix} \quad T = 1 - \left\lfloor \frac{X}{2} \right\rfloor = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Step-a Step-b Step-c

Figure-7: Steps to get twill matrix from a left circulant matrix

Steps in Figure-7 are summarized below.

Step-a: Generate a left circulant matrix X of appropriate size.

Step-b: Find the remainder of X when divided by 4 to get X'

Step-c: Invert the matrix obtained by flooring when X' is divided by 2 to get the twill matrix T

The method given in Figure-7 can be used to generate a balanced twill weave. However, for generating unbalanced weaves,we need to do some necessary modification to make parallel lines of zeros and ones for different ups and downs number of the unbalanced twill weaves.

***Algorithm for generation of Twill weave:***

1. Get the twill weave up/down numbers
2. Determine the repeat size
3. Generate the circulant matrix Cof size 2 to 3 times the repeat size
4. Find the remainder Cr of the circulant matrix when divided by the repeat size
5. Find the flooring of the remainder matrix when divided by the downs number
6. Inverse the matrix in step-5 by subtracting it from 1 and make all negative values to 0.
7. Display the matrix in step-6 as image.

The above algorithm is a generalized one in the sense that it can be used for generating balanced and unbalanced twill weaves. In the above algorithm, when we divide the circulant matrix by a small down number, some of the quotients become greater than 1 in step-5 which when subtracted from 1 result in negative values in step-6 which correspond to 0 lines. So, it is necessary to make zeros for all negative values in Step-6. Such negative values do not occur when the down number is greater than or equal to ups numbers. Following is the Scilab code for generating twill weave.

```
functiony=twill(m, u, d)
rp=u+d;
y=leftcirc(m);
y=1-floor(modulo(y,rp)/d);
y(y<0)=0;
endfunction
```

Using this twill function we can generate the balanced and unbalanced twill weave matrices of any desired size. Figure-8(a) show the unbalanced 3/2twill weave graph corresponding to a weave matrix generated using the twill(8,3,2). Similarly, Figure-8(b) shows the 2/3 twill weave generated using twill(8, 2, 3).

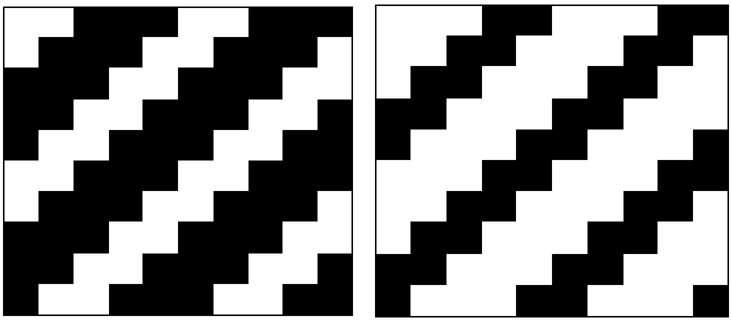


Figure-8(a): 3/2 Twillweave   Figure-8(b): 2/3 Twill weave

**Generation of S-Twill Weaves:**

The twill weaves discussed or generated so far having rib lines parallel to off-diagonal line are known as Z-Twill weaves. There are also twill weaves which have rib lines parallel to diagonal line. Such twill weaves are known as S-twill weaves. For drawing S-Twill weaves, drawing starts from the bottom of the right end. These two weave types i.e., Z-twill and S-twill are the reflected images of each other. That is, S-twill weaves are obtained by flipping the Z-twill weaves in the left-right direction.

If Y is a weave matrix of u/d Z-twill weave, then u/d S-Twill weave matrixW, can be generated in Scilab using the following command for performing flipping left-right direction.

```
W=filpdim(Y,2);
```

So, we do not need separate program code for generating S-twill weaves. The same twill function given for generating Z-twill weave will first be used which will be followed by flipping along the second dimension, i.e., along columns. Figure-9(a) shows the 3/2 Z-twill weave and the corresponding 3/2 S twill weave is shown in Figure-9(b).

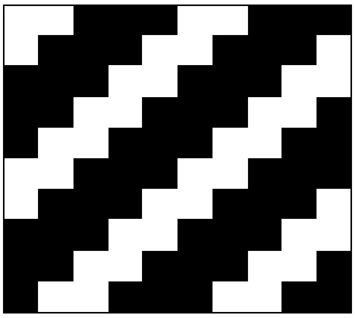


Figure-9(a): 3/2 Z twill weave

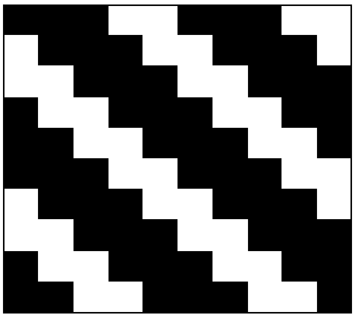


Figure-9(b): 3/2 S-twill weave

Combining Z and S twill weaves in different ways, different interesting weave patterns such as herring bone weave, wave weave, diamond weaves etc. can be created. Moreover, we can combine plain weaves together with twill weaves to generate more and more interesting weave patterns.

## IV. EXPERIMENTAL RESULTS

To test the methods of generating plain and twill weaves from the left circulant matrix, we apply the functions given in the paper on various types and size of weaves. It is found that circulant matrix can be used without many problems for generating weave matrices of size up to 3 times the repeat size. If weave matrix size of a twill weave is more than 3 times the repeat size, it gives extra rib lines giving wider rib line of 1s or 0s. This is really not a problem as twill weaves are drawn basically for a repeat size. Both Z and S twill weaves of any size can be generated using the twill weave function.

Table-1 shows the weave graphs of Z and S twill weaves of repeat sizes 8. First row of the table shows the possible unbalanced and balanced Z-twill weave patterns for a repeat size of 8. It may be noted that only the 4/4 is the balanced twill weave for repeat size-8. Similarly, all possible S-twill weaves having repeat size 8 are shown in the second row.

Table-2 shows the weave patterns which can be formed by Twill weaves of repeat size 8 and plain weaves of corresponding size. These weaves have plain weaves along the rib lines of the corresponding twill weaves. These makes the weaves pattern more dense rib lines as compared to the pure twill weaves.

Table-1: Twill weaves for repeat size 8

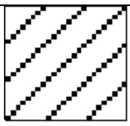
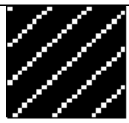
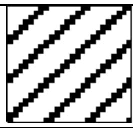
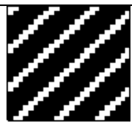
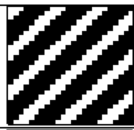
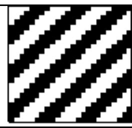
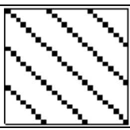
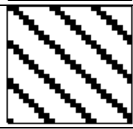
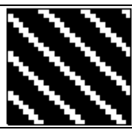
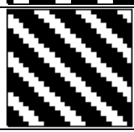
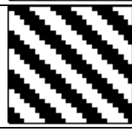
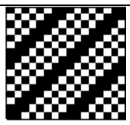
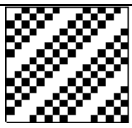
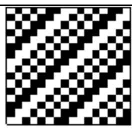
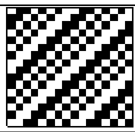
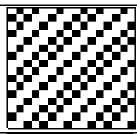
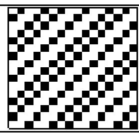
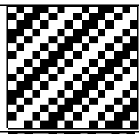
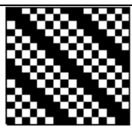
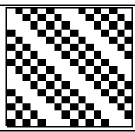
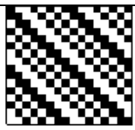
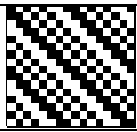
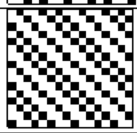
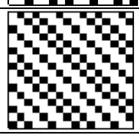
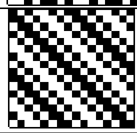
| Type | 1/7 | 7/1 | 2/6 | 6/2 | 3/5 | 5/3 | 4/4 |
|---|---|---|---|---|---|---|---|
| Z-Twill |  |  |  |  |  |  |  |
| S-Twill |  |  |  |  |  |  |  |

Table-2: Twill weaves combined plain weaves

| Type | 1/7 | 7/1 | 2/6 | 6/2 | 3/5 | 5/3 | 4/4 |
|---|---|---|---|---|---|---|---|
| Z-Twill |  |  |  |  |  |  |  |
| S-Twill |  |  |  |  |  |  |  |

Many more interesting weave patterns such as herringbone weaves, diamond weaves etc. can be formed by combining Z and Z-twill weaves given in Table-1 and 2.

## V. CONCLUSION

In this paper, we describe about circulant matrices and methods for generating left circulant matrices. Two different ways of generating circulant matrices, i.e., generating a circulant matrix from a vector (1-dimensional array) and a given number specifying only the size. Circulant matrices that are generated from a given number are formed by circular shifting of natural numbers and are more suitable for generating weave patterns. Some effective ways of generating plain and twill weave patterns from left circulant matrices are also described. Using these methods, we can easily generate weave pattern of desired size in an easier and faster way which will save significant amountof time and efforts for manually drawing the patterns. It has been tested for various types and sizes of balanced and unbalanced twill weave patterns. All twill weave patterns can be generated as expected. Also, it has been tested that plain weaves can be combined with twill weave patterns to generate more interesting new twill weave patterns which can subsequently be used to generate twill weave derivative patterns such as herringbone, diamond weaves etc.

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