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Heat Transfer Analysis of TiO_2 Nanofluid over Unsteady Stretching Surface

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Abstract
In this article, the flow and heat transfer of electrically conductingTiO_2 nanofluidover a stretching surface is manipulated. The governing equations are converted intoa nonlinear system of boundary value problems using suitable transformations. Thenonlinear system is then solved numerically for different physical parameters such as,Prandlt, Eckert, biot, volume fraction, Slip and unsteadiness. Later, graphical simulationsof energy profile of different shapes of nanoparticles (NPs) are constructed forthese parameters. Moreover, the numerical computation of both skin friction C_f andNusselt number Nu are prepared and discussed in Tables 3-4.

Keywords: *Thin film; Shape factor; Heat transfer; Convective conditions.*

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I. Introduction

A nanofluid is a suspension of nanoparticles (NPs) (with at least one dimension less than$100nm$) in a base fluid, such as water, alcohol, oil, or refrigerant. Nanofluid has attracted alot of attention in the fields of nanotechnology, thermal engineering, and other applicationsover the last three decades. Several researchers have revealed quantitative evidence of thermalconductivity increase and better heat transfer performance of various nanofluids. Thetendency of NPs to cluster in the suspended state causes nanofluid to unstable. Becauseof its high surface activity, the NP suspension tends to agglomerate [1]. Due toits interesting features, nanofluid research has exploded in recent years, and different researchgroups have contributed significantly to this subject by conducting experimental ortheoretical studies on various aspects of nanofluids studied in [2]. More detail on nanofluidand construction of NPs can be found in [3]. Nanofluids have a broader range of uses inthermal energy storage systems for cooling and heat transfer to boost heat transfer rate [4]and thermal energy absorption [5, 6, 7].

The heat transfer performance of the plate heatexchanger has been investigated using different nanofluids $(CeO_2, Al_2O_3, TiO_2, SiO_2)$for various volume flow rates and wide range of concentrations by Sajadi [8]. Vasheghani*et al.* measured the thermal conductivity of micro and nanofluids using hot wire method.They also discussed the enhancement of thermal conductivity and viscosity by adding $3wt.$of TiO_2 (either nano or micro) to engine oil. Alias *et al.* [9] reported the enhancementof thermal conductivity by increasing the concentration of NPs, and

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temperature because of Brownian motion. Yanni *et al.* [10] investigated three-dimensional numericalnanofluid thermocapillary convection around a gas bubble.

The aim of this article is to manipulate the development of thermal conductivity in TiO_2nanofluid using different NPs with convective condition.

Nomenclature

κ_f	Thermal conductivity of the base fluid	ρ_f	Density of the base fluid (water)
κ_{nf}	Thermal conductivity of the nanofluid	ρ_{nf}	Density of nanofluid
C_p	Specific heat of fluid	μ_f	Dynamic viscosity of water
Nu	Nusselt number	μ_{nf}	Dynamic viscosity of nanofluid
Re	Reynolds number	ν_f	Kinematic viscosity of water

Pr	Prandtl number	ν_{nf}	Kinematic viscosity of nanofluid
α_f	Thermal diffusion of water	σ_{nf}	Electrical conductivity
α_{nf}	Thermal diffusion of nanofluid	$(\rho C_p)_{nf}$	Heat capacity of nanofluid

II. Modeling of the problem

In this paper, TiO_2-nanofluid film over a stretching surface is considered. The stretchingsurface is placed along xy – plane and y – axis is taken normal to the surface attached witha slit at x = 0. The stretching of the surface causes flow along x– axis with velocity$U_w = \frac{bx}{1-\alpha\, t}$. Thesurface temperature $T_s$is given by

$$T_s = T_0 - T_r\left(\frac{bx^2}{2\nu_f}\right)(1-\alpha t)^{-\frac{3}{2}}.$$

where b and α are dimensional constants. T is the temperature of the nanofluid, while T_r representsconstant reference temperature with slit temperature T_0 and $\nu_f$is the kinematicviscosity of pure fluid. The uniform magnetic field acting normal to the surface is defined by $B(t) = \frac{B_0}{\sqrt{1-\alpha t}}$.

Based on these assumptions, the model is governed by the following equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{nf}}{\rho_{nf}}B^2(t)u, \quad (2)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{\rho_{nf}C_p}\left(\frac{\partial T}{\partial y}\right)^2, \quad (3)$$

with respect to the boundary conditions

$$u = U_w + A\nu_f\frac{\partial u}{\partial y}, \qquad v = 0, \qquad -\kappa_{nf}\frac{\partial T}{\partial y} = h_f(T_s - T), \qquad \text{at } y = 0, \quad (4)$$

$$u \to 0, \qquad T \to T_0, \text{ as } y \to \infty, \quad (5)$$

where $h_f$is the convective heat transfer coefficient, and A is the proportionality constant.

In the above system, $\rho_{nf}, \mu_{nf}, \sigma_{nf}, \alpha_{nf}$ are the density, viscosity, electrical conductivity, and diffusivity of the nanofluid defined in [11], as

$$\alpha_{nf} = \frac{\kappa_{nf}}{(\rho C_p)_{nf}}, \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \mu_{nf} = \mu_f(1 + A_1\phi + A_2\phi^2), \sigma_{nf} = (1-\phi)\sigma_f + \phi\sigma_s,$$

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s. \quad (6)$$

and

$$\kappa_{nf} = \kappa_f\left[\frac{\kappa_s + (m-1)\kappa_f + (m-1)(\kappa_s - \kappa_f)\phi}{\kappa_s + (m-1)\kappa_f - (\kappa_s - \kappa_f)\phi}\right], \quad (7)$$

where ϕ, A_1, A_2, and $(\rho C_p)_{nf}$ are the volume fraction, thermal conductivity coefficients, and.

viscosity enhancement heat capacity, respectively. The thermal conductivity andshape factor of NPs are denoted by $\kappa_s$and m, respectively. The subscripts *f*, *nf*, and *s*denote the thermo-physical aspects of the base fluid, nanofluid, and solid NPs. The valuesof coefficients of viscosity enhancement A_1, A_2 and shape factor $m$of multi-shape NPs ofTiO_2 nanofluid are listed in Table 1, while Table 2 shows the thermo-physical properties ofthese NPs.

Table 1: Values of shape factors with viscosity coefficients

Physical parameters	Particle Shapes				
	Blade	Brick	Cylinder	Platelets	Sphere
A_1	14.6	1.9	13.5	37.1	2.5
A_2	123.3	471.4	904.4	612.6	6.5
m	8.26	3.72	4.82	5.72	3.0

Table 2: Thermo-physical properties of base fluid and nanofluids [12]

Physical properties	$\rho\ (kgm^{-3})$	$C_p\ (Jkg^{-1})$	$\kappa\ (Wm^{-1}K)$	$\sigma\ (Sm^{-1})$
H_2O	997.1	4179	0.613	5.5
TiO_2	4250	686.2	8.9538	0.125

Introducing the similarity transformations, as

$$T = T_0 - T_r\left(\frac{bx^2}{2\nu_f}\right)(1-\alpha t)^{-\frac{3}{2}}\theta(\eta), \qquad \eta = \left(\frac{b}{\nu_f(1-\alpha\, t)}\right)^{\frac{1}{2}}, \qquad \psi = \left(\frac{b\,\nu_f}{1-\alpha\, t}\right)^{\frac{1}{2}} x\, f(\eta). \tag{8}$$

The continuity equation (1) remains constant, while the PDEs (2–3) and boundary conditions(4–5) have been converted into a system of nonlinear ODEs using the transformationstated in (8), as

$$\epsilon_1 f'''(\eta) - \epsilon_3 M f'(\eta) + \left[f(\eta)f''(\eta) - f'^2(\eta) - S\left(f'(\eta) + \frac{\eta}{2}f''(\eta)\right)\right] = 0, \tag{9}$$

$$\frac{\epsilon_2}{Pr}\theta''(\eta) + \epsilon_1 Ec f''^2(\eta) + \left[f(\eta)\theta'(\eta) - 2\theta(\eta)f'(\eta) - \frac{S}{2}\left(3\theta(\eta) + \eta\theta'(\eta)\right)\right] = 0, \tag{10}$$

subject to

$$f(0) = 0, \qquad f'(0) = 1 + Kf''(0), \qquad \theta'(0) = -\frac{\kappa_f}{\kappa_{nf}}\gamma\left(1-\theta(0)\right), \qquad \text{at } \eta = 0,$$

$$f' \to 0, \qquad \theta \to 0, \qquad \text{as} \qquad \eta \to \infty, \tag{11}$$

where $K = A\sqrt{\frac{\nu_f U_w}{x}}$ and $\gamma = \frac{h_f}{\kappa_f}\left(\frac{x\nu_f}{U_w}\right)^{\frac{1}{2}}$ are the respective slip parameter and biot-number.

The Eckert number Ec, magnetic parameter M, Prandtl number Pr, and unsteadiness parameter S are the dimensionless constants, and symbolically written as

$Ec = \frac{U_w^2}{C_p(T_s - T)}$, $M = \frac{\beta_0^2\sigma_f}{b\,\rho_{nf}}$, $Pr = \frac{(\rho C_p)_f \nu_f}{\kappa_f}$, and $S = \frac{\alpha}{b}$.

The constants $\epsilon_i, i = 1, \ldots, 3$ are defined as

$$\epsilon_1 = \frac{1 + A_1\phi + A_2\phi^2}{1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)}, \qquad \epsilon_2 = \frac{\left(\frac{\kappa_{nf}}{\kappa_f}\right)}{1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)}, \qquad \epsilon_3 = \frac{1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)}{1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)}. \tag{12}$$

The solid volume-fraction and density of the nanofluid are described by ϕ and ρ. In engineering discipline, the skin friction and Nusselt number both have great interest which aredefined by

$$C_f = \frac{\tau_w}{\rho_f U_w^2}, \qquad \text{and} \qquad Nu = \frac{x\, q_w}{\kappa_f(T_s - T_0)},$$

where

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \qquad \text{and} \qquad q_w = -\kappa_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{13}$$

The Eqn. (13) can be represented in non-dimensional form, as

$$Re^{\frac{1}{2}} C_f = (1 + A_1\phi + A_2\phi^2)f''(0), \qquad \text{and} \qquad Re^{-\frac{1}{2}} Nu = -\frac{\kappa_{nf}}{\kappa_f}\theta'(0). \tag{14}$$

III. Solution methodology

Initially, the nonlinear system (9-11) is converted into a system of first-order equations withthe following assumptions

$$y_1 = f, \tag{15}$$
$$y_1' = y_2, \tag{16}$$
$$y_2' = y_3, \tag{17}$$
$$y_3' = \epsilon_1^{-1}\left[\epsilon_3 M y_2 - y_1 y_3 + y_2^2 + S\left(y_2 + \frac{\eta}{2}y_3\right)\right], \tag{18}$$
$$\theta = y_4, \tag{19}$$
$$y_4' = y_5 \tag{20}$$
$$y_5' = \Pr\epsilon_2^{-1}\left[-Ec\,\epsilon_1 y_3^2 - y_1 y_5 + 2\, y_2 y_4 + \frac{S}{2}(3y_4 + \eta\, y_5)\right], \tag{21}$$
$$y_1(0) = 0, \qquad y_2(0) = 1 + Ky_3(0), \qquad y_5(0) = -\frac{\kappa_f}{\kappa_{nf}}\gamma\left(1 - y_4(0)\right), \qquad \text{at } \eta = 0, \tag{22}$$
$$y_2 \to 0, \qquad y_4 \to 0, \qquad \text{as} \qquad \eta \to \infty, \tag{23}$$

The above first-order system of equations is then solved on Matlab using numerical technique "**BVP4C**".

IV. Results interpretation

The interpretation of obtained results have been discussed in this section. Further, variationin shape of NPs of TiO_2 nanofluid over a stretching surface has also been studied for bothvelocity and temperature fields in the following subsections.

4.1 Velocity field

Figure 1 shows the effect of multi-shape NPs of TiO_2 nanofluids keeping other physicalparameters fixed. It is clearly seen that a drastic changed is observed in velocity field whilechanging shapes of NPs. It should be noted that the velocity is high for platelet shape, andlow at spherical shape NPs.

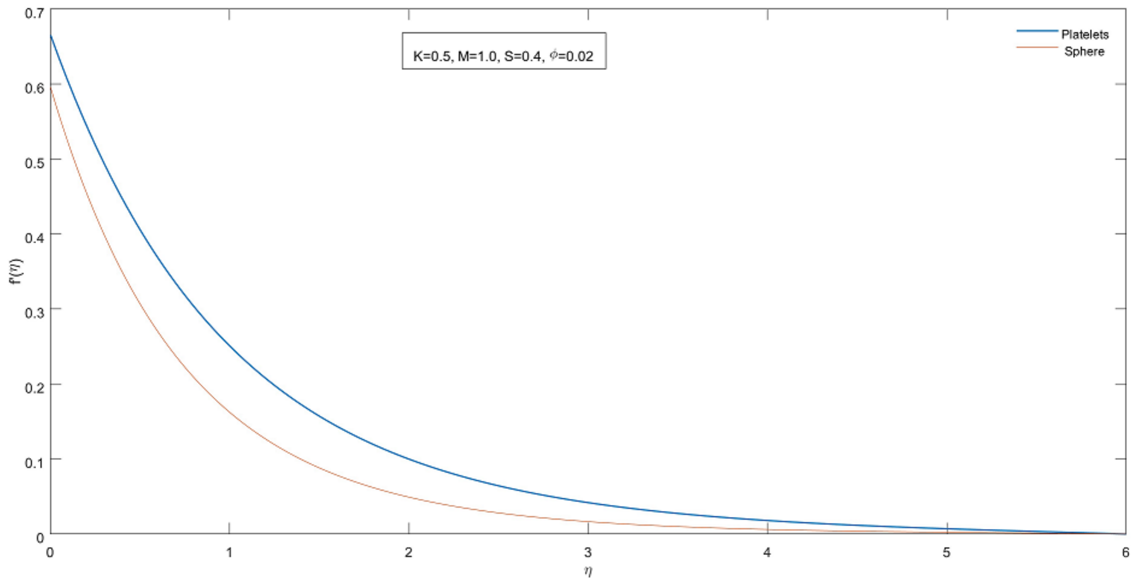

Figure 1: Effect of shape factor of NPs on velocity field

4.2 Temperature field

The effect of slip parameter K on temperature field distribution is plotted in Figure 2(a-b)that indicates the decay in temperature at different values of K. While in the case ofvolume-fraction parameter ϕ, the temperature field is getting increased for platelet shapeNPs, but the opposite trend is seen for spherical shape NPs as shown in Figure 3(a-b).Figure 4 (a-b) elucidates that the temperature field effected prominently for growing valuesof biot-number γ. Figure 5(a-b) represents the impact of the Eckert number on thetemperature field. By increasing the value of Eckert number, the temperature field gettinghigher. But in the case of Prandlt number, the temperature field reduces near the surface,whereas the opposite trend occurs near the free surface as shown in Figure 6(a-b). Finally,the effect of shape factor on temperature field are quite strong for platelet shape and lowfor spherical shape NPs, in Figure 7. Particularly, for multi-shape NPs, the dimensionless shear stress at the surface is calculatedand summarized in Table 3. The skin friction coefficient rises for magnetic M andunsteadiness S parameters, while the opposite is seen in the case of slip K and volumefractionparameters ϕ.

Table 3: Numerical values of skin friction

K	Physical ϕ	parameters M	S	Platelet $-Re^{\frac{1}{2}}C_f$	Sphere
0.5	--	--	--	0.66871951	0.80766488
1.0	--	--	--	0.49044125	0.56410362
1.5	--	--	--	0.38940407	0.43586071
--	**0.02**	--	--	0.66871951	0.80766488
--	**0.04**	--	--	0.56435350	0.80008536
--	**0.06**	--	--	0.48367517	0.79208929
--	--	**0.5**	--	0.61187959	0.74350633
--	--	**1.0**	--	0.66871951	0.80766488
--	--	**2.0**	--	0.75641958	0.90384095
0.5	0.02	1.0	**0.4**	0.66871951	0.80766488
--	--	--	**0.6**	0.68507079	0.82588767
--	--	--	**0.8**	0.70065034	0.84313550

Table 4 gives numerical results for heat transfer coefficients at the surface. It is obvioustthat increasing both the Prandtl-and Eckert-numbers gradually reduces the Nusselt number,whereas the rise in other physical parameters results the increase in Nusselt value.

Table 4: Numerical values of Nusselt number

Physical parameters						Platelet	Sphere
K	ϕ	Ec	S	Pr	γ	$Re^{-\frac{1}{2}}Nu$	
0.0	0.02	1.0	0.4	6.0	0.1	0.041573534	0.046972874
0.5	--	--	--	--	--	0.069456143	0.076476894
1.0	--	--	--	--	--	0.079386332	0.084760810
0.5	**0.00**	1.0	0.4	6.0	0.1	0.075735930	0.075735930
--	**0.02**	--	--	--	--	0.069456143	0.076476894

--	**0.04**	--	--	--	--	0.064506794	0.077090813
0.5	0.02	**0.0**	1.0	0.4	6.0	0.097035480	0.096898433
--	--	**0.5**	--	--	--	0.083245811	0.086687663
--	--	**1.0**	--	--	--	0.069456143	0.076476894
0.5	0.02		**0.4**	6.0	0.1	0.069456143	0.076476894
--	--		**0.6**	--	--	0.071838482	0.078550290
--	--		**0.8**	--	--	0.073740496	0.080176809
0.5	0.02	1.0	0.4	**4.0**	0.1	0.070815276	0.077512326
--	--	--	--	**6.0**	--	0.069456143	0.076476894
--	--	--	--	**8.0**	--	0.068473325	0.075677514
0.5	0.02	1.0	0.4	6.0	**0.2**	0.134912770	0.148352530
--	--	--	--	--	**0.4**	0.255134070	0.279866810
--	--	--	--	--	**0.6**	0.362939750	0.397255620

V. Conclusion

Effects of multi-shape NPs on electrically conducting TiO_2 nanofluids over exponentiallystretching surface have been investigated, numerically. The influence of the pertinent parameters on velocity and temperature fields have been displayed graphically and discussed,in detail. The findings of the present study are, as under

• An increment in slip parameter K results in decay of skin friction value.

• The velocity and thermal conductivity of TiO_2 nanofluid is maximum at plateletshape, and minimum at spherical shape NPs.

• The temperature profile has been increased by increasing the biot-number (γ).

• The skin friction coefficient is getting decreased for slip and volume fraction parameters, while increased at magnetic-and unsteadiness-parameters.

• The Nusselt number is getting increased for slip, unsteadiness, and biot number atboth Platelet and spherical shape NPs.

• By Changing the value of Prandtl-and Eckert-numbers yield the decrease in Nusseltnumber, while the opposite trend is seen in the case of volume fraction parameter.

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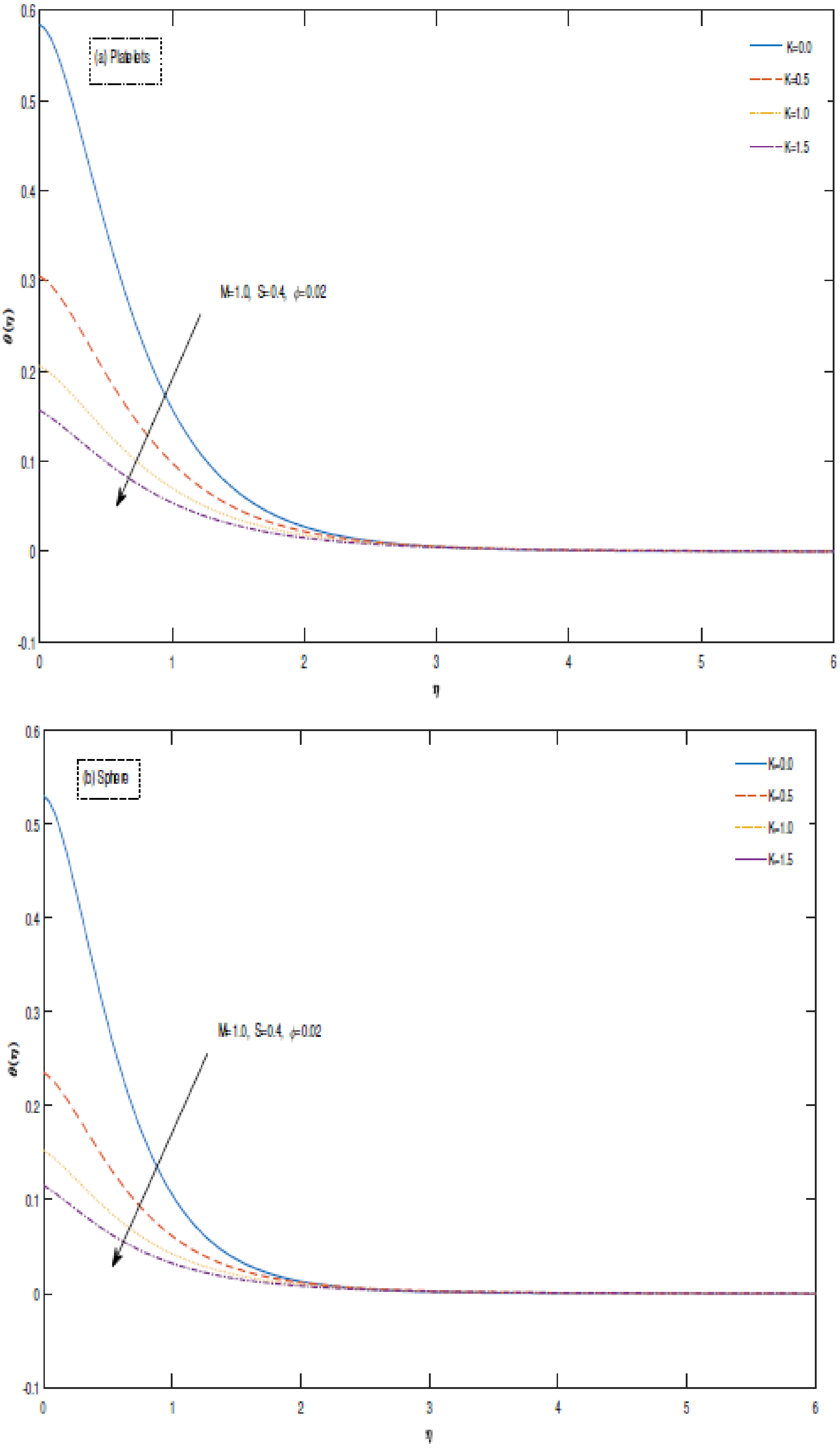


Figure 2: Effect of temperature profile changing slip K

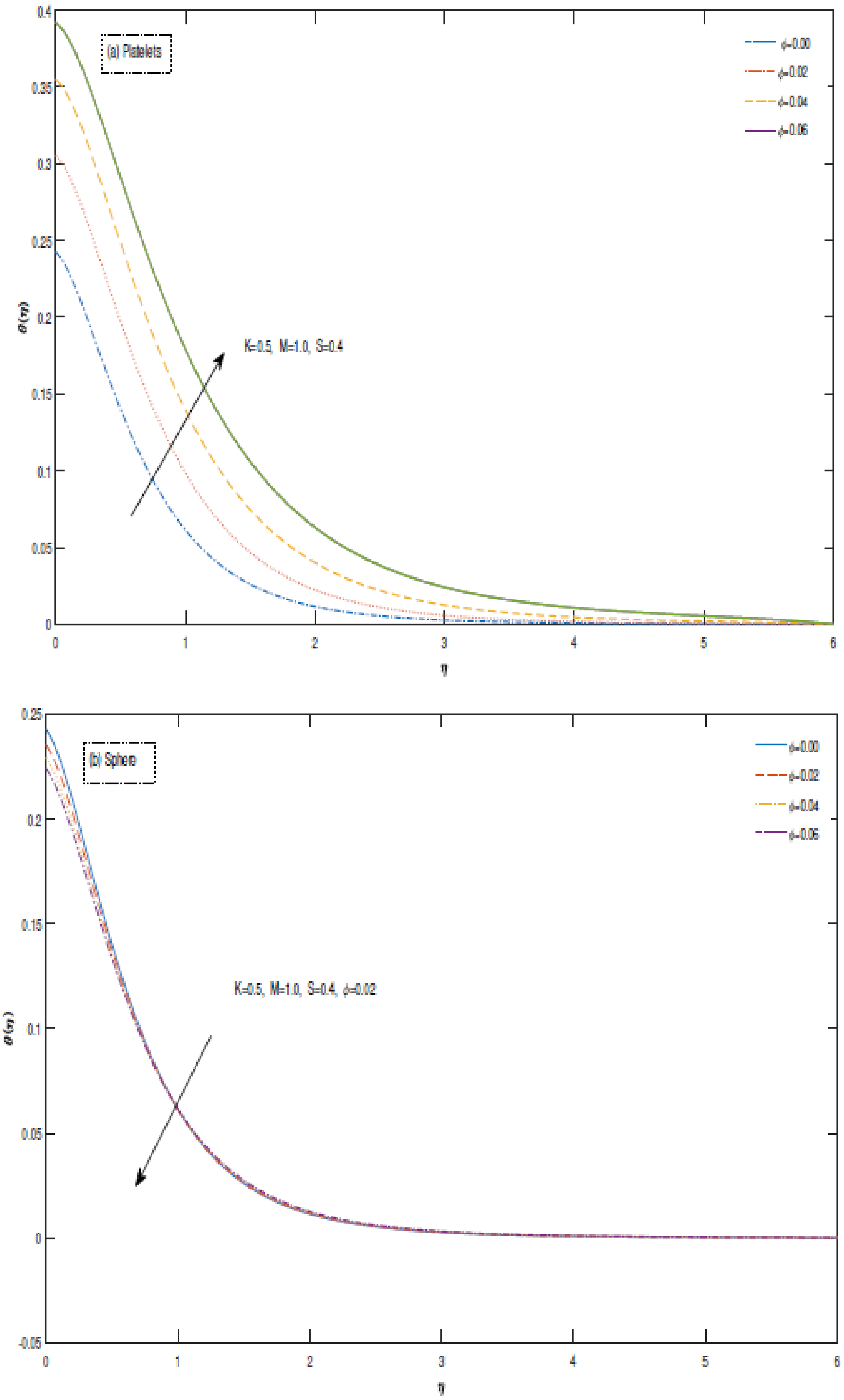
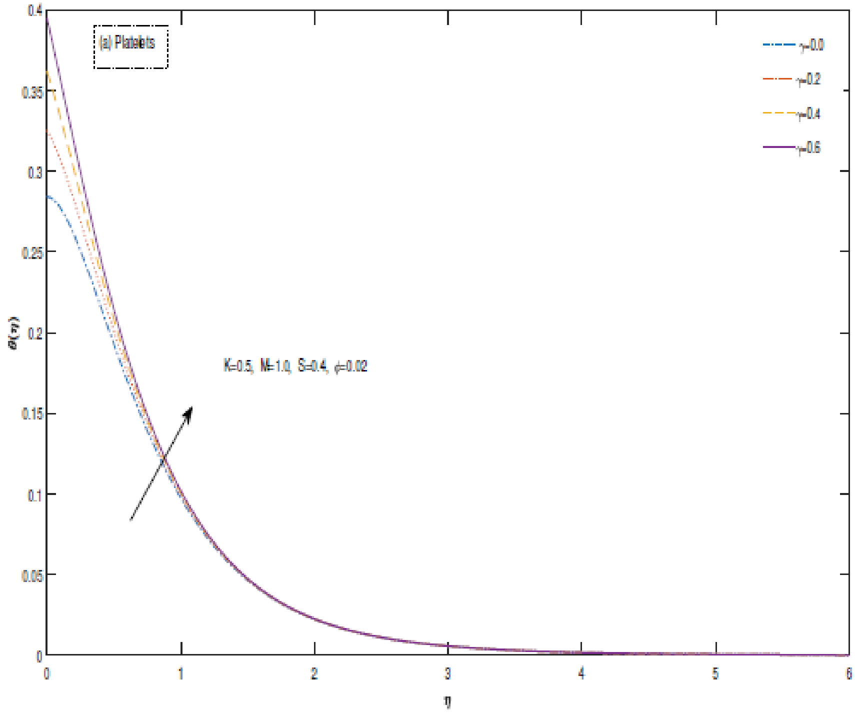


Figure 3: Effect of temperature profile changing volume fraction ϕ



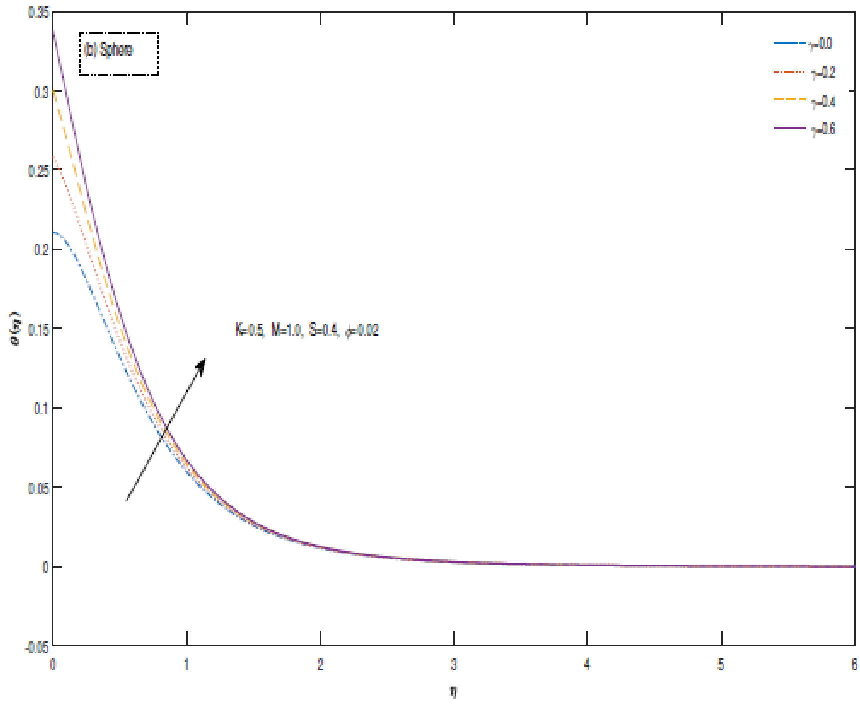


Figure 4: Effect of temperature profile changing biot γ

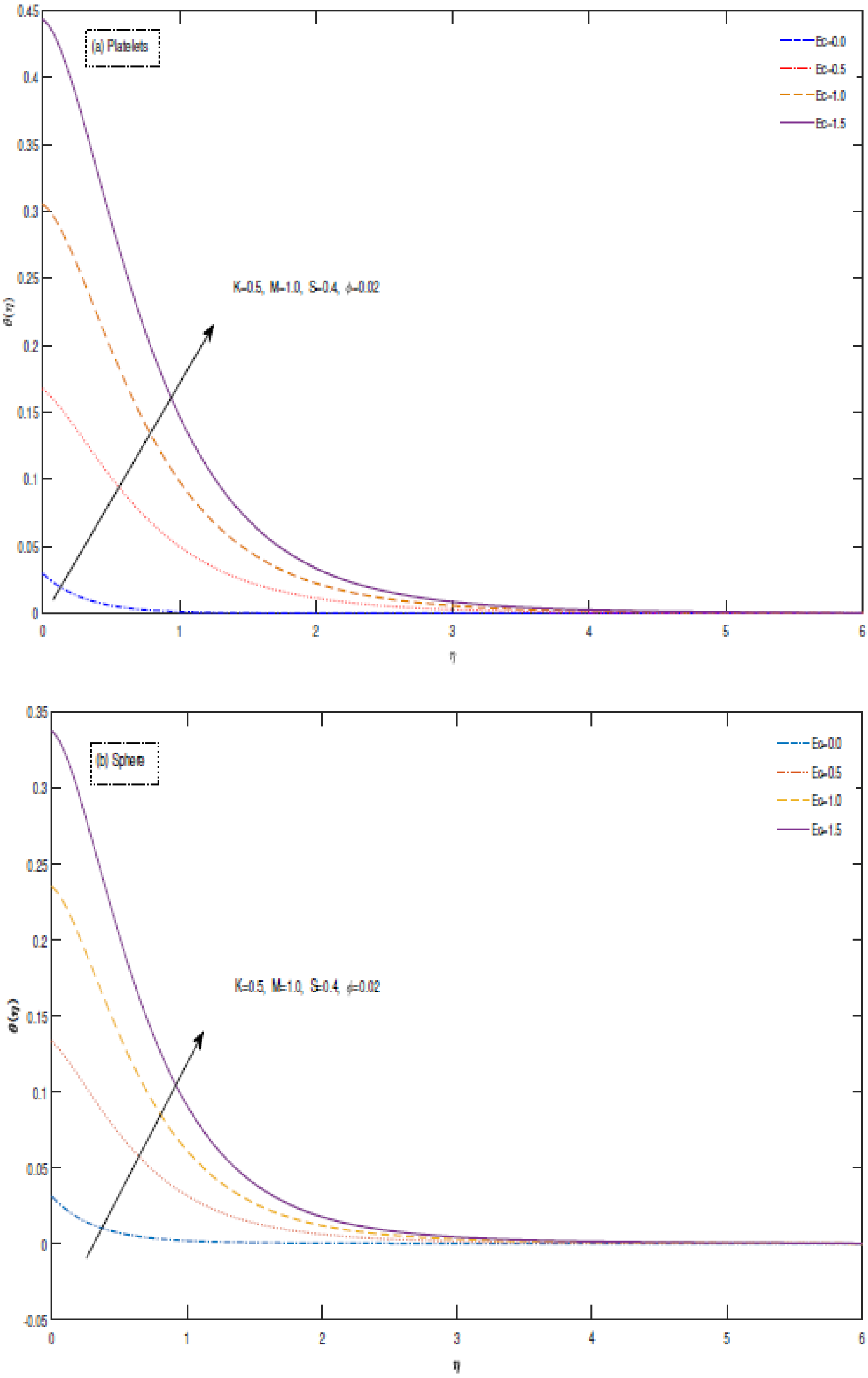


Figure 5: Effect of temperature profile changing Eckert *Ec*

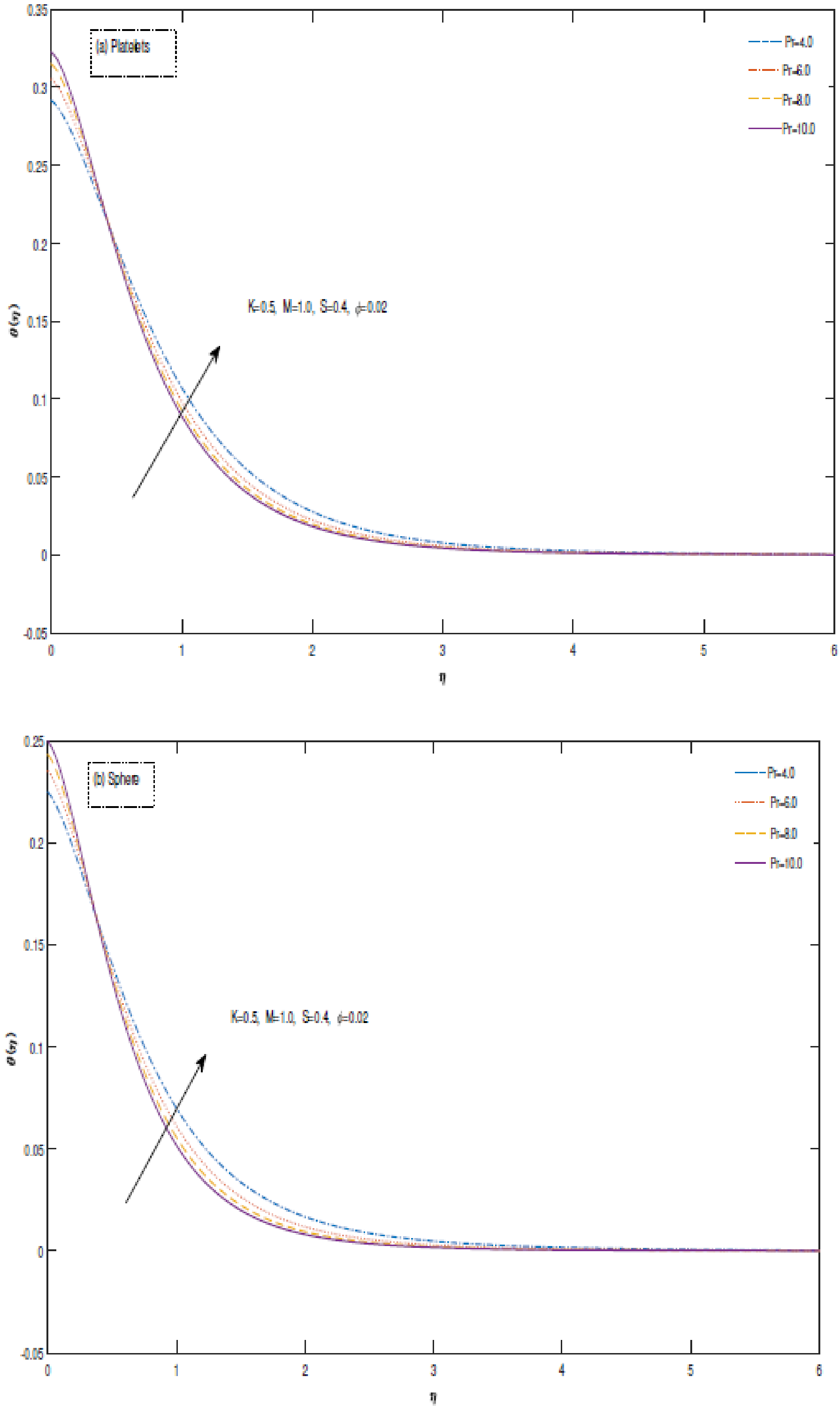


Figure 6: Effect of temperature profile changing Prandlt *Pr*

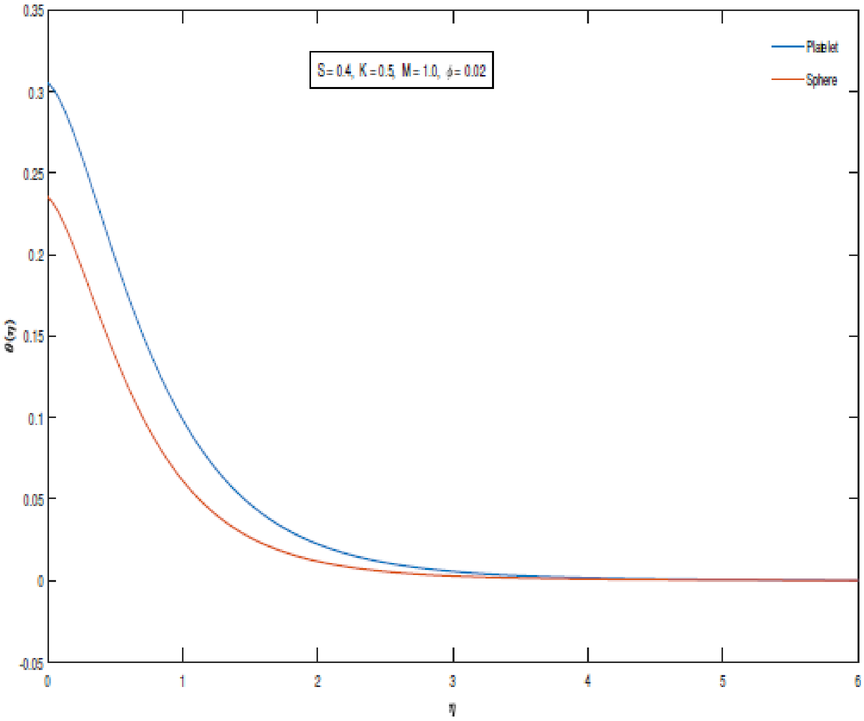


Figure 7: Effect of shape factor of NPs on Temperature field