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The Physics of Rolling Motion

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Abstract
Motion is the universal truth. From atomic scale to cosmological, the common state to both, all the physical objects and energy is the motion. The rest is simply a perception. The motion of energy is inform of wave while the motion of the objects is either pure translation, pure rotation or a combination of both which is known as rolling motion. In this communication, we are going to present the physics of rolling motion of a symmetrically mass distributed body about its centre of mass CM point (ring, disc, spherical shell, sphere etc.) or CM line (hollow cylinder, solid cylinder etc.) where CM, point or line as the case may be, is in pure translatory motion with respect to the surface and the body is in pure rotational motion with respect to its CM point or line.
Keywords: *Rolling Motion, Sliding forward, Sliding backward, Friction.*

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I. Introduction

In day-to-day life, every transport vehicle on road is engineered under the rolling motion of wheels attached. Along with the strength of theengineered axels to which the wheels are attached, the physics of rolling motion should be well understood to select the materials to design the wheels as well as roads and tracks such that the rolling motion should be efficient on one hand and in control of the operator on the other at the need of the hour. The concepts of physics involved in describing the rolling motion are: **centre of mass and its motion**, **angular motion about the CM** and **relation between angular motion and translational motion**of the point of contact between the rolling body and the surface, which may be **horizontal** or **inclined,smooth** or **rough**. Along with these, the **concept of frictional force** as sliding friction, is very crucial because in creating or maintaining rolling motion, it plays a very pivotal role. The frictional force always opposes the sliding tendency or such motions, but it can create motion too depending on how the system is designed and the frictional force appears, for example: frictional forces on bicycle wheels during paddling.

II. Motion of a Body

In case of a bound system, the motion of all parts of the system takes place about their common centre of mass and spin motion takes place about their individual centre of mass, point or line. In rotational motion there has to be a centripetal force according to the Newton's laws of motion to maintain the motion because in such motions at least there is a centripetal acceleration due to the change in the direction of velocity. In case of an open system, both, the bodies and surfaces are independent of each other till they are not in contact. When they come in contact, the contact forces come into picture (normal reaction, friction, etc.) in form of action-reaction. Out of these forces, the component of reaction force on the body parallel to the surfaces in contact appears in form of the frictional force called as sliding friction. It attains the maximum value known as limiting friction and if tangential pulling or pushing force component is just exceeding this limiting friction, themotion begins.

When motion begins, the friction is known as kinetic frictionwhich is slightly less than limiting friction. The kinetic friction is constant throughout the motionif surfaces are frictional. In case of smooth surfaces, the sliding friction as well as kinetic both are absent.For the inertial observer on the stationary surface, the translationalmotion of CM, point or line, of even a rolling body, is same as the translational motion of a single particle of velocity $\vec{V}_{CM}$.In case of a rigid body, this translational velocity $\vec{V}_{CM}$of CM of the body is for each point of the body irrespective of where is the point within or on the surface of the body. But, for the observer at CM i.e., CM observer's frame, all the points of the body (within and on the surface) are rest as for as translational motion is concerned. Hence, for a rigid body, in case of pure translational motion of a body, all the points of the body translate with the CM velocity $\vec{V}_{CM}$ for an inertial rest observer outside and at rest for the CM observer.

Suppose the axis of CM is fixed and the body can only rotate about it, then for outside inertial observer at rest, the $\vec{V}_{CM}$ is zero for all the points of the body including CM. Now, let's allow only rotational motion of the body about its CM axis, then we have as shown in Fig. 1:

- $\vec{V}_{CM} = \vec{0}$, for outside inertial observer at rest for all the points of the body.
- $\vec{V}_{CM} = \vec{0}$, for the CM observer.
- $\vec{V}_Q$ of any point Q other than CM is not equal to zero because of the angular velocity $\vec{\omega}$ of the body about its CM axis.
- At any instance of time, the linear velocity of any general point, say Q, is $\vec{V}_Q = \vec{\omega} \times \vec{r}$.

Since $\vec{\omega}$ and $\vec{r}$ are mutually perpendicular, we have $V_Q = \omega.r$. Hence, for the contact point P, we have $V_P = R\omega$ perpendicular to $\vec{R}$at P with respect to the CM and always tangential to the surface of the body at point contact.
Now, suppose the axis of CM is free to translate also and the body is free to spin about its CM axis, then in this case, the velocity of any general point Q with respect to the outside inertial observer at rest, we have:
$\vec{V}_Q = \vec{V}_{CM} + \vec{\omega} \times \vec{r}$ (1)

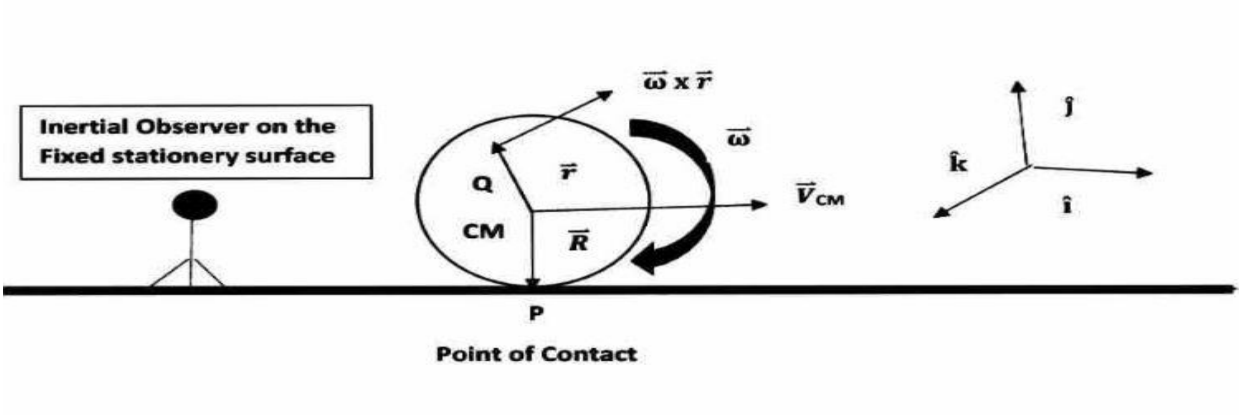


Fig.1: Motion on a symmetrical body on a stationary horizontal surface.

Knowing $\vec{V}_{CM}$, $\vec{\omega}$ and $\vec{r}$ we can find out the velocity of any point Q of the body. For the point of contact P between the surface and the body, we have:

$\vec{V}_P = \vec{V}_{CM} + \vec{\omega} \times \vec{R}$

or $\vec{V}_p = \vec{V}_{CM}\,\hat{i} + \omega\hat{k} \times R(-\hat{j})$

or $\vec{V}_p = \vec{V}_{CM}\,\hat{i} + R\omega(-\hat{i})$

or $\vec{V}_p = [\,V_{CM} - R\omega]\hat{i}$ (2)

Here, the quantitative values of V_{CM} and $R\omega$ along with the sense of spin (clockwise or anti-clockwise) with which the body is released, as $V_{CM} > R\omega$, $V_{CM} < R\omega$ or $V_{CM} = R\omega$ gives the various types of rolling motion on horizontal or inclined surfaces, smooth or frictional as:

- Rolling motion with sliding forward(clockwise spin or anti-clockwise spin).
- Rolling motion with sliding backward (clockwise spin or anti-clockwise spin).
- Pure rolling motion (clockwise spin or anti-clockwise spin).

III. Rolling Motion of a Body

3.1 Horizontal Surface:

Consider a horizontal surface at rest and uniform body of radius R. With respect to the surface, the CM of body may have linear velocity V_{CM}, from left to right,which will be the linear speed of all the points of the body hence that of P also, the point of contact of the body and the surface. If the body spins about its CM axis with angular velocity ω **clockwise** with respect to the reader, then the point P has another linear velocity due to spin of the body given by Rω and it will be opposite to the direction of V_{CM}. Hence, the net velocity of P with respect to the surface is given by Equ. (2) where V_P positive means along $\hat{i}$ direction i.e., forward-direction and $V_P$negative means along $(-\hat{i})$ direction i.e., backward direction. If VP = 0, the point of contact P is at rest with respect to the surface i.e., no sliding motion. We have three cases of rolling motion at t = 0:

i. When $V_P > 0$: Rolling motion with sliding forward [$V_{CM} > R\omega$]
ii. When $V_P < 0$: Rolling motion with sliding backward [$V_{CM} < R\omega$]
iii. When $V_P = 0$: Pure rolling motion [$V_{CM} = R\omega$]

If the body spins about its CM axis with angular velocity ω **anti-clockwise** with respect to the reader, then the point P has another linear velocity due to spin of the body given by Rω and it will be along the direction of V_{CM}

andV_P = (V_{CM} + Rω) in the forward direction. Hence, the net velocity of P with respect to the surface is in the forward direction. In this case, theV_P> 0 (always), and the bodywill always starts rolling motion with sliding forward.

CASE I: Smooth Horizontal Surface (μ=0):
Let the coefficient of friction (μ) of the surface and body is zero. In this case, even though there is a sliding motion, the frictional force, **f**will be zero.

(a) **$\vec{V}_{CM}$ = 0, and $\vec{\omega}$= 0att = 0:** The body will neither spin nor translate. Since, eithermotion is absent,hence the rolling motion will absent and body will be at rest as shown in Fig.2(a).

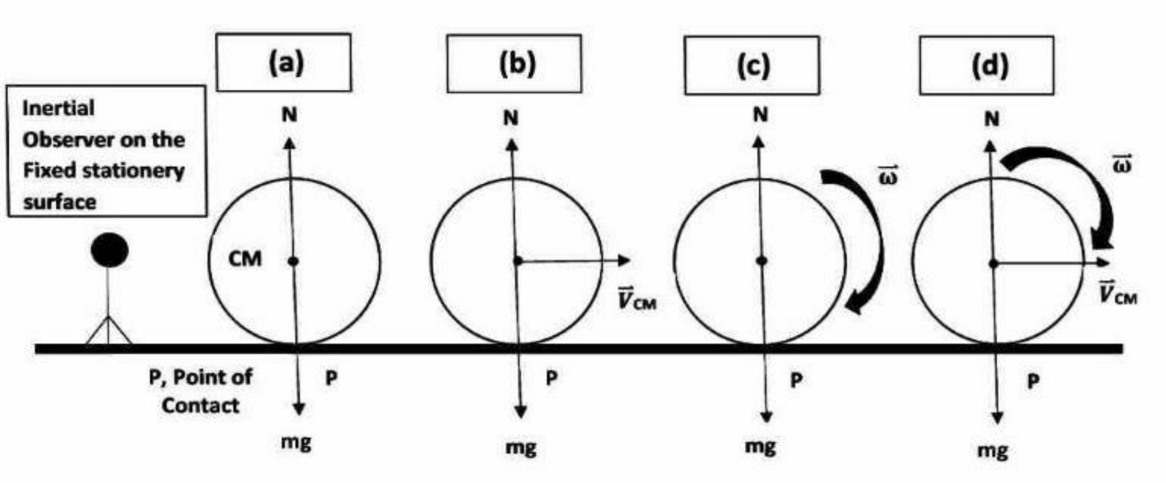


Fig.2: Rolling motion on a smooth horizontal surface: (a) $\vec{V}_{CM}=\vec{0}$, $\vec{\omega}=\vec{0}$;(b)$\vec{V}_{CM}\neq\vec{0}$,$\vec{\omega}=\vec{0}$; (c)$\vec{V}_{CM}=\vec{0}$, $\vec{\omega}\neq\vec{0}$; (d)$\vec{V}_{CM}\neq\vec{0}$, $\vec{\omega}\neq\vec{0}$

(b) $\vec{V}_{CM}\neq\vec{0}$ **and**$\vec{\omega}=\vec{0}$**at** $t=0$: As shown in Fig.2(b), in this case, forthe point of contact P of the body, the net velocity is given by, V_P = V_{CM}in the forward direction and the motion of the body will be purely sliding motion in the forward direction and it will continue.

(c) $\vec{V}_{CM}=\vec{0}$**and** $\vec{\omega}\neq\vec{0}$**at** $t=0$:In this case,if the spin is **clockwise**, at the point of contact, we have, V_P = -Rω, and the body will continue to spin clockwise with angular velocity ω at the same coordinate of CM of the body. Hence,purely rotational motion about CM axis with the passage of time, as shown in Fig.2(c). The same is true if the spin is **anti-clockwise**

(d) $\vec{V}_{CM}\neq\vec{0}$ **and** $\vec{\omega}\neq\vec{0}$**at** $t=0$:As shown in Fig.2(d), if the spin is **clockwise**, at the point of contact P, we have, V_P = (V_{CM} – Rω). Now, if V_{CM}> Rω, the body will go under – rolling motion with sliding forward; when V_{CM}< Rω, it will go under – rolling motion with sliding backward and when V_{CM} = Rω, there is a pure rolling motion of the body.All these motions are going to sustain with the passage of time depending on the relation between V_{CM} and Rω as above. If the spin is **anti-clockwise**, we have, V_P = (V_{CM+} Rω). Hence, in this case there is always rolling with sliding forward motion.

(e)

CASE II: Rough Horizontal Surface

Clockwise Spin

(a) **V_{CM}> Rωat t=0:**In case of $\vec{V}_{CM}=\vec{0}$and $\vec{\omega}=\vec{0}$, there is no sliding motion at all,hence there is no frictional force and body will be at rest.But, when V_{CM}> Rω, thenin this case, by Equ. (2) the V_P> 0 and it will be in the forward directionand the body will move under rolling motion with sliding forward and the frictional force will act in the backward direction as shown in Fig3(a).

This frictional force**f** plays two roles:

- Decreases V_{CM} by producing linear acceleration (A_{CM}) oppositely, and
- Increases ω by producing angular acceleration (α) in the same direction.

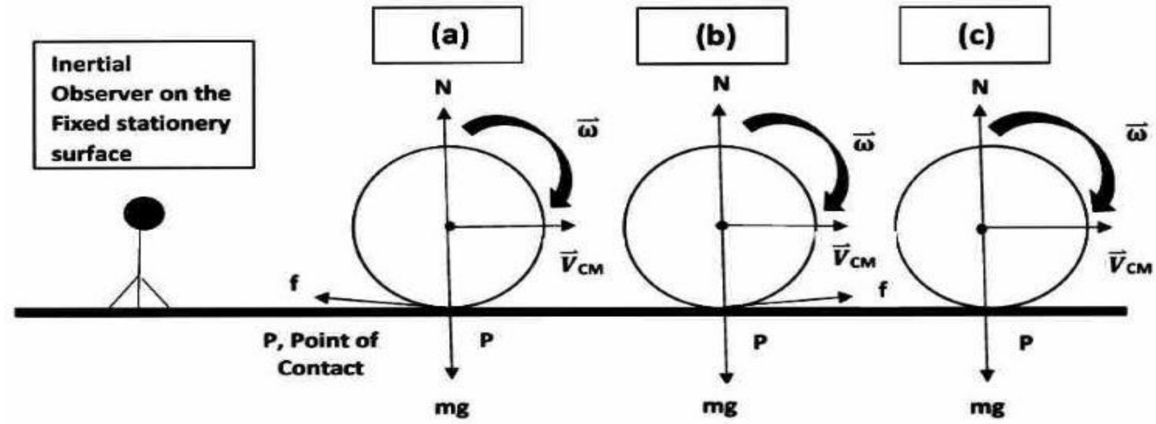


Fig.3: Rolling motion on a rough horizontal surface: (a) $\vec{V}_{CM} \neq \vec{0}, \vec{\omega} \neq \vec{0}, |\vec{V}_{CM}| > |R\omega|$; (b) $\vec{V}_{CM} \neq \vec{0}, \vec{\omega} \neq \vec{0}, |\vec{V}_{CM}| < |R\omega|$; (c) $\vec{V}_{CM} \neq \vec{0}, \vec{\omega} \neq \vec{0}, |\vec{V}_{CM}| = |R\omega|$

Hence, with the passage of time, V_{CM} decreases and $R\omega$ increases till the body attains $V_{CM} = R\omega$at the point of contact P i.e., $V_P = 0$. After this time,there is no sliding motion hence frictional force **f** seizes to exist and afterwards the body will go underpure rolling motionwith spinning clockwise and translating forward.

(b) $V_{CM} < R\omega$ at t =0: In this case as shown in fig.3(b), the body will move under rolling motion with sliding backward as by Equ. (2), $V_P < 0$. Now, the frictional force at P will be in forward direction and again**f** will play two roles same as in part (a). But, here, by producing A_{CM} it will increase $\vec{V}_{CM}$ and by producing α it will decrease the$\vec{\omega}$.That is, with the passage of time $\vec{V}_{CM}$ increases and $R\omega$ decreases and after sometime $V_{CM} = R\omega$. At this moment of time, sliding motion seizes and pure rolling motion starts and **f** seizes again in this case also. After this moment onwards, the body will go underpure rolling motion with spinning clockwise and translating forward.

(c) $V_{CM} = R\omega$ at t =0: Say $\vec{V}_{CM}$ and $\vec{\omega}$provided to the body at t = 0 are of such values that $V_{CM} = R\omega$, then there is no sliding at the point of contact P, at t = 0 itself because $V_P = 0$. Hence, **f** will be zero at t = 0 onwardsand V_{CM} and ω remain constant.Hence in this case, the body will move underpure rolling motion from t=0 itself and continue the same as shown in Fig.3(c).

Anti-clockwise Spin

(a) $V_{CM} > R\omega$ at t=0: In this case, $V_{CM}$and $R\omega$ both are in the forward direction and we have $V_P = V_{CM} + R\omega$. The motion begins as rollingwith sliding forward motion and anticlockwise spin. The **f**will act in the backward direction. Here,**f** will try to reduce both $V_{CM}$and ω.

- Let ω becomes zero before $V_{CM,}$ then body will still slide in the forward direction but seizes to spin at that moment. Now,**f** will spin it clockwise and $V_P = (V_{CM} - R\omega)$. With the passage of time, at some time, $V_{CM} = R\omega$ and $V_P = 0$. At this moment, no sliding and**f** seizes again and onwards the body will move as pure rolling motion in forward direction with clockwise spin.
- Let V_{CM} become zero before ω, then at that moment $V_P = R\omega$ and body will still slide forward but **f** will create V_{CM} in the backward direction. The body will roll with sliding forward direction and anti-clockwise spin. Now, we have,

$$V_P = (-V_{CM} + R\omega). \tag{5}$$

SinceRω is decreasing andV_{CM} is increasing, after some time, $V_P = 0$ and the body will move under pure rolling motion with anticlockwise spin and coming back towards the point of release.

In this case also at, t = 0, we have $V_P = (V_{CM} + R\omega)$ and body will start rolling motion with sliding forward and anticlockwise spin. The **f** will act in the backward direction which, tries to reduce both V_{CM} and Rω.

- Let ω become zeros before $V_{CM,}$ then at that moment $V_P = V_{CM,}$ the body will move assliding is forward and spin seizes.Now, **f** will start creating ω in clockwise sense and now we have, $V_P = (V_{CM} - R\omega)$. After some time, $V_{CM} = R\omega$ and $V_P = 0$, with no sliding motion and **f** seizes to exist. Here onwards, the body will move under pure rolling motion in forward direction with clockwise spin.
- In case, V_{CM} becomes zero before ω, then $V_P = R\omega$ i.e., sliding is still in forward direction and **f** will be still in the backward direction.Now, **f** creates V_{CM} in the backward direction and we have, $V_P = (-V_{CM} + R\omega)$, anticlockwise spin. After same time, when $V_{CM} = R\omega$ and $V_P = 0$, there is no sliding motion and **f** seizes to exist. Then onwards, there is a pure rolling motion with anticlockwise spin and moving back towards the point of release.

3.2 Inclined Surface:

Consider an inclined fixed surface with inclination angle θ with respect to the horizontal direction and a body of radius R is placed on the inclined surfaceas shown in Fig.4. The rolling type is determined by the nature of the surface as smooth or rough, the angle θ,V_{CM} and ω at t = 0,imparted to the body.

CASE I: Smooth Inclined Surface

In case of inclined surface, it is either smooth or rough, the components of weight mg as mgsinθ and mgcosθ act at the CM of the body and normal reaction component N passes through it. Hence, individually as well as collectively they are not capable to produce any spin motion about the CM of the body. There are two cases:

(a) $\vec{V}_{CM} = \vec{0}$ **and** $\vec{\omega} = \vec{0}$ **at t = 0:** Since there is no **f** to affect V_{CM} and ω in either way for increasing or decreasing because the surface is smooth. But A_{CM} = gsinθ along the surface of the inclined plane and with time it will create the V_{CM} as,

$V_{CM} = (0 + g\sin\theta \cdot t)$ (3)

Hence, as time passes, the body moves under-**purely uniform accelerated sliding motion** along the inclined plane with acceleration A_{CM} = gsinθ and α = 0.

(b) $\vec{V}_{CM} = \vec{0}$ **and** $\vec{\omega} \neq \vec{0}$ **at t = 0:** Described as in case (a) above, the V_{CM} will increase due to A_{CM} = gsinθ and ω will remain the same as at t = 0 because of **f=0**. If ω is in the sense that the body spins in **clockwise direction** with respect to the reader, then at the point of contact P,at any time t,

$V_P = (V_{CM} + R\omega)$ (4)

Hence, the body will- **roll down the slope under sliding forward motion with clockwise spin**. If ω is in the sense that the body spins in **anticlockwise direction** with respect to the reader, then at the point of contact P, at any time t,

$V_P = (V_{CM} - R\omega)$ (5)

Here ω is same but V_{CM} = (0 + gsinθ • t), which increases with time. Till $V_{CM} < R\omega$, the body will be rolling down the slope with sliding backward motion and anti-clockwise spin. At $V_{CM} = R\omega$, momentarily the rolling body attains the state of pure uniformly accelerated rolling motion. But just after that we have$V_{CM} > R\omega$, and now, the rolling body will go under-**uniformly accelerated rolling motion with sliding forward still with anti-clockwise spin.**

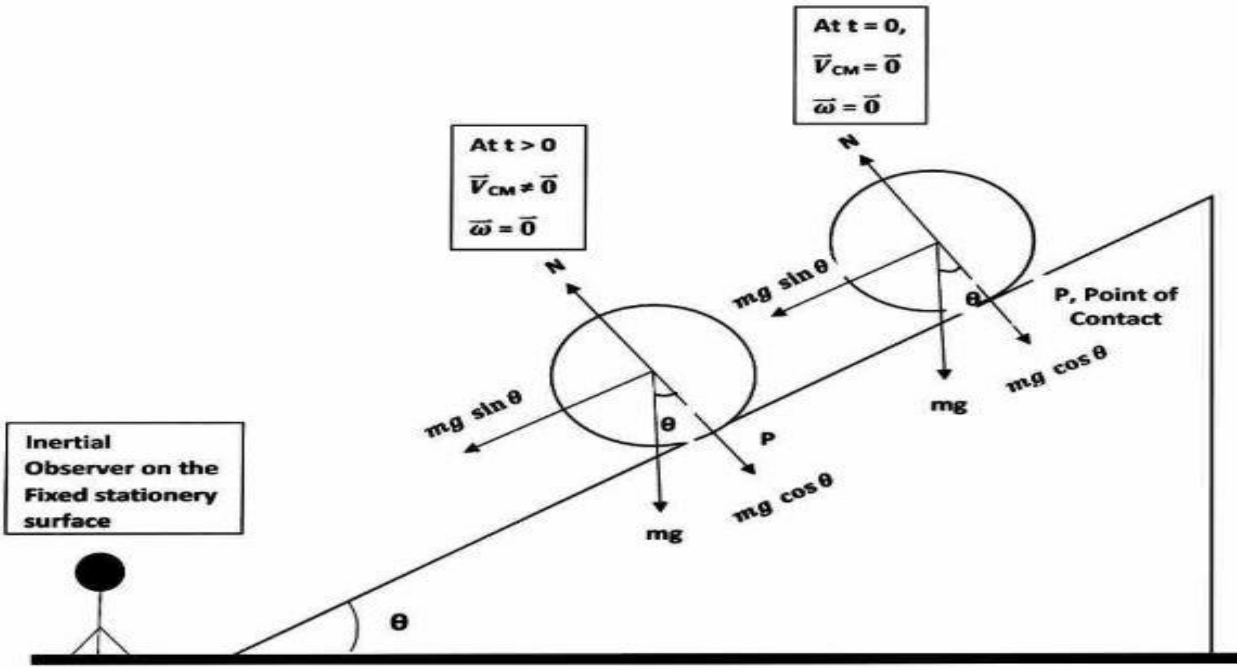


Fig.4: Rolling motion on a smooth inclined surface. (a)V_{CM} =0 and$\overline{\omega} = 0$, (b) V_{CM} =0 and $\overline{\omega} \neq 0$.

CASE II: Rough Inclined Surface

In case of rough inclined surface if there is any sliding motion of the body, there will be frictional force **f**. Since this force acts at the point of contact P, it will be able to create spin motion of the body about its CM axis by producing torque. The following cases are possible:

Clockwise Spin

(a) $\vec{V}_{CM} = \vec{0}$ **and** $\vec{\omega} \neq \vec{0}$ **at t = 0:** Since A_{CM} = gsinθ is always present from t = 0 itself, it will create V_{CM} with time. Along with this, due to the presence of ωin clockwise direction, the body will try to slide down the slope. The moment the body will slide down the slope, **f** will act at the point of contact P in the direction, up the slope as shown in Fig.5.

There are two situations: f = mgsinθor f < mgsinθ

- **Forf = mgsinθ:** The sliding friction**f**opposesthe mgsinθ and net force along the slope is zero. Further,by producing torque in anti-clockwise direction, it reduces$\vec{\omega}$initially provided. The body will slide due toω on the slope but it will not move down the slope and$\vec{V}_{CM} = \vec{0}$ with time. Hence, the body will spin at the point of placing in contact with the slope till $\vec{\omega}$becomes zero. After that, at any time t,$\vec{V}_{CM} = \vec{0}$, $\vec{\omega} = \vec{0}$, and the body will rest at the point of placement forever, as shown in Fig.5(a).
- **Forf < mgsinθ:** The sliding friction **f** stillopposes the mgsinθ but in this case, the net force along the slope is not zero.It will be directed down the slope and theV_{CM}will be producedwhere $V_P = (V_{CM}+R\omega)$. The body will slide down the slope and **f** will create the spin motion to the body about its CM axis opposite to the $\vec{\omega}$ initially provided and the body will **spin clockwise with sliding down** the slope with increasingV_{CM} and decreasingRω. Since, Rω is decreasing,at the time say t = $t_0$theRω becomes zero and the body attains **uniformly accelerated puresliding motion down** the slope.Just after that,**f** will spin the body in anti-clockwise direction with increasing ω and now, we have $V_P = (V_{CM} - R\omega)$ down the slope and the body will move as **uniformly accelerated sliding motion down**the slope with anti-clockwise spin.
- After some time say at t = t_1,V_{CM} = Rω andthere will be **uniformly accelerated pure rolling motion down**the slope with anti-clockwise spin. Here onwards V_{CM} is increases by A_{CM} and ω is increases by αwhere at any time t ≥ t,we haveA_{CM} = Rαand V_{CM} = Rω and it will be maintained throughout the motion. The equations of motion are:

$$mg\sin\theta - f = m \cdot A_{CM} \quad (6)$$
$$\tau\ (= I \cdot \alpha) = R \cdot f\sin 90^o \quad (7)$$
$$I = mk^2,\ A_{CM} = R \cdot \alpha \quad (8)$$
$$V_P = (V_{CM} - R\omega) = 0 \quad (9)$$
$$A_P = (A_{CM} - R \cdot \alpha) = 0 \quad 10)$$

where, we should have: $f \leq f_{limiting} (= \mu \cdot N)$.

Solving (6), (7) and (8), we get:

(a) $A_{CM} = \left[\frac{g\sin\theta}{1+\frac{R^2}{k^2}}\right]$

(b) $f = \left[\frac{mg\sin\theta}{1+\frac{k^2}{R^2}}\right]$ (11)

(c) $0 < \theta \leq \tan^{-1}\left[\mu\left(1+\frac{R^2}{k^2}\right)\right]$

where,V_{CM} = Rω, A_{CM} = Rα, V_P = 0 and A_P = 0 at uniformly accelerated pure rolling motion down the slope.

Now, the question is, when pure rolling will start if $\vec{V}_{CM} = \vec{0}$ and $\vec{\omega} = \vec{0}$? The first condition is that till: $f \leq f_{limiting}$ (= μmgcosθ) and this condition is formulated inform of the range of θ. When this condition is satisfied, at any time t, we have:

$$V_{CM} = 0 + A_{CM} \cdot t \text{ and } \omega = 0 + \alpha \cdot t$$

or $V_{CM} / \omega = A_{CM} / \alpha$ (12)

Substituting A_{CM} and α we get, $V_{CM} / \omega = R$ (13)

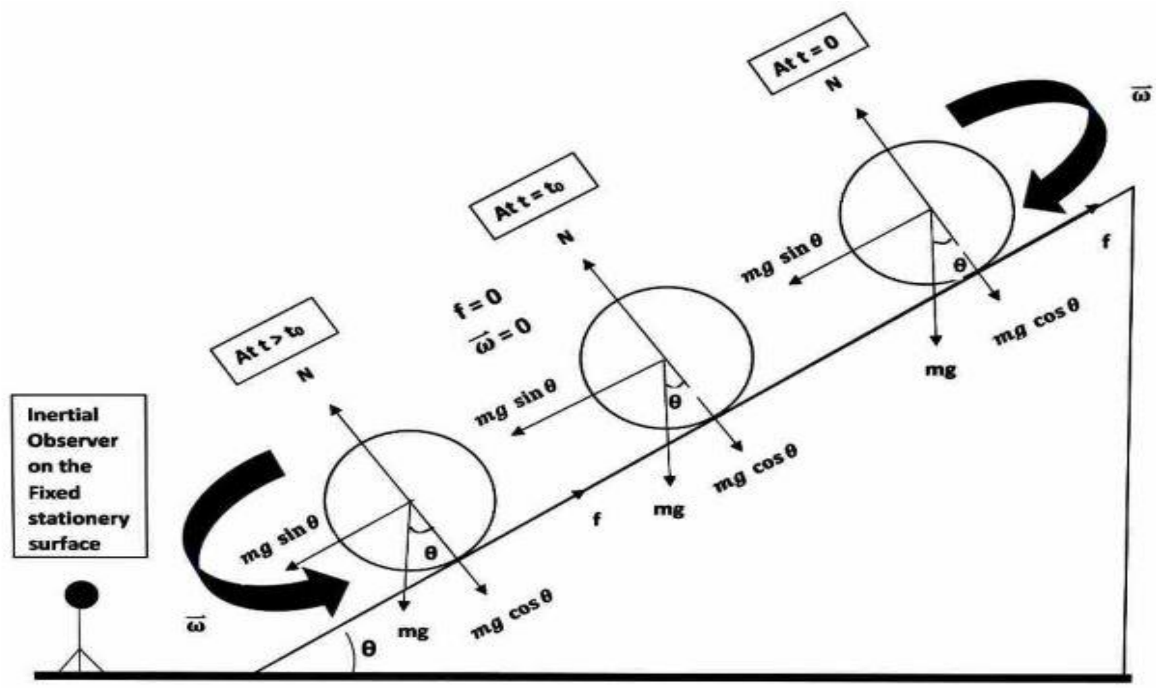


Fig.5: Rolling motion on a rough inclined surface $\vec{V}_{CM} = \vec{0}$ and $\vec{\omega} \neq \vec{0}$at t = 0 with clockwise spin, at t = t_0, $\vec{\omega}$ becomes zero and just after that uniformly accelerated rolling with sliding down the slope motion starts till t= t_1 at whichV_{CM} = Rω, A_{CM} = Rα and onwards uniformly accelerated pure rolling motion with anti-clockwise spin continues.

Therefore, if V_{CM}= 0, at t = 0, pure rolling starts from the beginning itself and V_{CM}= Rω and A_{CM} = Rα at any time t from the beginning when condition for θ is satisfied. Hence, the body will roll down the slope under uniformly accelerated pure rolling motion but because of the presence of A_{CM} and α both, with time, V_{CM} and ω increases but every time the relation of pure rolling V_{CM} = Rω and A_{CM} = Rα is satisfied. Hence, even at t = 0, V_{CM}= 0 and ω = 0, when $0 < \theta \leq \tan^{-1}[\mu(1+\frac{R^2}{k^2})]$, there is a uniformly accelerated pure rolling motion down the slope from the beginning of the motion itself.

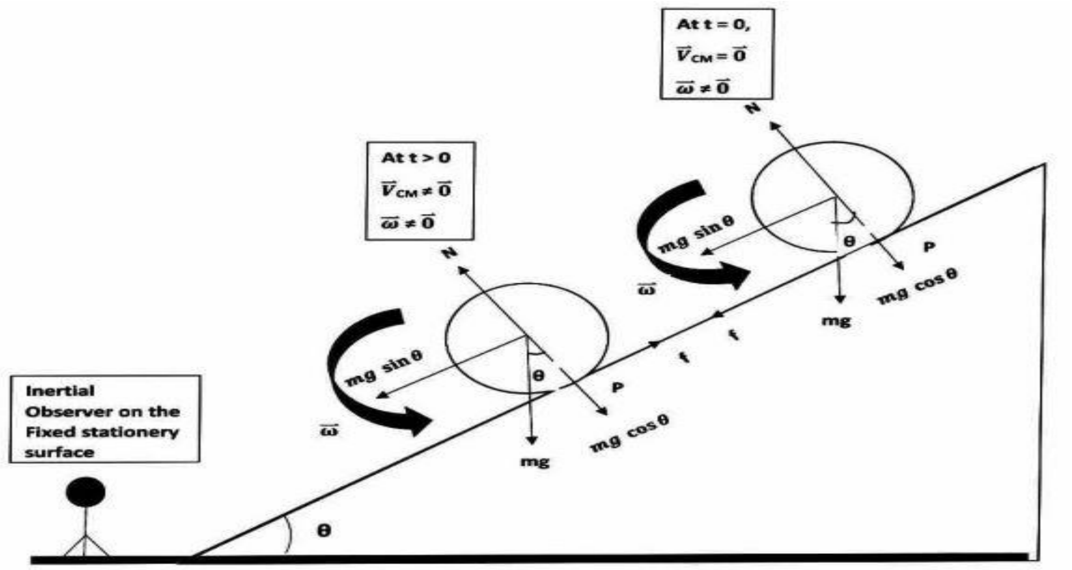
(b) **$\vec{V}_{CM} \neq \vec{0}$ and $\vec{\omega} = \vec{0}$ at t = 0:** The f will decrease the V_{CM} and increase ω in anti-clockwise spin with time. Till V_{CM}> Rω, there is a**uniformly accelerated rolling motion with sliding down** the slope. When V_{CM} = Rω is attained for $0 < \theta \leq \tan^{-1}[\mu(1+\frac{R^2}{k^2})]$ condition, there is a **uniformly accelerated pure rolling motion down** the slope with expressions given in Equ. (11)and onwards this motion will continue.

(c) **$\vec{V}_{CM} \neq \vec{0}$ and $\vec{\omega} \neq \vec{0}$ at t = 0:** In this case, as discussed above, depends on what is the sense of spin at which the body is released and what is the numerical relation between V_{CM} and Rω for the contact point P. Let, the body is released with clockwise spin, then we have: $V_P = (V_{CM} + R\omega)$

Hence, rolling with sliding down the slope and f will act up the slope. f will decrease both V_{CM} and ω. Let at time t,$\vec{V}_{CM} \neq \vec{0}$ and $\vec{\omega} = \vec{0}$ case (b) will appear onwards. Let at time t, $\vec{V}_{CM} = \vec{0}$ and $\vec{\omega} \neq \vec{0}$, case (a) will appear onwards. Let at time t, V_{CM} = Rω, the uniformly accelerated pure rolling motion takes place down the slope.

Anti-clockwise Spin

(a) **$\vec{V}_{CM} = \vec{0}$ and $\vec{\omega} \neq \vec{0}$ at t = 0:**With respect to reader point of view, say spin is anti- clockwise then at t = 0, f will act down the slope at the point of contact P, which will try to decrease ω by producing torque oppositely. But the A_{CM}will produce V_{CM} with time. Since at t = 0, $V_P = (0 - R\omega)$,the body will be in rollingmotion (positive down the slope and negative up the slope) with sliding up the slope till V_{CM} = Rω. As shown in Fig.6.



(b)

Fig.6: Rolling motion on a rough inclined surface in anti-clockwise direction. (a) $\vec{V}_{CM} = \vec{0}$ and $\vec{\omega} \neq \vec{0}$; $V_P = (V_{CM} - R\omega)$ (b) $\vec{V}_{CM} \neq \vec{0}$ and $\vec{\omega} \neq 0$ and $V_{CM} = R\omega$.

Now, at $V_{CM} = R\omega$, the rolling body will roll down the slope with sliding forward motion. After this when $V_{CM} = R\omega$ it will attain: **uniformly accelerated pure rolling motion down** the slope. Since, at t = 0, the rolling body spins anticlockwise, f will be down the slope. It will support the mgsinθ to have more A_{CM} hence increasing V_{CM} but decreasing ω. In this, at t = 0, we have $V_P = (0 - R\omega)$ and at time t, $V_P = (V_{CM} - R\omega)$ where V_{CM} increases from zero and ω decreases. At $V_{CM} = R\omega$, $V_P = 0$ and the rolling body will attain the uniformly accelerated pure rolling motion down the slope. Just after this, due to increase in V_{CM}, the body tries to slide down the slope and because of this the f will change the direction and becomes up the slope and increases ω and the relation $V_{CM} = R\omega$ is maintained. Onwards, the body moves as **uniformly accelerated pure rolling motion down** the slope.

(c) $\vec{V}_{CM} \neq \vec{0}$ and $\vec{\omega} \neq \vec{0}$ at t = 0: In this case, as discussed above, depends on what is the sense of spin at which the body is released and what is the numerical relation between V_{CM} and Rω for the contact point P. Let, the rolling body is released with anticlockwise spin, then we have: $V_P = (V_{CM} - R\omega)$

Now, if $V_{CM} > R\omega$: body slides down the slope with rolling motion, **f** will act up the slope and finally the motion attained will be uniformly accelerated pure rolling motion at $V_{CM} = R\omega$, because till this stage **f** will decrease V_{CM} and increase ω. Now if, $V_{CM} < R\omega$: rolling body slides up the slope with rolling motion down and **f** will act down the slope which will increase V_{CM} and decrease ω till $V_{CM} = R\omega$ and finally the motion attained is uniformly accelerated pure rolling motion.

IV. Conclusions:

i. In case of rolling motion on a smooth surface either horizontal or inclined there is no frictional force acting on the body, sliding forward, sliding backward or purerolling motion.

ii. In case of smooth surfaces, if the surface is horizontal, the spin and translational motion created at t = 0, sustains throughout the motion. But if the surface is inclined, the spin motion sustains as it was imparted at t = 0, but the translational motion attains the uniformly accelerated down the slope motion.

iii. $\vec{V}_{CM} = \vec{0}$ and $\vec{\omega} \neq \vec{0}$ with**clockwisespin** for rough surfaces, if the surface is horizontal, the motion is: rolling with sliding backward and finally attains the uniform pure rolling motion as $V_{CM} = R\omega$ and $V_P = 0$, in which, with passage of time V_{CM} and ω are constant and $A_{CM} = 0$ and $\alpha = 0$ because of the absence of **f** and after that pure rolling motion is attainedin the backward direction with clockwise spin.

iv. $\vec{V}_{CM} = \vec{0}$ and $\vec{\omega} \neq \vec{0}$ with**anti-clockwisespin**for rough surfaces, if the surface is horizontal, the motion is: rolling with sliding forward and finally attains the uniform pure rolling motion as $V_{CM} = R\omega$ and $V_P = 0$,in which, with passage of time V_{CM} and ω are constant and $A_{CM} = 0$ and $\alpha = 0$ because of the absence of **f** and after that pure rolling motion is attained in the forward direction with anti-clockwise spin.

v. In case of inclined smooth surfaces, the spin of release is maintainedbut there is uniformlydecelerated / accelerated motion up or down the slope depending on the initial sense of V_{CM}released respectively. But, the final state of rolling motion will be uniformly accelerated rolling motion with sliding down the slope.

vi. In case of rough inclined surfaces, it depends upon at t = 0 what is the sense of the spin and what are the values of V_{CM} and ω:

- $\vec{V}_{CM} = \vec{0}$ and $\vec{\omega} \neq \vec{0}$with**clockwise spin**of the body, **f** is always up the slope.**For, f = mgsinθ,**$\vec{V}_{CM} = \vec{0}$, ω decreases and by $V_P = (V_{CM} + R\omega)$, ω is going to vanish at some time and the body attains rest position at that point. **For, f < mgsinθ,** $\vec{V}_{CM}$ increases, ω decreases to zero and just after that spin reverses, ω increases and we get $V_P = (V_{CM} - R\omega)$ and there is a rolling with sliding down the slope motion. After some time $V_{CM} = R\omega$ and$V_P = 0$. Here onwards the body will move as uniformly accelerated pure rolling motion with anti-clockwise spin. In case of $\vec{V}_{CM} = \vec{0}$ and $\vec{\omega} \neq \vec{0}$with**anti-clockwise spin** at t = 0, $V_P = (0 - R\omega)$, **f** is down the slope. The V_P is negative till $V_{CM} = R\omega$. So, till $V_{CM} = R\omega$, the body will roll down with sliding up the slope and at $V_{CM} = R\omega$ and onwards**f** is up the slope and the body willmoveunder anticlockwise spin with uniformly accelerated pure rolling motiondown the slope.
- $\vec{V}_{CM} \neq \vec{0}$ and $\vec{\omega} = \vec{0}$, **f** will act up the slope, spin will be anti-clockwise, decreasing$\vec{V}_{CM}$and increasing ω. Since $V_P = (V_{CM} - 0)$ at t=0 and after some time t, $V_P = V_{CM} - R\omega$. So, at the time when $V_{CM} = R\omega$, onwards the uniformly accelerated pure rolling motion down the slope with anticlockwise spin.
- $\vec{V}_{CM} \neq \vec{0}$ and $\vec{\omega} \neq \vec{0}$, spin is clockwise or anticlockwise, by $V_P = V_{CM} + R\omega$ or $V_P = V_{CM} - R\omega$ respectively.The results will be either of $\vec{V}_{CM} = \vec{0}$, $\vec{\omega} \neq \vec{0}$ or $\vec{V}_{CM} \neq \vec{0}, \vec{\omega} = \vec{0}$but after some time,finally, in any situation there will be uniformly accelerated pure rolling motion down the slope with anticlockwise spin.

vii. In case of rough horizontal surface finally there is uniform pure rolling with clockwise spin if projected left to right and frictional force is absent.

viii. In case of rough inclined surface finally there is uniformly accelerated pure rolling motion down the slope with the anti-clockwise spin if the slope is left to right and clockwise spin if the slope is right to left with respect to the reader.

V. Nomenclature:

CM: Centre of mass of a rolling body.
$\vec{V}_{CM}$: Velocity of CM of the rolling body.
A_{CM}: Acceleration of the centre of mass.
$\vec{V}_Q$:Velocity of any point Q of the rolling body.
$\vec{\omega}$: Angular-velocity of the rolling body about its CM.
$\vec{r}$: Position vector of point Q with respect to the CM of the rolling body
$\vec{R}$: Radius vector of the point of contact P between rolling body and the surface.
$\vec{V}_P$: Velocity of the point of contact P of the rolling body with respect to the surface.
f: Sliding friction (assumed as kinetic friction too).
μ: Coefficient of friction.
m: Mass of the rolling body.
g: Acceleration due to gravity at the place.
k: Radius of gyration of the rolling body.
t: Time

VI. Suggested Work:

How to describe the rolling motion of a symmetrical body in shape, but due to asymmetrical mass distribution,CM of the body does not coincide with the axis of rotation.

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