

Fixed Point Theorems Under Strict Contractive Conditions In Fuzzy Metric Spaces

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ABSTRACT: *In this paper, by using property (E.A.), we prove common fixed point theorems for weakly compatible mappings satisfying strict contractive condition in fuzzy metric spaces.*

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I. INTRODUCTION:

In 1986, Jungck [7] introduced the concept of Compatible mapping and proved some common fixed point theorems of compatible mappings in metric space. The notion of fuzzy sets was introduced by Zadeh [17]. Various concepts of fuzzy metric spaces were considered in [8, 9]. Many authors have studied fixed point theory in fuzzy metric spaces. The authors [4, 5, 13, 14] have proved fixed point theorems in fuzzy (probabilistic) metric spaces. It is well known that the probabilistic metric space is an important generalization of metric space (see [14]). Fixed point theory in probabilistic metric spaces can be considered as a part of probabilistic analysis, which is a very dynamic area of mathematical research.

Definition 1.1:

A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t-norm if it satisfies the following conditions.

- 1) $*$ is associative and commutative,
- 2) $*$ is continuous
- 3) $a*1=a$ for all $a \in [0,1]$
- 4) $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$, for each $a,b,c,d \in [0,1]$.

Two typical examples of continuous t-norm are $a*b = ab$ and $a*b = \min(a, b)$.

Definition 1.2:

A 3-tuple $(X, \mathcal{M}, *)$ is called a fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm, and \mathcal{M} is a fuzzy set on $X^2 \times (0, \infty)$, satisfying the following conditions for each $x, y, z \in X$ and $t, s > 0$,

- 1) $\mathcal{M}(x, y, t) > 0$,
- 2) $\mathcal{M}(x, y, t) = 1$ if and only if $x = y$,
- 3) $\mathcal{M}(x, y, t) = \mathcal{M}(p\{x, y\}, t)$ (symmetry) where p is a permutation function,
- 4) $\mathcal{M}(x, y, t) * \mathcal{M}(y, z, s) \leq \mathcal{M}(x, z, t+s)$
- 5) $\mathcal{M}(x, y, \cdot): (0, \infty) \rightarrow [0,1]$ is continuous

Example 1.3:

Let X be a non-empty set and D is the D-metric on X . Denote $a*b = a.b$ for all $a,b \in [0,1]$.

For each $t \in (0, \infty)$, define $\mathcal{M}(x, y, t) = t / [t + D(x,y)]$ for all $x, y \in X$. It is easy to see that $(X, \mathcal{M}, *)$ is a \mathcal{M} -fuzzy metric space.

Definition 1.4:

Let A and S be mappings from a fuzzy metric space $(X, \mathcal{M}, *)$ into itself. Then the mappings are said to be weak compatible if they commute at their coincidence point, that is $Ax = Sx$ implies that $ASx = SAx$.

Definition 1.5:

Let A and B be two self-mappings of a fuzzy metric space $(X, \mathcal{M}, *)$. We say that A and B satisfy the property (E-A), if there exists a sequence $\{x_n\}$ such that $\lim_{n \rightarrow \infty} \mathcal{M}(Ax_n, u, t) = \lim_{n \rightarrow \infty} \mathcal{M}(Bx_n, u, t) = 1$ for same $u \in X$ and $t > 0$.

II. MAIN RESULTS:

Let \mathcal{F} be the set of all fuzzy set on $X^2 \times (0, \infty)$ that is $\mathcal{F} = \{f: X^2 \times (0, \infty) \rightarrow [0,1]\}$.

Definition 2.1:

Let f and $g \in \mathcal{F}$. The algebraic sum $f \oplus g$ of f and g is defined by

$$f(x, y, t) \oplus g(x', y', t) = \sup_{t_1+t_2=t} \min\{f(x, y, t_1), g(x', y', t_2)\}$$

Throughout this section Φ denotes a family of mappings such that for each $\varphi \in \Phi$, $\varphi: [0,1]^2 \rightarrow [0,1]$ is continuous and increasing in each co-ordinate variable. Also $\gamma(t) = \varphi(t, t) \geq t$ for every $t \in [0,1]$.

Now, we prove a common fixed point theorem using a property (E-A).

Theorem 2.2:

Let A, B, S and T be mappings from a fuzzy metric space $(X, \mathcal{M}, *)$ into itself satisfying the following conditions. $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$ → (2.1).

$$\mathcal{M}(Ax, By, t) \geq \varphi \left\{ \begin{array}{l} \mathcal{M}\left(Sx, Ty, \frac{2t}{k}\right), \mathcal{M}\left(Ax, Sx, \frac{2t}{k}\right) \oplus \mathcal{M}\left(By, Ty, \frac{2t}{k}\right), \\ \mathcal{M}\left(Ax, Ty, \frac{4t}{k}\right) \oplus \mathcal{M}\left(Sx, By, \frac{4t}{k}\right) \end{array} \right\} \rightarrow (2.2).$$

for all $x, y \in X$, $t > 0$, $\varphi \in \Phi$, and $0 \leq k < 2$. Suppose that one of the pairs (A, S) and (B, T) satisfies the property (E, A). (A, S) and (B, T) are weakly compatible and one of $A(X)$, $B(X)$, $S(X)$ and $T(X)$ is a complete subspace of X . Then A, B, S and T have a unique common fixed point in X .

Proof:

Suppose that the pair (B, T) satisfies the property (E, A) then there exists a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} \mathcal{M}(Bx_n, z, t) = \lim_{n \rightarrow \infty} \mathcal{M}(Tx_n, z, t) = 1$ for some $z \in X$ and all $t > 0$. Therefore $\lim_{n \rightarrow \infty} \mathcal{M}(Bx_n, Tx_n, t) = 1$ since $B(X) \subseteq S(X)$.

There exists a sequence $\{Y_n\}$ in X such that $Bx_n = SY_n$, hence $\lim_{n \rightarrow \infty} \mathcal{M}(Sy_n, z, t) = 1$.

We prove that $\lim_{n \rightarrow \infty} \mathcal{M}(Ay_n, z, t) = 1$. Using (2.2) we have

$$\begin{aligned} \mathcal{M}(Ay_n, Bx_n, t) &\geq \varphi \left\{ \begin{array}{l} \mathcal{M}\left(Sy_n, Tx_n, \frac{2t}{k}\right), \mathcal{M}\left(Ay_n, Sy_n, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right), \\ \mathcal{M}\left(Ay_n, Tx_n, \frac{4t}{k}\right) \oplus \mathcal{M}\left(Sy_n, Bx_n, \frac{4t}{k}\right) \end{array} \right\} \\ &= \varphi \left(\begin{array}{l} \mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right), \mathcal{M}\left(Ay_n, Bx_n, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right), \\ \mathcal{M}\left(Ay_n, Tx_n, \frac{4t}{k}\right) \oplus 1 \end{array} \right) \rightarrow (2.3). \end{aligned}$$

$$\begin{aligned} \text{Since, } \liminf_{n \rightarrow \infty} \mathcal{M}\left(Ay_n, Bx_n, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right) \\ \geq \liminf_{n \rightarrow \infty} \min \left\{ \mathcal{M}\left(Ay_n, Bx_n, \frac{2t}{k} - \epsilon\right), \mathcal{M}\left(Bx_n, Tx_n, \epsilon\right) \right\} \\ = \liminf_{n \rightarrow \infty} \mathcal{M}\left\{ Ay_n, Bx_n, \frac{2t}{k} - \epsilon \right\} \end{aligned}$$

letting $\epsilon \rightarrow 0$, in the above inequality, we get

$$\liminf_{n \rightarrow \infty} \mathcal{M}\left\{ Ay_n, Bx_n, \frac{2t}{k} \right\} \oplus \mathcal{M}\left\{ Bx_n, Tx_n, \frac{2t}{k} \right\} \geq \liminf_{n \rightarrow \infty} \mathcal{M}\left(Ay_n, Bx_n, \frac{2t}{k}\right).$$

$$\begin{aligned} \text{Also, by remark 2.2, } \liminf_{n \rightarrow \infty} \mathcal{M}\left(Ay_n, Tx_n, \frac{4t}{k}\right) \oplus \mathcal{M}\left(Sy_n, Bx_n, \frac{4t}{k}\right) &= \liminf_{n \rightarrow \infty} \mathcal{M}\left(Ay_n, Tx_n, \frac{4t}{k}\right) \oplus 1 \\ &\geq \liminf_{n \rightarrow \infty} \mathcal{M}\left(Ay_n, Tx_n, \frac{2t}{k}\right), \end{aligned}$$

hence letting $n \rightarrow \infty$, in inequality (2.3), we get,

$$\liminf_{n \rightarrow \infty} \mathcal{M}(Ay_n, z, t) = \mathcal{M}\left(\liminf_{n \rightarrow \infty} Ay_n, z, t\right) = \liminf_{n \rightarrow \infty} \mathcal{M}(Ay_n, Bx_n, t)$$

$$\begin{aligned}
 &\geq \varphi \liminf_{n \rightarrow \infty} \mathcal{M} \left(Bx_n, Tx_n, \frac{2t}{k} \right), \liminf_{n \rightarrow \infty} \left\{ \mathcal{M} \left(Ay_n, Bx_n, \frac{2t}{k} \right) \oplus \mathcal{M} \left(Bx_n, Tx_n, \frac{2t}{k} \right) \right\}, \\
 \liminf_{n \rightarrow \infty} \left\{ \mathcal{M} \left(Ay_n, Tx_n, \frac{4t}{k} \right) \oplus 1 \right\} &\geq \\
 &\quad \varphi \left(1, \liminf_{n \rightarrow \infty} \mathcal{M} \left(Ay_n, Bx_n, \frac{2t}{k} \right), \liminf_{n \rightarrow \infty} \mathcal{M} \left(Ay_n, Tx_n, \frac{2t}{k} \right) \right) \\
 \geq \varphi \left(\liminf_{n \rightarrow \infty} \mathcal{M} \left(Ay_n, Bx_n, \frac{2t}{k} \right), \liminf_{n \rightarrow \infty} \mathcal{M} \left(Ay_n, Bx_n, \frac{2t}{k} \right), \liminf_{n \rightarrow \infty} \mathcal{M} \left(Ay_n, Tx_n, \frac{2t}{k} \right) \right) \\
 &= \varphi \left\{ \mathcal{M} \left(\liminf_{n \rightarrow \infty} Ay_n, z, \frac{2t}{k} \right), \mathcal{M} \left(\liminf_{n \rightarrow \infty} Ay_n, z, \frac{2t}{k} \right), \mathcal{M} \left(\liminf_{n \rightarrow \infty} Ay_n, z, \frac{2t}{k} \right) \right\} \\
 &\geq \mathcal{M} \left(\liminf_{n \rightarrow \infty} Ay_n, z, \frac{2t}{k} \right) \\
 &\quad \vdots \\
 &\geq \mathcal{M} \left(\liminf_{n \rightarrow \infty} Ay_n, z, \left(\frac{2}{k} \right)^n t \right) \rightarrow 1.
 \end{aligned}$$

Similarly, $\limsup_{n \rightarrow \infty} \mathcal{M}(Ay_n, z, t) = \mathcal{M} \left(\limsup_{n \rightarrow \infty} Ay_n, z, t \right) = 1$, hence, $\lim_{n \rightarrow \infty} \mathcal{M}(Ay_n, z, t) = 1$.
 Assume that $S(X)$ is a closed subset of X . Then, There exists $u \in X$ s.t $Su = z$ using (2.2), we get

$$\begin{aligned}
 \mathcal{M}(Au, Bx_n, t) &\geq \varphi \left(\mathcal{M} \left(Su, Tx_n, \frac{2t}{k} \right), \mathcal{M} \left(Au, Su, \frac{2t}{k} \right) \oplus \mathcal{M} \left(Bx_n, Tx_n, \frac{2t}{k} \right), \right. \\
 &\quad \left. \mathcal{M} \left(Au, Tx_n, \frac{4t}{k} \right) \oplus \mathcal{M} \left(Su, Bx_n, \frac{4t}{k} \right) \right) \\
 &= \varphi \left(\mathcal{M} \left(z, Tx_n, \frac{2t}{k} \right), \mathcal{M} \left(Au, z, \frac{2t}{k} \right) \oplus \mathcal{M} \left(Bx_n, Tx_n, \frac{2t}{k} \right), \right. \\
 &\quad \left. \mathcal{M} \left(Au, Tx_n, \frac{4t}{k} \right) \oplus \mathcal{M} \left(z, Bx_n, \frac{4t}{k} \right) \right) \rightarrow (2.4).
 \end{aligned}$$

In addition, it is easy to verify that

$$\liminf_{n \rightarrow \infty} \mathcal{M} \left(Au, Su, \frac{2t}{k} \right) \oplus \mathcal{M} \left(Bx_n, Tx_n, \frac{2t}{k} \right) \geq \mathcal{M} \left(Au, Su, \frac{2t}{k} \right) \rightarrow (2.5).$$

In fact, for all $\epsilon \in \left(0, \frac{2t}{k} \right)$, we have

$$\mathcal{M} \left(Au, Su, \frac{2t}{k} \right) \oplus \mathcal{M} \left(Bx_n, Tx_n, \frac{2t}{k} \right) \geq \min \left\{ \mathcal{M} \left(Au, Su, \frac{2t}{k} - \epsilon \right), \mathcal{M} \left(Bx_n, Tx_n, \epsilon \right) \right\}$$

Since $\lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} Tx_n = Su$, the above inequality implies that,

$$\liminf_{n \rightarrow \infty} \left(\mathcal{M} \left(Au, Su, \frac{2t}{k} \right) \oplus \mathcal{M} \left(Bx_n, Tx_n, \frac{2t}{k} \right) \geq \mathcal{M} \left(Au, Su, \frac{2t}{k} - \epsilon \right) \right)$$

Letting $\epsilon \rightarrow 0$, in the above inequality, we get (2.5). Also by remark (2.2), we get

$$\begin{aligned}
 \liminf_{n \rightarrow \infty} \left(\mathcal{M} \left(Au, Tx_n, \frac{4t}{k} \right) \oplus \mathcal{M} \left(Su, Bx_n, \frac{4t}{k} \right) \right) &= \liminf_{n \rightarrow \infty} \mathcal{M} \left(Au, Tx_n, \frac{4t}{k} \right) \oplus 1 \\
 &\geq \liminf_{n \rightarrow \infty} \mathcal{M} \left(Au, Tx_n, \frac{2t}{k} \right) \\
 &= \mathcal{M} \left(Au, z, \frac{2t}{k} \right)
 \end{aligned}$$

So, letting $n \rightarrow \infty$, in inequality (2.4), we get

$$\begin{aligned}
 \mathcal{M}(Au, z, t) &\geq \varphi \left(1, \mathcal{M} \left(Au, z, \frac{2t}{k} \right), \mathcal{M} \left(Au, z, \frac{2t}{k} \right) \right) \\
 &\geq \varphi \left(\mathcal{M} \left(Au, z, \frac{2t}{k} \right), \mathcal{M} \left(Au, z, \frac{2t}{k} \right), \mathcal{M} \left(Au, z, \frac{2t}{k} \right) \right) \\
 &\geq \mathcal{M} \left(Au, z, \frac{2t}{k} \right) \\
 &\quad \vdots \\
 &\geq \mathcal{M} \left(Au, z, \left(\frac{2}{k} \right)^n t \right) \rightarrow 1.
 \end{aligned}$$

Hence, $\mathcal{M}(Au, z, t) = 1$.

That is, $Au = Su = z$. Since, $A(x) \subseteq T(x)$, There exists $v \in X$ s.t $z = Tv$. Using (2.2), and Remark (2.2), we have $\mathcal{M}(z, Bv, t) = \mathcal{M}(Au, Bv, t)$

$$\begin{aligned}
 &\geq \varphi \left(\mathcal{M} \left(Su, Tv, \frac{2t}{k} \right), \mathcal{M} \left(Au, Su, \frac{2t}{k} \right) \oplus \mathcal{M} \left(Bv, Tv, \frac{2t}{k} \right), \right. \\
 &\quad \left. \mathcal{M} \left(Au, Tv, \frac{4t}{k} \right) \oplus \mathcal{M} \left(Su, Bv, \frac{4t}{k} \right) \right) \\
 &= \varphi \left(1, 1 \oplus \mathcal{M} \left(Bv, z, \frac{2t}{k} \right), 1 \oplus \mathcal{M} \left(z, Bv, \frac{4t}{k} \right) \right) \\
 &\geq \varphi \left(\mathcal{M} \left(Bv, z, \frac{2t}{k} \right), \mathcal{M} \left(Bv, z, \frac{2t}{k} \right), \mathcal{M} \left(z, Bv, \frac{2t}{k} \right) \right)
 \end{aligned}$$

$$\begin{aligned} &\geq \mathcal{M}\left(Bv, z, \frac{2t}{k}\right) \\ &\vdots \\ &\geq \mathcal{M}\left(Bv, z, \left(\frac{2}{k}\right)^n t\right) \rightarrow 1. \end{aligned}$$

Hence $z = Bv = Tv$.

Since the pairs (A, S) and (B, T) are weakly compatible, we obtain $Az = Sz$ and $Bz = Tz$ using the inequality (2.2), we have

$$\begin{aligned} \mathcal{M}(Az, z, t) &= \mathcal{M}(Az, Bv, t) \\ &\geq \varphi\left(\mathcal{M}\left(Sz, Tv, \frac{2t}{k}\right), \mathcal{M}\left(Az, Sz, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bv, Tv, \frac{2t}{k}\right), \right. \\ &\quad \left. \mathcal{M}\left(Az, Tv, \frac{4t}{k}\right) \oplus \mathcal{M}\left(Sz, Bv, \frac{4t}{k}\right)\right) \\ &= \varphi\left(\mathcal{M}\left(Az, z, \frac{2t}{k}\right), 1 \oplus 1, \mathcal{M}\left(Az, z, \frac{4t}{k}\right) \oplus \mathcal{M}\left(Az, z, \frac{4t}{k}\right)\right) \\ &\geq \varphi\left(\mathcal{M}\left(Az, z, \frac{2t}{k}\right), \mathcal{M}\left(Az, z, \frac{2t}{k}\right), \mathcal{M}\left(Az, z, \frac{2t}{k}\right)\right) \\ &\geq \mathcal{M}\left(Az, z, \frac{2t}{k}\right) \\ &\vdots \\ &\geq \mathcal{M}\left(Az, z, \left(\frac{2}{k}\right)^n t\right) \rightarrow 1. \end{aligned}$$

Then $Az = Sz = z$.

Similarly, we can prove that $z = Bz = Tz$. Therefore z is a common fixed point of A, B, S and T .

Now, we prove the **uniqueness**:

Let if possible $w \neq z$ be another common fixed point of A, B, S and T . Then by inequality (2.2), we have

$$\begin{aligned} \mathcal{M}(z, w, t) &= \mathcal{M}(Az, Bw, t) \\ &\geq \varphi\left(\mathcal{M}\left(Sz, Tw, \frac{2t}{k}\right), \mathcal{M}\left(Az, Sz, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bw, Tw, \frac{2t}{k}\right), \right. \\ &\quad \left. \mathcal{M}\left(Az, Tw, \frac{4t}{k}\right) \oplus \mathcal{M}\left(Sz, Bw, \frac{4t}{k}\right)\right) \\ &\geq \varphi\left(\mathcal{M}\left(z, w, \frac{2t}{k}\right), 1 \oplus 1, \mathcal{M}\left(z, w, \frac{4t}{k}\right) \oplus \mathcal{M}\left(z, w, \frac{4t}{k}\right)\right) \\ &\geq \varphi\left(\mathcal{M}\left(z, w, \frac{2t}{k}\right), \mathcal{M}\left(z, w, \frac{2t}{k}\right), \mathcal{M}\left(z, w, \frac{2t}{k}\right)\right) \\ &\geq \mathcal{M}\left(z, w, \frac{2t}{k}\right) \\ &\vdots \\ &\geq \mathcal{M}\left(z, w, \left(\frac{2}{k}\right)^n t\right) \rightarrow 1. \end{aligned}$$

which is a contradiction. Hence $w = z$ is a unique common fixed point of A, B, S and T .

Remark: 2.3:

Putting $B = A$, and $T = S$ in the above theorem, we get the following corollary 2.6.

Corollary 2.4:

Let A and S be mappings from a fuzzy metric space $(X, \mathcal{M}, *)$ into itself satisfying the following condition:

- (i) $A(x) \subseteq S(x)$
- (ii) $\mathcal{M}(Ax, Ay, t) \geq \varphi\left(\mathcal{M}\left(Sx, Sy, \frac{2t}{k}\right), \mathcal{M}\left(AX, Sx, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Ay, Sy, \frac{2t}{k}\right), \right. \\ \left. \mathcal{M}\left(Ax, Sy, \frac{4t}{k}\right) \oplus \mathcal{M}\left(Sx, Ay, \frac{4t}{k}\right)\right)$

for all $x, y \in X$, and $t > 0$, Where $0 \leq k < 2$. Suppose that the pair (A, S) satisfies the property $(E-A)$, (A, S) is weakly compatible and one of $A(x)$ and $S(x)$ is a complete subspace of X . Then A and S have a unique common fixed point in X .

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