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SPECIAL r-TM CONNECTION

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ABSTRACT -
In this paper we will discuss Typical four connection r- CΓ, r- HΓ, r- RΓ, r- BΓ are special r-TM (r-M(0)) Connection.
Keywords: *Berworld's Connection, Cartan's Connection and Rund's Connection.*

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I. INTRODUCTION :-

The theories of connections on Finsler Space have been studied by many authors from their own Stand point. A well known connection are Berworld's Connection, Cartan's Connection and Rund's Connection. In all these connections the deflexion tensor and torsion tensor vanishes H. Yalsuda [4], [5], [6], has considered connection on Finsler Space with given deflexion and torsion tensor field. Prasad et at [1], [2] [3] have introduced a Finsler Connection with respect to which metric tensor is h-recurrent or v-recurrent. In this paper the connections will be discuss for which following condition are satisfied.

(i) The connection is h and v-recurrent.
(ii) Their deflexion tensors do not vanish.
(iii) The torsion tensor field do not vanish
(iv) They are closely similar to the connection introduced in [1]

II. MATSUMOTO'S CONNECTION :-

Let M be n dimensional Finsler Space with fundamental function L (x,y). We shall consider one of the most general connection on M. This connection may represented by

(2.1) $\Gamma = (\Gamma^i_{jk}, \Gamma^i_k,\ \tilde{C}^i_{jk})$ where $\Gamma^i_{jk}, \Gamma^i_k,\ \tilde{C}^i_{jk}$ are positively homogenous of degree 0, 1 – 1 in y^i respectively and are called h- connection, non linear connection and v-connection of Γ respectively. The *hv* torsion tensor.

(2.2) $\tilde{P}^i_{kj} = \ \Gamma^i_{k||j} - \Gamma^i_{jk}$ where symbol $||$ denote partial differentiation by y^j. We put $Q^i_{jk} = -P^i_{kj}$ then h-connection is expressible in.

(2.3) $\Gamma^i_{jk} = \Gamma^i_{k||j} + Q^i_{jk}$ where Q^i_{jk} is (o)-P homogerious tensor. If we denote non linear connection of Cartan's (or Barwald's) by G^i_k than non linear connection Γ^i_k of Γ is expressible as -

(2.4) $\Gamma^i_k = G^i_k + T^i_k$ for T^i_k is (1) p homogeneous tensor.

Now applying (2.4) after Partial differentiation with respect to y^j on (2.3) we get,

(2.5) $\Gamma^i_{jk} = G^i_{jk} + T^i_{jk} + Q^i_{jk}$ where $T^i_{jk} = T^i_{k||j}$

let Three tensors $T^i_k, Q^i_{jk}, \tilde{C}^i_{jk}$ are given as follows :-

(2.6) (a) T^i_k is (1) – P homogenous (1, 1) tensor.
(b) Q^i_{jk} is (0) P homogenous (1, 2) tensor.
(c) $\tilde{C}^i_{jk}$ is a (-1) P homogenous (1, 2) tensor.

Then a connection Γ on M is uniquely determined by (2.1), (2.4), (2.5) First we shall give important axiom concerning connection M Γ in Finsler geometry.

$F_1 -\ M\Gamma$ is L recurrent w.r.t. recurrence vector K_K i.e.

(2.7) $L_{|k} = K_K$L

$F_2 -$ The defluxion tensor

(2.8) $D^i_k = y^j\ \Gamma^i_{jk}$ - Γ^i_k

$F_3 -$ MΓ is v-recurrent with respect to recurrence vector b_k and v-symmetric i.e. , (2.9) $g_{ij|k} = bkg_{ij}$.

(2.10) $C^i_{jk} = \ C^i_{kj}$.

$F_3' -$ The V-connection of $M\Gamma$ vanishes i.e. $\tilde{C}^i_{jk} = 0$.

$F_4 -$ With respect to Γ the absolute differentiation Dy_i of y_i $(g_{ij}\ y^i)$ is given by $Dy^j = g_{ij}Dy^j$.

F_5 − Paths w.r.t. to Γ are always geodesics of M.
F_6 − MΓ is h recurrent with respect to recurrence vector a_k
(2.11) $g_{ij}|_k = a_k\, g_{ij}$

F_7 − Γ is h symmetric that is h torsion tensor
$\bar{\Gamma}^i_{jk}$ (= Γ^i_{jk} - Γ^i_{kj}) vanishes
F_8 The hv-torsion tensor P^i_{kj} (= − Q^i_{jk}) of Γ vanishes

III. SPECIAL r-TM CONNECTION :-

Typical four connection r-C Γ, r-H Γ, r-R Γ, r-B Γ are special r-TM (r-TM (0)) connection. First we will discuss r-C Γ connection it is given by axiom F_2, F_3, F_6, F_7 from F_3 we get equation.

(3.1) $\bar{C}^k_{ij} = C^k_{ij} - \frac{1}{2}(b_j\, \delta^k_i + b_k\, \delta^i_j - g_{ij}\, b^k)$.

From axiom F_6 we get equation.

(3.2) $a_k g_{ij} + 2P_{ijk} + \frac{\partial g_{ij}}{\partial y^h} T^h_k + T_{ijk} + T_{jik} + Q_{ijk} + Q_{jik} = 0$.

Applying Christoffel process to (3.2) we get,

(3.3) $\frac{1}{2}(a_k \delta^i_j + a_j \delta^i_k - a^i g_{jk}) + (C^i_{kr}\, T^r_j + C^i_{jr} T^r_k + g^{im} C_{jkr}\, T^r_m)$
$+ P^i_{jk} + T^i_{kj} + Q^i_{kj} = 0$

Now contracting (3.3) by y^k we get

(3.4) $T^i_j = \frac{1}{2}(a^i y_j - a_j y^i - a_o \delta^i_j - L^2\, C^i_{jr}\, a^r)$

From equation (3.3) and (3.4) we get

(3.5) $Q^i_{kj} = -T^i_{kj} - P^i_{kj} - (C^i_{kr} T^r_j + C^i_{jr} T^r_k - g^{im} C_{jkr} T^r_m)$
$- \frac{1}{2}(a_k \delta^i_j + a_j \delta^i_k - a^i g_{jk})$

So r-C Γ is determined by equation (3.1), (3.4) and (3.5)
Next we will discuss r-R Γ whose characterizing axiom are $F_2, F'_3\, F_6 F_7$ by F'_3 , we get
(3.6) $\tilde{C}^i_{jk} = 0$
by axiom F_6 equation (3.4), (3.5) so r-R Γ connection is determined by equation (3.6), (3.4), (3.5). Now we will discuss r-H Γ whose characterizing axiom are F_2, F_3, F_7, F_8 and F_1. From axiom F_8 we get.
(3.6) $Q^i_{jk} = 0$ by axiom F_1 , we get
(3.7) $\partial_i F = (\Gamma^{\,o}_{\,i} + K_i\, L^2)$ where $F = \frac{L^2}{2}$.
So non linear connection G^i_j of Berwald is given by $G^i_j = \partial_i\, G^i$
(3.8) $G^i = \frac{1}{2} g^{ij}\, [\, y^r\, \partial_j\, \partial_r F - \partial_j F\,]$.
Putting (4.6) in (4.7) and using axiom F_2, F_8 we get
(3.9) $G^i = \frac{1}{2}\Gamma^{\,o}_{\,i} + \frac{L^2}{2} g^{ij}\, K_o||_j - L^2\, K^i + K_o y^i$
Differentiating (3.9) w.r.t. y^i and using axiom F_2 we get

$\Gamma^{\,i}_{\,j} = G^i_j + (L^2\, K^i + K_o y^i)||_j - \frac{1}{2}(L^2\, g^{ir}\, K_o||_r)||_j$

So from (2.4) it follows that
(3.10) $T^i_j = (L^2\, K^i + K_o y^i)||_j - \frac{1}{2}(L^2\, g^{ir}\, K_o||_r)||_j$.
So r-H Γ is determined by eq. (3.1), (3.6), (3.10) Now we consider r-B Γ whose characterizing axioms are F_1, F_2, F'_3, F_7, F_8. From axiom F'_3 and F_8 follows that
$C^i_{jk} = 0$, $Q^i_{jk} = 0$ and from axiom F_1, F_2, F_8 the tensor T^i_j is given by eq. (3.10) so r-B Γ is determined.
If M is a Riemannian Space then torsion tensor C_{ijk} vanishes. In this case fundamental function L (x, y) is given by.
L (x, y) = $(g_{ij}(x)\, y^i\, y^j)^{\frac{1}{2}}$ and Riemannian Connection is given by.
RN Γ = [$\left\{ {i \atop j\ k} \right\}$, $y^j \left\{ {i \atop j\ k} \right\}$, 0] Where $\left\{ {i \atop j\ k} \right\}$ is christoffel symbol fermed with $g_{ij}(x)$. Now we consider ar-RN Γ whose characterizing axioms are F_2, F'_3, F_6, F_7
and
F_0 : Fundamental tensor g_{ij} is independent of y^i
$C_{ijk} = 0$ from axioms F'_3, F_6 we get

(3.11) $\frac{\partial g_{ij}}{\partial x^k} - g_{hj}\, \Gamma^{\,h}_{\,ik} - g_{ih}\, \Gamma^{\,h}_{\,jk} = a_k\, g_{ij}$

Apply Christoffel process to his equation and using axiom F_7 we get

(3.12) $\Gamma^{\,h}_{ij} = \left\{ \begin{matrix} h \\ i \quad j \end{matrix} \right\} - \frac{1}{2}\,(a_i\,\delta^h_j + \,a_j\,\delta^h_i - a^h\,g_{i\,j})$ From axiom F_2 it follows that $\Gamma^{\,i}_{\,k} = \Gamma^{\,i}_{jk}\,y^i$ so r-RN Γ is using determined.

IV. CONCLUSION :-

In this paper we have obtained Typical r-C Γ, r-H Γ, r-R Γ, r-B Γ are special r-TM (r-M (o)) connection r-C Γ is given by (3.1), (3.4), (3.5), r-R Γ connection is given by (3.6), (3.4), (3.5) r-H Γ connection is given by (3.1), (3.6), (310), r-B Γ is given by (3.10) r-RN Γ is given by (3.12), For, (3.11).

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