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Prime Ring With d^{3n+1} Contained In The Nucleus.

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ABSTRACT: In this paper we show that if R is a non associative ring with a derivation d then $d^{3n+1}(R) \subseteq N$ and $Rd^{3n+1}(R) \subseteq N$, using this it is show that if R is a non associative prime ring such that $d^{n}(R) \subseteq N$. Where n is a fixed positive integer then either R is associative or thederivatives which are in arithmetic progression becomes zero i.e d^{3n+1}=0.
KEYWORDS: Nonassociative,ring, prime ring,centre, nucleus, derivation.

I. INTRODUCTION

R is called a prime ring if the product of any two nonzero ideals of R is non zero with an additive mapping d in R is called a derivation if $d(xy) = d(x)y + xd(y)$ for all x, y in R .Suh [1] prove that if R is a prime ring with a derivation such that $d(R) \subseteq N$ then R is associative or $d^{3} = 0$. Yen [2] generalized this result for non associative rings .In this paper we prove that $d^{3n+1}(R) \subseteq N$ and R $d^{3n+1}(R) \subseteq N$ we use this to show that if R is a prime ring with a derivation d such that $d^{n}(R) \subseteq N$ where n is fixed positive integer. Then either R is associative or $d^{3n+1} = 0$.

II. PRELIMINARIES:

Let R be a non associative ring we shall denote the associator by $(p.q.r) = (pq)r - p(qr)$ for all (p,q,r) in R and commutator $(pq) = pq - qp$ where p,q in R. The nuclei are a collection of sub rings followed by

Left nucleus $N_{\alpha} = \{\alpha \in R / (\alpha, \beta, \beta) = 0\}$

Middle nucleus $N_{\beta} = \{\alpha \in R / (\beta, \alpha, \beta) = 0\}$

Right nucleus $N_{\gamma} = \{\alpha \in R / (\beta, \beta, \alpha) = 0\}$

Then the nucleus N is defined as

Nucles $N = \{\alpha \in R / (\alpha, \beta, \beta) = 0 = (\beta, \alpha, \beta) = (\beta, \beta, \alpha)\}$ i.e $N = N_{\alpha} \cap N_{\beta} \cap N_{\gamma}$.

The commutative centre C is defined as $C = \{C \in R / (C, R) = 0\}$

An additive mapping d on R is called a derivation or product rule if

$d(xy) = d(x)y + xd(y)$ for all x, y in R.

If the characteristic is not two, the linear zed relation implies the flexible property valid in any ring known as Teichmiller identity

$$(\omega p, q, r) - (\omega, pq, r) + (\omega, p, qr) = \omega\,(p,q,r) + (\omega, p, q)r \text{ ---------------- } (1)$$

for all ω p,q,r in R.

Put $\omega = n \in N_{\alpha}$,

$$(np,q,r) - (n,pq,r) + (n,p,qr) = n(p,q,r) + (n,p,q)r$$
$$\Rightarrow (np,q,r) = n(p,q,r) \text{ ------------------ } (2)$$

with $r = n \in N_{\gamma}$ in (1)

$$(\omega p, q, n) - (\omega, pq, n) + (\omega, p, qn) = \omega(p, q, n) + (\omega, p, q)n$$

$$(\omega, p, qn) = (\omega, p, q)n \qquad \text{--(3)}$$

As a consequence of (1) (2) and (3) we have that N is an associative sub ring of R

Through this section we assume that R is a prime ring, d is a derivation of R and n is a fixed positive integer such that the following property

$$d^{n}(R) \subseteq N. \qquad \text{-----------------------(4)}$$

Note that $d^{(i)}(R) \subseteq d^{(n)}(R) \subset N$ for all integers $i \geq n$

III. MAIN RESULTS

Lemma: $d^{3n+1}(R)R \subseteq N$ and $Rd^{3n+1}(R) \subseteq N,$

Proof : The method is implicit in (2) using (4) and by induction on n we obtain Leibnitz' s theorem. This gives the n^{th} derivative of a product function as a series of terms

$$D^{n}(uv) = (D^{n}u)v + n(D^{n-1}u)(Dv) + \frac{n(n-1)}{1.2}(D^{n-2}u)(D^{2}v) + --- + n(Du)(D^{n-1}v) + u(D^{n}v)$$

Where $D^{m} f$ means that the function f has to be differentiated m times, this can be rewritten as its symmetric for

$$D^{n}(UV) = \sum_{i=0}^{n} n_{c} m(D^{n-m}u)(D^{m}v) --------------(5)$$

Replacing u by $D^{n+1}(u)$ and v by $D^{n}(v)$ in (5) respectively, we get $D^{-2n+1}(u)D^{n}(v) \in N$ for $u, v \in R$. We know that C is an associative sub ring of N and N is an associative sub ring of $R.$

Again replacing u by $D^{n+2}(u)$, v by $D^{n-1}(v)$ D n-2 (v) in (5) respectively, we have $D^{2n+2}(u)D^{n-1}(v) \in N$ for $u, v \in R.$

Continuing in this manner we finally obtain $D^{2n+i+1}(u)D^{n-i}(v) \in N$ for $u, v \in R.$ ------------ (6).

And all $i \in \{1,2,3,---n\}$ In particular for $i = n$ $D^{3n+1}(R)R \subseteq N$ Remark here that $D^{n}(u) = u$ for all $u \in R$ similarly $RD^{3n+1}(R) \subset N$ -- (7)

Since the associator and commutator ideal I of R is the smallest ideal which contains all associators and commutators in R.

The associator ideal is zero if and only if $R.$ is associative similarly the commutator ideal is zero if and only if R is commutative if considere the commutative case also as in (1) I can be characterized as all the finite sums of right or left multiples of associators hence

$$I = ((R, R, R) + (R, R, R)R = (R, R, R) + R(R, R, R) -------------(8)$$

By using (4) (2) and (6) we get $d^{3n+1}(R)(R, R, R) = 0$ and so $d^{3n+1}(R)((R, R, R)R) = 0$

Applying (8) these two equalities imply $d^{3n+1}(R).I = 0$ ------------------------------(9)

Similarly by (7) $I.d^{3n+1}(R) = 0$.

Lemma -2: The ideal F of R generated by $d^{3n+1}(R)$ is F = $d^{3n+1}(R) + d^{3n+1}(R)R + Rd^{3n+1}(R) + R.d^{3n+1}(R)R.$

Proof: obviously, F is an additive sub group of $(R,+)$

By (4), $d^{3n+1}(R)$ since $3n+1 \geq n$

Also by lemma (1) $d^{3n+1}(R)R + Rd^{3n+1}(R) \subseteq N$

Thus by (5) and lemma (1) F is an ideal of R

Theorem : If R is a prime ring with a derivation d such that $d^{n}(R) \subseteq N$ where n is a fixed positive integer ,then either R is associative or $d^{3n+1} = 0$

Proof: since $3n+1 \geq n$

We have $d^{3n+1}(R) \subseteq N$ by lemma (1) and (9) we obtain $d^{3n+1}(R) + d^{3n+1}(R)R + Rd^{3n+1}(R) + R.d^{3n+1}(R)R.$

Hence $F.I = 0$

by the primness of R either $I = 0$ or $F = 0$

Thus either R is associative or $d^{3n+1} = 0$

Thus the derivation which are in arithmetic progression contained in the nucleus.

This completes the proof.

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